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OF

THERMODYNAMICS,

WITH

SPECIAL APPLICATIONS TO HOT-AIR, GAS AND STEAM ENGINES.

BY

ROBERT RÖNTGEN.

TEACHER IN THE POLYTECHNIC SCHOOL AT REMSCHEID.

TRANSLATED, REVISED, AND ENLARGED

BY

A. JAY DU BOIS, PH.D.,

PROFESSOR OF DYNAMIC ENGINEERING IN THE SHEFFIELD SCIENTIFIC SCHOOL OF YALE COLLEGE.

IN TWO PARTS.

PART I. GENERAL PRINCIPLES—HOT-AIR AND GAS ENGINES.
PART II. HEAT, STEAM AND STEAM ENGINES.

WITH 103 WOOD-CUTS IN THE TEXT.

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PREFACE.

In the presentation of the present work to the Engineering Profession and to our technical Schools, a few words of introduction seem necessary.

There are several excellent works upon the subject of Thermodynamics, in English, but none with which the writer is acquainted, sufficiently wide in its scope and practical in its applications, and at the same time adapted in its mode of treatment to the needs of *beginners*. The subject is thus one of the most difficult for the student to get hold of in the scheme of our engineering schools, and the effort to teach it, so far as the writer's experience goes, is seldom productive of satisfactory results.

It is to meet this want that the present work is offered to the public. The writer has used the work of RÖNTGEN in his classes for several years, and with good success. The treatment is full and practical and the presentation such as to offer but little difficulty to an earnest reader. The notation employed is that used by ZEUNER, so that the book forms a good introduction to the "*Wärme-theorie.*"

During these years the work of RÖNTGEN has grown upon the writer's hands into its present proportions, and it becomes proper here to point out at least those portions for which the German author is *not* responsible. In general the work of Röntgen is comprised by the large print only, while all the rest is from other sources.

Of these, apart from the writer's own additions, the most noteworthy are the two lectures by PROF. VERDET, which have been introduced as an introduction to the work. They form, with the *Notes and Additions*, an admirable summary of the whole field, and being in a popular form, will, it is hoped, awaken an interest in one of the most important of the more recent developments of physical science.

In Chapter XIII. we have given a very excellent abstract of MONS. PERNOLET'S work "*L'Air Comprimé*," for which we are indebted to Mr. BAILEY WILLIS, M. E. It will be found of great interest, and the diagram given at the end of the chapter will be found of great practical value.

In the Appendix to Chapter XXIII., we have given ZEUNER'S *theory of superheated steam*—by far the best and latest work upon this very important subject.

As to other additions, we have added here and there to the text of our author, matter which seemed desirable, distinguishing all such additions by fine print and brackets ; have appended "questions for examination" to many chapters, as well as added many selected "examples for practice," reduction tables, etc., all of which are calculated to aid the teacher and student. The steam tables at the end of the work are taken from ZEUNER'S "*Wärme-theorie.*"

As regards the extent of the work, it will be found considerably more than can be read completely by any class. This need cause no trouble to the instructor. The principles are completely laid down in the first six chapters of the first and second parts. The rest, consisting merely of the applications of these principles, can be pursued at such length as may seem proper in any case. We consider it a positive advantage to the student, who is expected to make use of the principles he acquires, to have a text book so full and comprehensive that it shall serve as a book of reference as well, and point out the method to be pursued in the investigation of any problems which may occur in future practice.

For several reasons it has not been thought well to convert the French measures into English. Those who wish to become familiar with the literature of the subject, *must* be able to use the French system easily. No graduate of our Technical Schools should be without a thorough knowledge of it. For the practical use of the formulæ and results in daily work, the reduction tables we have given will be found to answer every requirement.

The method of the author requires only a knowledge of algebra and no use is made of the calculus. This fact will perhaps gain for the work readers who have long desired to obtain some insight into the subject, but have been unable to read the works hitherto published upon it.

The effort throughout has been to aid both teacher and student in their work, and to impart such a knowledge of the subject as shall render it practically serviceable.

Sheffiel Scientific School of Yale College,
June 11th, 1880.

PREFACE TO SECOND EDITION.

VERY considerable additions have been made to the present edition. The application of the Calculus to the subject has been given with sufficient fullness, in the shape of additions to Chapter VII., page 208, and Chapter XVIII., page 462. All examples and formulæ are given throughout the work in both French and English units, and the Steam Tables of Zeuner are given complete, both in their original French units, and also reduced throughout to English units. Many new examples have been added.

It is hoped that these changes will better adapt the work to the needs of teachers and pupils, as well as render it more valuable as a work of reference for the engineer.

The student will thus find two Courses presented, one with and the other without the aid of the Calculus. The first six chapters of Part I. and Part II. form by themselves a short elementary and practical Course.

The writer has found it decidedly of advantage that students should thus first become familiar with the main features of the subject. Upon review, the remaining portions can then be taken with profit, and the application of the Calculus will be better understood.

It is believed that such a Course will open up the subject, so that the student can pursue it readily in higher works, while the numerous examples and complete Tables will make the book a valuable aid in practice.

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LECTURE I.

I.

Thermodynamics, or the Mechanical Theory of Heat, is that science which treats of the mechanical effects of heat, and of those mechanical processes by which heat is generated. This science is yet in its infancy. It is not more than 40 years ago[*] that Sadi Carnot pointed out its first problems, and scarcely 30 years have elapsed[*] since Julius Robert Mayer indicated the methods by which their solution might be attempted.

Nevertheless, this science has already reached a great development, and has attained points of contact with almost all the other sciences. We shall endeavor, in what follows, to obtain a comprehensive view of this rapid progress.

The new science rests upon a few fundamental principles of mechanics, and to these let us first, for a moment, direct our attention.

The law, according to which the velocity of any material point acted upon by a constant force changes, is well known (it is $v=\sqrt{2gh}$). We also know that the square of the velocity attained in any given time is equal to twice the product of the moving force and the distance passed over, divided by the mass of the moving point $\left(\text{or } v^2 = \frac{2Fh}{M}\right)$.

The velocity increases or is accelerated when the moving force acts in the direction of the original velocity of the point, and it is retarded when the moving force acts in the opposite direction.

The product of the force into the distance passed over, we call the "MECHANICAL EFFECT" or "WORK" of the force. We call this work positive or negative, according as the force causes motion or opposes motion of the point, *i.e.*, according as it acts in the direction of the initial velocity or the reverse.

We call the half product of the mass and square of the velocity ($\frac{1}{2} Mv^2$) the "VIS VIVA" or "LIVING FORCE," and by the aid of these two definitions we may express the foregoing principle in the following manner:

When a body moves with uniformly accelerated or retarded motion, the

---

[*] The two introductory lectures which follow were delivered by Prof. Verdet before the Chemical Society of Paris, in the year 1862. The dates above should therefore now be 58 and 48 respectively. As a popular and yet scientific exposition of the subject, these lectures are still unrivaled, and to the beginner who desires to get clear general ideas of the scope and spirit of the science they will, it is thought, prove both interesting and valuable, and render the proper comprehension of the technical discussion which follows much easier.

"work" during any given time, performed upon the body or performed by the body, is equal to the change in the "living force"—[*i.e.*, $Work = \frac{1}{2} M(V_2^2 - v_1^2)$].

This principle, which follows directly from the above definitions and principles, enables us to measure forces by the resulting velocities, and may be easily generalized.

Thus, by the aid of the Calculus, we may remove the limitation as to constant force, which we have introduced for the sake of clearness. Then the limitation as to direction may be removed by finding the component of the inclined force in the direction of the motion, and taking its work. Finally, we may consider any system whatever of forces and bodies, and show that in all cases *the sum of the works performed in any given time is equal to the change in the sum of the living forces in that time.* This is the principle known as the equality of work and living force, upon which rests the entire theory of machines.

Mechanical effect, or work, is expressed numerically by means of a conventional unit. Thus that amount of work may be taken as unity which is performed in lifting one unit of weight, as one pound or one kilogram, against the force of gravity, through the vertical distance of one unit of length, as one foot or one meter. Work is thus measured in "foot-pounds" or "meter-kilograms." If, thus, we say that the work of any system is *positive* and equal to 100 ft. lbs., we mean that by means of this system we can perform the same work as would be performed by a weight of 100 lbs. descending through a height of one foot, or, regarding the force of gravity as constant, by the descent of one pound through 100 feet. In like manner, 100 meter-kilograms signifies the work performed by the descent of one kilogram through the distance of 100 meters.

Inversely, a *negative* work of 100 ft. lbs. denotes an expenditure of work by the system equal to that expended in raising a weight of one pound through 100 feet, or 100 lbs. through one foot, the final velocity being zero.

It is not our purpose to indicate here how the entire theory of machines follows from this equation of work; but it is necessary to direct attention to two general conditions which the motion of any machine must satisfy, and which are expressed in this equation.

First, in every machine whose motion has become constant, or, in general, in any system whose velocity is independent of the time, the sum of the living forces is constant, and hence in any period which we consider, the sum of the works zero. In other words, the work of the moving forces is constantly equal to the work of the resistances, and has a contrary sign. If the velocities indeed are not constant, but periodic in their variation, as, for example, is the case in a machine with a reciprocating motion, then, although equality no longer exists between the work of the moving forces and of the resistances for any arbitrary interval of time, still it does exist for the duration of a full period, or for any entire number of such periods.

If, further, the forces which act upon a system have at one time an effect opposed to the action of the individual points of this system upon each other, if therefore they act in the directions of the lines joining these points, and are dependent only upon the distances apart of the points, and if at another time they proceed from a center, and are subject to the same conditions; then, the sum of the living forces is the same both at the beginning and end of a time such that the bodies of the system return to their first positions. The sum of the works of the forces during this time is zero. These conditions are satisfied by every case which occurs in nature.

This law, which rests upon the surest ideas which we have concerning the operation of the forces of nature, is nothing more than the principle of THE IMPOSSIBILITY OF PERPETUAL MOTION.

According to this it is impossible by any combination of natural forces to make a machine whose parts being once set in motion and then left to the operation of gravity, or other similar forces, and their own mutual action, shall later return to their original positions with greater velocities than they at first possessed.

A perpetual motor means, then, a machine which, being put in motion and left to itself will, in a certain time, regain its original velocity, and at the same time impart to some body, originally at rest, a certain velocity. It is clear that both cases of impossibility are identical.*

It does not appear at sight easy, proceeding from these principles, to make any new discoveries. The theory of simple machines is firmly founded, and all analyses of deceptive discoveries of a perpetual motion are to-day completely devoid of interest. Nevertheless, it is from a new application of these apparently thoroughly explored principles that the entire mechanical heat theory has arisen.

It will, for our purposes, be sufficient to give heed to these two rules :

First, always to recognise, together with the outward and visible motions of any machine, those less perceptible interior motions of the atoms of bodies which escape observation by our senses.

Second, whenever, following customary theories, we meet with a force whose mode of action does not agree with the general laws of action of natural forces, we must regard this force as a mathematical fiction, and seek to establish its true nature.

Without these two maxims every theory of machines must lead astray; every machine in motion must appear as a direct contradiction of the law of equality of the work of the motive power and the work of the resistance, or as a solution of the problem of perpetual motion. The only means of avoiding such contradiction would be to propound views as to the nature and mode of action of heat, whose scope would exceed that of the simple circle of phenomena which first suggested them.

II.

Next, we assert that in no machine which has attained a state of uniform motion can the work of the resistances be equal to the work of the moving forces. Although this assertion appears, in view of the above, paradoxical, yet it simply expresses what at bottom can be found in any text-book upon mechanics. It is nothing more than the true interpretation of the preponderance of the work of the moving forces compared with that which we call the "useful work."

Let us consider an hydraulic machine which is designed to raise water, *i.e.*, to produce an effect similar to that which it receives. This will simplify the comparison of the two kinds of work. In a machine of this kind a certain quantity of water enters in a given time, falls through a certain distance performing work, and, if the machine is perfect, departs with the velocity which it had before falling. The product of the weight of water and the height of fall is evidently the work of the moving force. In the same time, the machine takes a certain quantity of water from some reservoir, it may be from the very stream which furnishes the motive water itself, and raises it up to another and higher reservoir. This negative work, against gravity, is the product of the weight of water raised, into the difference of level between the two reservoirs, and is what we call the "*useful work.*"

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* See Note 1, at the end of these lectures.

Now, every one knows that this useful work is only a fraction of the work of the moving force. This fact is ordinarily accounted for by the consideration of what we call the "passive resistance," that is, by the assumption of forces which oppose the motion of the machine, and thus perform a negative work exactly equal to the excess of the work of the moving force above the useful work. Let us see what value this explanation has.

There is one part of the passive resistance of somewhat indefinite amount. To this belong every contribution of velocity to surrounding bodies, either to the air or to the foundations, which theory, of course, assumes as firm. All these constitute a useless development of living force at the expense of, and equivalent to, a certain fraction of the work of the moving force. In by far the greater number of cases, however, these constitute the least portion of the work of the passive resistances. Much the greatest portion must nearly always be attributed to the action of a certain special force which bears the name of "friction," and to this force we now wish to call special attention.

What, then, is friction? It is purely a resisting force, incapable of causing motion in the machine, or of increasing its velocity. It is a force which always tends, when two surfaces in contact move with different velocities, to diminish the velocity of the fastest.

It is not an elementary work, but the result of actions which occur between the molecules of the rubbing surfaces. We know little or nothing of these actions, more than that they must obey the general laws which we have laid down just now, while speaking of perpetual motion. We do not need, however, to know anything whatever about them, in order to demonstrate that they can furnish no work, and hence can give no information as to the facts to be accounted for. In machines, ordinarily, rubbing surfaces are ground down, also the lubricating materials undergo a change. We might suppose that the work corresponding to such molecular changes was the exact equivalent of that portion of the excess of the work received over that performed, which we ascribe to friction. But it is easy to conceive of a machine whose rubbing surfaces are so smooth and of such hard material as not to rub down perceptibly in a long time; it would not indeed be difficult to practically realize such a machine. If we consider, in such a case, the work of molecular forces, which is the cause of the friction, during the period between two precisely identical positions of the machine, it is at once evident that this work must be zero, because at the beginning and end of the period the relative positions of the molecules is the same.

Where, then, does the ordinary explanation of the excess of the work received over the useful work lead us? Can we recognise in it anything else than a pure mathematical fiction, which may perhaps be useful as a temporary presentation of an unknown process, but which must be rejected by every one not prepared to deny the most certain conclusions of science? Must we not conclude, that in every case where we have friction without change of surface, there must be some unobserved change, which is, in fact, equivalent to work, and which seems to absorb the friction?

To the eye of the pure mechanic no such change may be apparent, the physicist, however, without doubt, will recall a well-known phenomenon, familiar even to ordinary experience, and which has already more than once been the subject of scientific investigation. I speak of the *increase of temperature* which always takes place when surfaces are rubbed, and which is more considerable the greater the friction; or, what amounts to the same thing, the greater the unexplained loss of work.

Without pausing to recapitulate the laws of this phenomenon, let us direct attention to its essential character. It is a heating which corresponds to no cooling of any other part of the machine. It is not another distribution of heat which already existed, but it is a generation, or, still better, an actual creation of heat. What is more natural than to recognize in this the equivalent of the excess of the work received over the useful work, which we are otherwise at a loss how to account for?

III.

In order to estimate the value of this supposition, let us consider a kind of action entirely different from that which takes place in machines—*viz.*, the phenomena of radiant heat. Let us recall the experiments of Delaroche, Bérard, Melloni, Knoblauch, Tyndall, and other physicists, upon what are called, both in popular and in scientific parlance, "heat rays."

These experiments are in complete accord with those by which the true nature of light is revealed, and as the view held to-day as to the nature of light is held by all, we are in like manner forced to conclude that heat rays are nothing else than a certain vibratory motion of the ether of space. Thus, in accordance with experiment, we say, that if a body is brought near to another of lower temperature, certain vibrations are generated, which follow certain laws. Upon these vibrations depend the phenomena of the imparting of heat, and under certain circumstances they are capable of acting upon our eyes so as to give rise to the phenomena of light also. We have no reason to suppose that the two kinds of phenomena have different causes.

This fundamental identity of radiant heat and light was demonstrated twenty years ago * by Melloni in his paper "Upon the Identity of Rays of all Kinds," read before the Academy at Naples, February 2, 1842. Still, Melloni recognized that an important step remained to be made to complete the demonstration. The *interference* of heat rays had not yet been experimentally shown; no one had yet succeeded by two rays of heat in producing cold, as by two rays of light, under proper conditions, darkness had been caused. Five years later, Fizeau and Foucault detailed, in a paper before the Academy, experiments by which the interference of heat was made as evident as that of light. (*Comptes rendus*, Vol. XXV., *and Poggend. Annalen, Bd.* 72.)

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* These lectures were delivered by Verdet in 1862.

After this, not a single doubt remained to oppose to a theory which recognized in the heat rays a system of vibrations. We shall consider it as an undoubted fact, that in a body which is brought to a higher temperature, vibrations of its molecules are caused; in other words, *the body contains a certain amount of living force.* While another body of higher temperature, which serves as a source of heat, causes this development of living force, it cools gradually itself.

Inversely, when the vibrations which constitute a system of heat rays meet a body which possesses the property, as we say, of "absorbing heat," and become diminished or disappear, the body becomes heated.

The cooling of a body by radiation, therefore, corresponds to the generation in other bodies of a certain amount of living force; the heating of a cold body by the absorption of radiant heat, on the other hand, corresponds to a diminution of living force in others. Heating and cooling, therefore, are phenomena of the same kind, whatever may be their cause. They must, in all cases, be considered as pure mechanical operations. Heating can only be the sum total of those changes which take place during the disappearance of a certain amount of living force; *i.e.*, either a performance of work or a development of living force, or a combination of both. It is evident that heating corresponds to mechanical work.

Heat acts to change the volume of bodies, the molecules are forced farther apart against their forces of attraction, and thus a negative work is performed. At the same time occurs that change of the properties of the body which we call rise of temperature, and it is easy to see in it the effect of the change in the sum of the living forces of the molecules.

It makes very little difference whether we accept or reject these last conclusions; it is none the less certain that the heating of a body represents a certain performance of work and the development of a certain amount of living force, or still better, *is* such force. The work in question consists of molecular disturbances which, indeed, escape observation, and are only visible, finally, in the change of form and dimensions of the body; the living force is also as difficult of direct observation, and consists neither of the motion of the body as a whole, nor of directly visible motions of its parts, such, for example, as constitute sound phenomena. It consists, in all probability, in vibrations of the smallest particles of matter, and eludes our senses. Considered from a mechanical standpoint, these speculations have no importance, and cannot prevent our recognizing in the heating of a body, mechanical work, just as plainly and as certainly as in the raising of a weight or the motion of a projectile.

IV.

Let us return now, in the light of these new principles, to the consideration of the machine which we have instanced, and those questions which then arose will now find immediate solution. The *heat* which is developed at those points where friction occurs is a mechanical phenomenon, a combination of mechanical work and living force in a relation which we shall determine more precisely hereafter. It is evident that this heat may be equivalent to the difference which exists between the work of the motive forces and the useful work, the explanation of which we have been seeking. I say *may be*, and you will, perhaps, be inclined to add, *must be*. The equation of work must necessarily be satisfied at every instant, only we must not limit it to those living forces or visible effects which are usually alone considered, but we must also include those living forces, or that work, which we know in the shape of heat. If we neglect this term of the equation of works, the fundamental theorem of applied mechanics may indeed appear incorrect, but by its introduction all difficulties disappear.

Having now arrived at this point, we can submit the correctness of our conclusions to the test of experiment. We may seek, for instance, whether it is really true that the heat generated by friction in a machine is exactly equivalent to the unexplained difference between the useful work and that received. Although, indeed, it may be impossible to measure this quantity of heat in the condition of living force or work, as we measure, for instance, the work of gravity upon a body of one pound weight which falls through one foot, still we may measure it *relatively* by comparison with another quantity of heat, which may be sharply defined and taken as unity.

The result of such a procedure will give us this quantity of heat *expressed in terms of these units*, and then if we know the ratio of this unit of heat to the unit of work, that is, the number of foot-pounds or meter-kilograms corresponding to each heat unit, we can easily find the work equivalent to the heat, which will be the difference between the useful work and the work originally imparted. This constant ratio will therefore determine the mechanical value of those heat effects which we assume as constituting one unit of heat.

This has been established by experiment. The physicist Joule, who has perhaps contributed more than any one else to the science of Thermodynamics, has investigated friction of various kinds in such a way as to determine the amount of heat developed in comparison to the work expended. He used a very simple mechanism, which, by means of a falling weight, set in motion a small paddle-wheel, which turned while immersed in water or mercury, the motion of the liquid being prevented by partitions. The friction of the liquid particles upon each other, upon the partitions and upon the paddles, generated a certain amount of heat, which could be estimated from the rise of temperature of the various parts of the apparatus. The work corresponding to this heat was given by the fall of the weight used, due regard being had for the friction of those parts of the machine out of the liquid. Thus was determined the ratio of the mechanical work expended to the heat produced. Experiments with water gave for every unit of heat, that is, for every kilogram of water raised one degree Centigrade, the equivalent work of 424 meter-kilograms. (If we take as the unit of heat, *one pound* of water raised one degree *Centigrade*, we have about 1,390 foot lbs. If we take one pound of water raised one degree *Fahrenheit*, we have about 772 foot lbs. All three equivalents are in use.)

Experiments with mercury gave 425 meter-kilograms, or very precisely the same as water. Joule made still a third determination, with an iron ring instead of a paddle-wheel, which ring he caused to rub upon an iron plate immersed in water, and found in this case 426 meter-kilograms.

You will doubtless be surprised at the close correspondence of these three numbers. When I add that each is the mean of a large number of determinations, it will be readily confessed, that in this work of Joule, classic even to-day, is found the experimental verification of our new principle. You will admit that the mean of the determinations for water, which are regarded as the most reliable, or 424, represents with tolerable exactness the quantity of work which is the equivalent of that living force among the particles of a body, to which we give the name of "one heat unit." Let us pause a moment to consider the significance of this number. It expresses, that from a mechanical standpoint, we produce two equivalent effects, whether we generate one unit of heat, or whether we raise 424 kilograms through one meter. In other words, in every application of the equation of works, in which we take account of both the living force of heat, and, at the same time, of the work of the visible forces, we must, for every unit of heat, add 424 units to the negative work or to the living forces. This relation is independent of the special method of the production of heat by friction. It follows from principles, whose generality has already been proved, that 424 can be *in every case* regarded as the MECHANICAL EQUIVALENT OF HEAT.

If it should possibly appear rather premature to consider this definite numerical value as absolutely correct, still no objections can be urged, and no doubt remain as to the entire correctness of the principle of the equivalence of work and heat; for, making allowance for errors unavoidable in all experiments, the most diverse determinations all agree in giving us the same value.

V.

We find the first confirmation of Joule's experiments in the researches of Favre upon the friction of steel on steel: but we shall, for a moment, pass over such confirmations, in order to direct attention to still another contradiction which seems to exist between the usual theory of machines and general mechanical laws, which is, in a certain sense, the opposite of the preceding, and which only disappears when we apply those principles which we have already deduced. It can be easily shown, that if we depart from these principles, every machine which is moved by heat can be regarded as a perpetual motion, which continually generates living force in surrounding bodies, without any change in its own, without, in fact, a positive work of the motive forces equivalent to the living forces generated.

Let us take as an example that most important and well-known machine of our civilization—the steam-engine. Consider, then, with me a machine which has arrived at the condition of its normal activity, and in order to fix our ideas, let us take a condensing engine. What takes place during one revolution or one double stroke of the piston? A certain quantity of water of low temperature is forced by the feed-pump into the boiler, it is there heated and converted into saturated steam of a temperature above 100° C. The water in this new condition enters the cylinder, raises the piston, expands in volume, and finally returns to the condenser, where it retakes its original condition, viz., water of low temperature. We have thus, at the end of this cycle of changes, everything in the same condition as at first. Not only are all parts of the machine in the same relative position, but the moving agent also has returned to precisely its original condition. (The amount of water which is injected into the condenser, in order to condense the steam, need cause no confusion; this water is simply a cooling agent, which might be replaced by any other without changing the operation of the machine. Thus, for example, we may have for the condenser a spiral pipe immersed in cold water, and which, therefore, contains only that amount of water which is used in one stroke of the piston.) In such a case, it is at once evident that at the beginning and end of each period of the machine, the conditions both of the fluid motor and of the mechanism are precisely the same, and we can at once conclude that the sum of the works during that period within the machine must be zero. This separation of the motive and cooling fluids is, in fact, actually accomplished in those engines worked by ether or chloroform steam; it would be in like manner allowable, in principle at least, in ordinary steam-engines. The motive work of the steam is computed, like the work of friction, by an empirical expression for an imperfectly known fact. In reality, the sum of the works of all the elementary forces, *i.e.*, the work of the mutual actions exerted by the molecules of the liquid, of the steam and of the parts of the machine, are equal to zero. And yet the machine is continually imparting living force to exterior bodies, raising weights, shaping metals, in short, performing work. Perpetual motion seems accomplished. The outer work of the machine does not seem to correspond either to an equivalent work within the machine, nor yet to a disappearance of living force.

Such, at least, is the state of things so long as we recognize in the steam-engine purely mechanical processes only; so long, at least, as we do not search for other living forces than those possessed by the visible portions of the machine. The difficulty, however, vanishes as soon as we take into account the living force of the heat. During the action of the machine, the steam generated takes away heat with it from the boiler, and gives up heat in the condenser, where it becomes water again. If these two quantities of heat were equal, the difficulty already noticed would still hold in full force. If, however, they are not equal; if the quantity of heat received by the condenser, or carried away by the cooling water, is less than that furnished by the boiler, then the difficulty is solved. The disappearance of a certain quantity of heat during the cycle of changes, corresponds, in fact, according to our new principle, to the disappearance of a certain amount of living force.

In the same time in which outer work is performed by the machine, or living force is developed, an equivalent quantity of living force disappears within the machine, and the general laws of mechanics hold good.

In order to confirm this conclusion experimental proof is necessary. We must measure, on the one hand, the work of the machine, and on the other, the loss of heat within the machine, and then, if our conclusions are correct, there must be found between the two a certain constant relation.

The necessity for the existence of such a constant relation will be evident, without repeating the considerations which in the case of friction led us to a similar conclusion.

For every unit of heat which disappears in the machine, it must furnish 424 units of outer work, or it must generate an equal quantity of living force.

The experiment is difficult, much more so than the experiments of Joule upon friction; but it has been performed, and without going into details, I will endeavor to point out the various operations which compose it. In an engine whose motion has become constant, the quantity of steam used for a certain number of strokes is measured; the physical condition of this steam as it enters the cylinder from the boiler is exactly determined by measuring its temperature and pressure; we also so arrange that it shall enter the cylinder without carrying with it, mechanically suspended, any appreciable amount of watery particles, and without being heated above its point of saturation. All these conditions being satisfied, we have, in connection with our knowledge of the total heat required for vaporization (made known by the experiments of Regnault), all the data which we need in order to calculate the amount of heat used in a given time in order to convert the water in the condenser into steam.

On the other hand, we can find, without great difficulty, the quantity of heat which in the same time is given up in the condenser. It is sufficient to determine the quantity of water used for cooling, which is necessary to preserve the temperature of the condenser constant in spite of the continual admission of steam, and also the temperature of the condenser and that of the reservoir from which the cooling water is taken.

Thus, in the first case, if the water to be converted into steam enters the boiler with the temperature of 0°, and if T is the temperature of the steam, we have, according to Regnault's experiments, for the formation of each unit in weight of steam,

$$606.5 + 0.305T^\circ$$

heat units. But the water comes from the condenser where the temperature is t°; the heat received, therefore, is less than that which is necessary to raise each unit in weight of water from 0° to T° by the amount which is necessary to raise it from 0° to t°, or by t heat units. This is, regarding the specific heat of the water as constant, tolerably exact for those limits of temperature which the condenser never exceeds.

In the second case, let t° be the temperature of the condenser, and θ the temperature and p the weight of the injection water in a given time. The heat given out by the concentration of the steam must be equal to that absorbed by the injection water. This heat is, then, that which is necessary to raise p units weight of water from θ° to t°, or it is equal to

$$p\,(t^\circ - \theta^\circ).$$

The calorimetric part of the experiment is completed by determining and adding the losses of heat due to conduction, radiation, and disturbances of the air. The most difficult part of the experiment is the mechanical. In determining the total work of the machine we cannot use the friction brake. If we do, we shall only determine the useful work, to which we should then have to add the work absorbed by the passive resistances, which latter are almost impossible of exact determination. We must therefore adopt an entirely different method. Thus, by means of the steam indicator, we can find the mean pressure of the steam upon the piston, and then, knowing the length of stroke, can determine accurately the total work performed.*

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* Thus we may conceive, instead of the actual pressure of the steam upon the piston, a sinking weight and pulley so arranged that the motion of the machine is unchanged. The work of the falling weight is that which we call the motive work of the machine, and is exactly that which we obtain by the measurement in the text. In reality there is a complete compensation between the positive and negative work of the forces in the machine. The rising of the piston is a constant action, and the mechanism requires for this action a force of definite intensity. We can, therefore, without contradiction, continue to speak of the work of the steam in the machine. We shall further see that, very probably, the steam raises the piston by imparting a portion of the living force of its molecules. See Note 11, at the end of these lectures.

The necessity of determining successive values of a pressure which varies rapidly, prevents the use of the ordinary manometric apparatus, by which the elastic force of steam may be measured with almost absolute exactness. We have to make use of the steam indicator. In spite of the inaccuracies of this instrument, which was constructed to meet practical needs rather than those of scientific investigation, the results obtained answer unmistakably the question at issue.*

This long and laborious work, the chief steps of which we have thus detailed, has been successfully performed by Hirn, who knew how to make use of a large factory for the solution of an abstract question of science. His measurements were made, not upon a miniature model in a scientific collection, nor in the laboratory; but upon engines of 100 and 200 horse-power, and in the very halls of industry. These circumstances had two especial advantages. On the one hand they met the objections of practical men, who are disposed to regard with distrust what they call "cabinet experiments;" on the other hand, and this is still more important, by reason of the large dimensions of apparatus and the long duration of the experiments, those thousand accidental disturbances which ever attend new discoveries could be eliminated.

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* The indicator consists of a small cylinder *C*, Fig. 1, within which moves the piston *K*, attached to the spring *f*. The space below the piston communicates with the cylinder of the engine. As the pressure increases or diminishes, the piston rises or falls. A pencil *G*, which partakes of the motion of the piston, describes a line upon a strip of paper wound around the drum *G*. This drum is made to revolve about its vertical axis by means of a cord and pulley. The cord leads to the cross-head or some reciprocating part connected with the piston. By means of a spring within the drum, it is made to revolve back during the return stroke. The pencil *G* thus traces a closed curve, the length of which represents the stroke, and the varying height of which represents the pressure at corresponding positions of the piston. If the piston were without friction, the area of this curve, measured from the straight line described when the pressure on both sides of the piston is constant and equal to the atmospheric pressure, will be proportional to the total disposable work. We see that it is impossible to correct the influence of friction by any graduation.

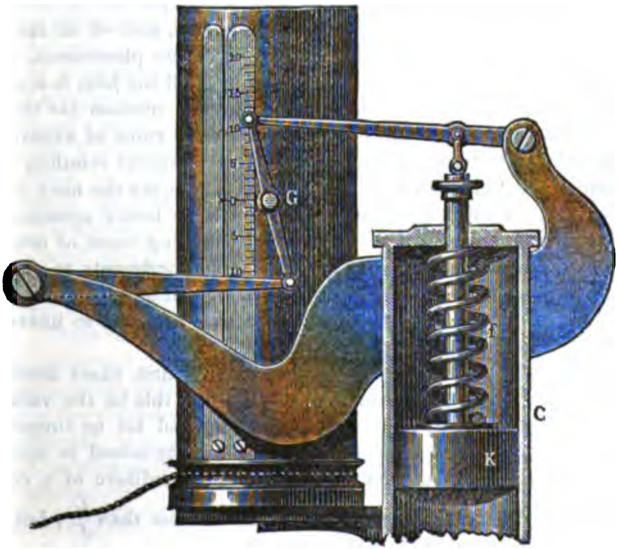

Fig. 1.

Rightly interpreted, the experiments of Hirn furnished results which you, no doubt, can anticipate. They showed that the steam actually gave up less heat in the condenser than it had received in the boiler, and that the heat absorbed in the machine was proportional to the actual work of the steam. The ratio of these two quantities furnished a new determination of the mechanical equivalent of heat, which nearly approached the determinations of Joule and Favre. Thus, although individual results vary within considerable limits, the mean of Hirn's experiments gives 418, or just the same as that found by Favre for the friction of steel on steel, and but very little different from the results of Joule. It must be confessed that Hirn drew entirely different conclusions from his experiments; but you will, I think, hardly be inclined to coincide with his views. He compared the heat consumption of his engine, not with the entire work of the steam, but only with that portion corresponding to its expansion. You will also, I think, agree with me, that such a division into two parts of the work is equivalent to the assumption that, in the period preceding expansion, while the machine works with full steam pressure, its work is nothing, and that appreciation which is the just due of the skillful experimenter will not, I trust, blind you to the error of his conclusions.* (It is but just to add that in later works Hirn has acknowledged his error and correctly interpreted his experiments.)

VI.

You will now, I trust, follow with confidence the generalizations which I shall lay before you. We have now, in fact, arrived at the same results by two entirely different ways. The study of two phenomena, of entirely different character, has shown us, that as soon as all the heat is converted into work, in both cases, we obtain the same numerical relation for the transformation. I might now, without trespassing against the rules of experimental methods, expect you to recognize in this a perfectly general relation. I might remind you that the greatest scientific discoveries are, for the most part, the result of no larger number of experiments, and a no better agreement of results. I would like, however, to remove every lingering trace of doubt, and prove to you that it is impossible for two different experiments to give as a value for the mechanical equivalent two essentially different results, *i. e.*, two values, whose difference, if any, cannot be entirely ascribed to unavoidable errors of observation.

In honor of Joule, to whom we owe its first exact determination, let us denote the mechanical equivalent by J. Let this be the value as determined by observations upon the steam engine, and let us suppose for a moment that this value does *not* coincide with that determined in some other manner. That is, suppose that we are able by the expenditure of a certain amount of work, L, to generate a greater amount of heat than $\frac{L}{J}$. Let this amount of heat be

$$\frac{L}{J}(1 + h),$$

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* See Notes 2 and 3 at close of these lectures, also Note 38.

and let us suppose that it is then applied in a steam engine for the production of work. The work obtained will then be

$$L(1 + h),$$

or, what is the same thing, we have stored up in the fly-wheel of the engine a living force

$$L(1 + h).$$

This living force we can now convert back into heat by the first process, and we thus obtain the heat

$$\frac{L(1 + h)}{J}(1 + h) = \frac{L}{J}(1 + h)^2.$$

Again, using this quantity of heat in the steam engine, we obtain in the fly-wheel a living force

$$L(1 + h)^2,$$

and accordingly a velocity greater than before.

But the steam engine and the other apparatus, whatever it may be, by which the work is transformed into heat, may be considered as forming one system. It follows, therefore, from our supposition, that in any period during which all the moving parts have returned to their initial positions, the living force has increased from

$$L(1 + h) \quad \text{to} \quad L(1 + h)^2.$$

Perpetual motion is therefore accomplished. The supposition is therefore impossible.

Inversely, let us consider a process by which heat is transformed into work, and assume that it is possible by the use of a quantity of heat, Q, to generate a greater amount of work than QJ. The consequence of this supposition will be a contradiction similar to the preceding. For this process it may be remarked that the steam engine is a reversible apparatus. Ordinarily it transforms heat into work, but by the application of outer forces its action may be reversed, and work may be transformed into heat. The motion of the piston, caused by outer forces, will gradually vaporize the water in the condenser, and the steam thus generated will be compressed in the cylinder until it is changed into saturated steam of the temperature of the boiler, and finally, this steam will be compressed into water of the boiler temperature.

The steam actually brings then more heat to the boiler than it starts with in the condenser. There is an expenditure of work and a generation of heat. In order to obtain perpetual motion nothing more is necessary, then, than to unite in one system a steam engine, whose action is reversed, with an apparatus which, according to the supposition, can produce from the quantity of heat Q a greater work than QJ. I need hardly add that in a precisely similar way we can prove that no process can give for the mechanical equivalent any other value than the constant one, J. Our conclusions have thus brought us to a perfectly general natural law. Let us endeavor to formulate these conclusions into a series of principles which shall accurately express their essence and make evident their application.

1. To "generate heat" means to impart to the ponderable or imponderable molecules of one or more bodies a certain amount of *living force;* if the bodies

thereby change their volume, a certain work is performed which is equivalent to a certain quantity of living force.

2. In every application of the equation of work it is necessary to take into account not only the visible living forces, but also, by means of the mechanical equivalent, the heat absorbed or set free.

3. In all cases in which we fail to find equilibrium between the sum of the works of the forces and the change in the living forces, or when such equilibrium can apparently only be effected by the introduction of an empirical term, as, for example, by introducing the work of friction, or by the assumption of a loss of living force, as in the impact of bodies, we must have, together with the mechanical, heat phenomena also, which restore the equilibrium.

4. When the sum of the works of the forces exceeds the increase in the sums of the living forces, we have a generation of heat of just so many heat units as 424 is contained in such excess.*

5. If the sum of the work of the moving forces is less than the increase in the sum of the living forces, we have a disappearance of heat of just so many heat units as 424 is contained in such difference.

Is it necessary to insist upon the importance of these principles? Who does not recognize that their influence extends through the whole range of science? Who can fail to see that every process which is based finally upon motion falls under the scope of these mechanical laws and includes in it an application of the equation of living forces? Who will not at once perceive that every application in which these laws are not regarded, must be at once rejected, as soon as it is known, or even suspected, that heat phenomena are bound up with the mechanical? I venture to assert that there is not a natural science which can elude the necessity of this new test. Physiology and astronomy have need of it equally with physics and chemistry.

This revision of scientific results is, moreover, not merely a laborious work of correction, which at most allows the hope of discovering in certain phenomena the influence of certain disturbing causes whose effect may be more or less difficult of calculation; or which renders more exact the determination of some numerical coefficient; it constitutes one of the most fruitful studies which true science can undertake, and is especially suited to bring to light relations between apparently the most diverse phenomena. The single example of friction teaches us what the new theory can accomplish in directions which are generally supposed to have been thoroughly investigated already.

VII.

Let us now endeavor to test the value of these considerations, so far as is possible within the narrow limits to which we must confine ourselves. We shall see that, from the very first step, they will lead us, not merely to superficial approximations, but to exact relations which may be verified by experiment. The consequence of such comparisons must be a continual verification, *à posteriori*, of the absolute generality of our new principles. Let us first consider, as is most natural, the changes which heat causes in the volume and condition of bodies.

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* See Note 4.

I scarcely need remind you that every body when it changes in temperature changes also in volume, and that when the temperature reaches a certain point for each body, that sudden change takes place which we call *liquefaction* or *vaporization*. The body passes from the solid condition to the liquid, or from the liquid to the gaseous, or the reverse. No part of science has been oftener investigated, and yet, in the absence of our new principles, no part seems to have made less progress. The chapters which treat of this subject, even in the most recent text-books, contain little more than a presentation of the most exact experimental methods of determining the most reliable coefficients of expansion, the specific heat and latent heat of substances, and tables in which these numerical values are given. All these phenomena are given as if entirely independent of each other.

This want of connection between the various properties of the same body, or between similar properties of different bodies, is certainly very unsatisfactory. So long as no bond of union exists between isolated facts, even the best observations can no more constitute a science than carefully cut stones, arranged in order of size and shape, can constitute a building.

It is, moreover, worth observing, that the actual progress of science has, at certain periods, rather made this condition of things worse than better. The condition in physics has gradually become what it might have been in astronomy, if the perfection of methods of observation had progressed more rapidly than the progress in theory—if, for example, the discovery of achromatism or the improvement in circle graduation of recent times had followed immediately the publication of Keppler's laws, instead of following, as they did, long after the discovery of the universal law of gravitation. For about thirty years science possessed, or thought that it possessed, in Mariotte's laws, the laws of the expansion of gases,* and the laws of Dulong, Petit, and Neumann relating to specific heat, laws analogous to those of Keppler. The marvelous improvement in experimental methods since that time, recalled by the mere mention of the names of Rudberg, Magnus, and Regnault, led, as a natural and direct consequence, to a knowledge of the deviations of these laws from the reality, and there were no theoretical views which could reconcile these disagreements, and refer both laws and deviations back to the same causes. The importance of these laws themselves soon seemed less than that of empirical formulæ, which represented approximately and with more or less exactness the general features of the phenomena. Thus it was that science seemed, little by little, to destroy itself. The Mechanical Theory of Heat has changed all this. It has not only put a new phase upon the phenomena themselves, but it has fundamentally changed our conception of them; in many cases it has even pointed out the reasons of variation. If we assume a certain amount of heat imparted to a body, the volume changes, and so also does the totality of its properties, which we express by saying that its "temperature is increased." If, however, in the degree that a body is heated, we increase the outer pressure upon its surface, we can completely prevent its expansion, and we find that in this case the amount of heat necessary to raise its temperature is *less* than before. If the rise of temperature is in both cases the same arbitrary unit of some thermometric scale, then the two quantities of heat are, the one, the specific heat by constant pressure, the other, the specific heat by constant volume. Their difference is the latent heat of expansion. The expression "latent heat" means simply the heat imparted to the body which has no effect upon the thermometer.

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* See Note 5.

What, now, is the mechanical view of this process? To heat a body—to draw heat from a certain source and cause it to enter another body—means to diminish the living force of the source by a certain amount, and to cause in the body mechanical processes which are equivalent to this diminution. If the volume is unchanged, we have simply an increase of the sum of the living forces of the particles (rise of temperature), and it may be, a certain work due to a change in the relative position of the molecules.* If the pressure is constant, the volume increases, and there is a *new* work which we may divide into two parts. First, the distances of the molecules are increased, while their mutual actions tend to keep them in their old positions. We have, therefore, a work performed in thus separating them, which we may call "disgregation work," and regard as negative, since the molecular forces oppose the displacements. Second, the body expands against the outer pressure of the atmosphere. This constitutes another work which is also negative, and which we may call "outer work." The excess of the specific heat by constant pressure over the specific heat for constant volume, or the latent heat of expansion, is therefore that amount of heat which is withdrawn from the source while these works are performed. Expressed in heat units, it must be equal to the quotient of the sum of both these works divided by the mechanical equivalent.

Consider now the double result of our conclusions. First, we have learned what latent heat is. We have seen that it is that heat which disappears when work is performed, and which reappears again when, by means of outer forces, an equal work of opposite sign is performed.

In the second place, we can determine a numerical relation between two physical constants which are apparently independent of each other, and also the mechanical-work corresponding to a given change.

Unfortunately this relation, in the form in which it occurs, is of no use. Of the two terms which form the left side of the equation, only one, that which gives the "outer work," can be accurately determined. This is evidently equal to the product of the pressure and of the increase of volume, and is accordingly quite considerable for gases and vapors, and very small for liquid and solid bodies. The disgregation work, on the other hand, in the present state of science, eludes every attempt at determination, and will, without doubt, do so for a long time to come. We must have a complete knowledge of the interior constitution of the body, in order to determine it, and it is impossible to say how far the more or less plausible ideas held to-day represent the actual state of things. A great error is committed if, as sometimes happens, it is sought to establish an equivalent relation between the heat absorbed by a body and the outer work. The error may be diminished, but not eliminated, by replacing the disgregation work of a body by the work of outer forces, which cause a deformation equal to the expansion. It cannot but be a cause of wonder, if determinations of the mechanical equivalent based upon such a method of determination, have given results which closely agree with the true results.*

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* See Note 6.

In view of these difficulties, it would seem as if the theory must soon cease developing, and as if the discovery of exact relations whose numerical value can be checked by experiment, must be delayed till such a time as the science of physics shall have said its last word as to the nature of all things. We can, however, avoid such difficulties by means of a method or artifice which we owe to Sadi Carnot. Thus, we may, without knowing anything of the interior structure of bodies, establish such relations between the mechanical and thermal properties of bodies as shall be of value, by considering such a sequence of changes as takes place during a process in which the initial and final conditions are alike, and hence the disgregation work zero.

Let us consider any solid, liquid, or gaseous body, which has the temperature t, the pressure p, and the volume v. Let us call the state of the body, as determined by these three conditions, the state T, and represent the volume v by the abscissa OA, Fig. 2, the tension p by the ordinate AT. Now, suppose the outer pressure to diminish, and while the body expands let us impart heat to it, so that its temperature changes according to any given law. Let this continue till the body comes to the state T', for which it has the temperature t', the volume v', and the tension p'. Let $OB = v'$, $BT = p'$, and let the abscissa and ordinate of the curve TMT' at any point be the volume and pressure at any intermediate state. Call the change of state from T to T', D. During this change a certain amount of heat, Q, is imparted to the body, and a certain outer work is performed, L. Both quantities can be calculated, if, for the limits of temperature t and t', the influence of the outer pressure upon the volume of the body and the amount of heat which the body requires for a given change in volume and temperature, are given by experiment. These quantities may be expressed theoretically, in terms of the coefficient of elasticity and the two specific heats, provided that we regard these two elements as functions of the temperature and the volume. The work L is therefore given in the figure by the area between the curve TMT' and the axis OB and the two extreme ordinates AT and BT'.

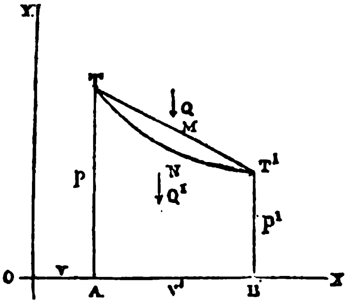

FIG. 2.

Let us now assume that, by a gradual increase of the outer pressure, the body is brought back to its original state, and that during this change, which we may call D', we continually subtract heat from the body as it is compressed, so that its temperature for any given volume is less than during the change D, except at the beginning and end of the entire experiment. The body thus finally comes back to its original condition, but at all intermediate states of the change D', the pressure corresponding to a given volume is *less* than during the change D. The curve TNT', which gives this second relation between pressure and volume, consists throughout, with the exception of the first and last points, of less ordinates than the curve TMT'. The area between the curve TNT', the axis OA, and the extreme ordinates, gives the work L' performed upon the body while compressing it, and evidently we must have

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* See Note 7.

$$L' < L.$$

We may also compute L' and Q' in the same way as L and Q.

These two operations, D and D', may be regarded as parts of one process in which the initial and final conditions are identical. The relative position of all the elements of the body are the same at the beginning and end. It follows from the general laws of mechanics, that there must be a complete compensation between the work of the molecular forces ; that the inner work, corresponding to the transformation D, must be exactly equal and opposite to that which corresponds to the transformation D'. We have then nothing to do with it. Still, L' is less than L. We see, therefore, that the body, in the cycle of changes to which it is subjected, moves in a determined law from its initial state to another, and then, according to *another* determinate law, returns to its original state, and during this cycle it performs an outer work equal to $L - L'$, which is represented by the area $TMT'NT$, that is, by the difference of the two areas which represent the works L and L'. No inner work is performed, no sensible living force has disappeared, therefore a certain amount of heat must have disappeared equivalent to the work done. It follows, therefore, first, that the body during the change D has received more heat than during the change D' it has given up. Further, the ratio of the work $L - L'$ to the heat absorbed, $Q - Q'$, is equal to the mechanical equivalent. The formula

$$L - L' = J(Q - Q'),$$

which we thus obtain, gives us a numerical relation between the mechanical and thermal phenomena, the study of which is usually relegated to two different departments of physics, since L and L', Q and Q', are determined by means of the coefficients of elasticity, the two kinds of specific heat, the temperatures and the volumes. We may obtain as many special relations as we suppose cycles of changes. In order to obtain a general equation which shall include all these cases, it will be sufficient to consider the change as infinitely small. Then the above formula will become a differential equation, whose integration will give us the law of expansion of the body under all circumstances. Two other differential equations, obtained by analogous reasoning, and containing other elements, give the laws for melting and vaporization.*

VIII.

The character of these lectures forbids any use of the Calculus. Without noticing any further, therefore, these differential equations or their consequences, let us direct our attention to a certain class of bodies of which we can give an almost complete account, simply by the consideration of the *outer* work which they perform under the action of heat. It has for a long time been noticed that the similarity of the mechanical and thermal peculiarities of different gases seem to indicate that in these bodies the influence of the mutual actions of the molecules is not noticeable.

The older text-books of physics held generally the hypothesis that heat was something material, and ascribed the elastic force of gases to the repulsive force of the absorbed heat upon the molecules. Laplace himself deduced from such views the law of Mariotte, as well as that of the diffusion of gases and of their expansion (*Méc. Célèste*, liv. xii., chap. 2). At the present time, when the views as to the nature of heat have undergone such great changes, the demonstration of Laplace no longer holds good; but the point of departure remains, however, the same. The simplest way of explaining how it can be possible that mechanical action and heat produce almost the same effects upon various gases, is to assume that at the distances which separate the molecules of such bodies their mutual actions are imperceptible. The laws of the diffusion of gases seem indeed to impart to this conception the character of necessity. If the molecular forces in gases had any appreciable intensity, those existing between two molecules of the same kind, and between two molecules of different kinds, would not be the same. The properties of a mixture of two gases must, then, be different from those of a simple gas. Every one knows, for example, that from a physical standpoint no other differences exist between oxygen and air except the density and the coefficient of refraction; while all those properties which depend upon the mutual interaction of the molecules are exactly the same. From this follow two consequences: First, if in gases the molecular forces are almost zero, it is impossible to frame any conception of the constitution and general properties of such bodies, without assuming that their molecules possess a considerable velocity, which is greater the higher the temperature, and that these molecules by their impact cause pressure. Second, the change of volume of a gas is not accompanied by any disgregation work at all comparable with the outer.

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* See Notes 8 and 9.

The development of the first of these consequences has given rise to views upon the constitution of gases which have replaced those of Laplace. I make here merely this passing reference, as I do not wish to lay down anything in these lectures which rests at bottom upon hypothesis.* The second consequence is susceptible of direct confirmation by experiment. Thus, if we allow a gas to expand without overcoming any outer resistance, that is, without performing any outer work, and if the disgregation work is also zero, and the gas, both at the beginning and end of the experiment, is at rest, there can be neither absorption nor generation of heat.

This assertion may excite astonishment, since it appears at variance with well known facts. All of us are familiar with the simple experiment of putting a thermometer under the receiver of an air-pump, in order to observe the decrease of temperature which occurs at the very first stroke of the pump. We also know that when air which has been greatly compressed in a reservoir is allowed to issue in a jet into the room, it may cool to such a degree that the vapor contained in it is frozen and deposited upon surrounding bodies in the shape of frost.

In view of such facts it appears surprising when we assert that a gas may, under certain conditions, expand without cooling. It is nevertheless really so. In a metallic reservoir *R*, Fig. 3, communicating by a pipe and cock with the equal reservoir *E*, Joule has compressed air under a pressure of 22 atmospheres, while the reservoir *E* was exhausted. Both were then immersed in a vessel full of water and the cock *D* opened. The air in *R* rushed in to *E*, and its volume was thus doubled, while there was, of course, no resistance to its expansion, except the small amount of air which might remain in *E* after exhaustion. Although the tension of the air decreased from 22 to 11 atmospheres, there could be then no outer work performed, since at both the beginning and end of the experiment all parts of the apparatus and of the gas were at rest. In perfect accord with theory, it was found that there was no absorption of heat. The most sensitive thermometer immersed in the water which surrounded both *R* and *E* showed not the least change when the cock *D* was opened.

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* See Note 10.

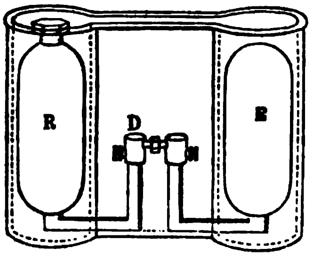

FIG. 3.

It is not difficult to see why under the receiver of the air-pump, or in efflux into the air, the expansion is accompanied by absorption of heat. If we consider closely the case of the air-pump, we see that a part of the work required to work it is furnished by the pressure of the air removed. Outer work is thus performed at every stroke, and heat correspondingly absorbed. We could not, therefore, have a better confirmation of our new principles. In the efflux of air it rushes with great velocity from the reservoir, driving the outside air before it and thus performs work. Hence the cooling with which we are familiar.

If we alter Joule's experiment so as to perform outer work or generate living force, we shall find that heat is absorbed. Thus, if we remove the reservoir *E*, and fasten to *D* a hose, and thus allow the air to discharge into a large bell-glass filled with water and inverted in the pneumatic bath, the water will be forced out against the pressure of the atmosphere, and a thermometer in *R* will show a decrease of temperature due to the disappearance of heat corresponding to the outer work performed. We can easily see that such an experiment may lead to a determination of the mechanical equivalent. In this way Joule found 441, a result very closely agreeing with his others, the deviation being completely attributable to the unavoidable errors of observation. Thus disappears the apparent contradiction between what we may call the old and new physics.

In order, however, to leave not the slightest doubt or uncertainty upon so important a point, let me try to meet in advance an objection which has, without doubt, already occurred to you. Let us look somewhat deeper into this process. Conceive in the reservoir *R* that portion of the gas which, after the experiment is completed, just fills this reservoir. Why does not *this* portion cool during the expansion? It is in every respect similar, and in similar circumstances, to the same portion of the gas in the second experiment, where its expansion *was* accompanied by a decrease of temperature. In both cases this portion expands against the pressure of the rest of the gas. To say that in the one case it preserved its temperature, and in the other case loses it, would seem to imply that it knew what was going on outside, and was gifted with intelligence and choice of action.

We do not, in general, willingly receive anything against a theory which is regarded by the highest scientific authorities as correct. It seems a thankless task to give audible expression to difficulties like the above; yet, at the bottom of one's heart they must still remain and cause a silent distrust of all science. Let us, therefore, see if we can lay this doubt.

As a matter of fact, that portion of the air, in Joule's experiment, which remains behind in the reservoir *R*, must, and *does* lose heat and cool, because during the experiment it continually imparts living force to that portion of the air which rushes with considerable velocity into the reservoir *E*. But this living force immediately disappears. The velocity of the gas entering *E* is destroyed by the friction of its own molecules upon each other, by their impact upon the walls of the reservoir, and by friction in the communicating pipe. As soon as the gas ceases to enter, therefore, all is at rest. But this living force cannot be destroyed without a *generation* of heat exactly equal to that which disappears in the reservoir *R*. In Joule's experiment, then, no change of temperature was observed, because there is a perfect compensation; the friction in *E* replaces the heat disappearing in *R*. We have no need, then, to ascribe to the gas any inconceivable properties; we do not even need to suppose properties any different from those long known. We can also easily prove our conclusions by experiment, by having the reservoirs *E* and *R* *in separate vessels*, when *R* will be found to absorb heat and *E* to give out a precisely equal amount.

This remarkable experiment of Joule, performed in 1845, more than any other directed attention to the new theory. Regnault repeated it in every shape, with all the precautions which his long experience in calorimetric researches rendered available. He notified the Academy in April, 1853, that he had completely confirmed it, and from that moment he counted himself among the advocates of the new views.

No further doubt can remain. In gases the disgregation work which accompanies expansion or compression is zero, or at least is imperceptible to ordinary calorimetric methods.* Heat when imparted to a gas causes only two effects, a rise of temperature (" vibration work ") and outer work. If the rise of temperature is one degree, while the gas expands under a constant pressure, then the outer work is equal to the product of this pressure into the increase of volume. If V_0 is the original volume, and if α is the coefficient of expansion, then for a rise of one degree the increase of volume will be αV_0, and the new volume will be $V = V_0 (1 + \alpha t)$ for a rise of t degrees, or $V_0 = \frac{V}{1 + \alpha t}$, where t is the temperature for the volume V.

The change of volume, then, or αV_0, is $\frac{\alpha V}{1 + \alpha t}$, and this multiplied by the pressure p, gives us for the outer work

$$p V \frac{\alpha}{1 + \alpha t}.$$

If the weight of the expanding gas is equal to one unit, then the value of the outer work is the mechanical equivalent of the excess of the specific heat by constant pressure over the specific heat by constant volume. If J represents the mechanical equivalent, we have

$$(C_p - C_v) J = \frac{p V \alpha}{1 + \alpha t},$$

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* See Note 11.

or if V_0 is the volume for the temperature zero and pressure p_0, we have

$$(C_p - C_v) J = \alpha p_0 V_0 .$$

This gives us, for all gases which follow Mariotte's law, a numerical relation between the coefficient of expansion, the two specific heats, the volume of the unit of weight under the given circumstances, and the mechanical equivalent of heat. We may make use of it in order to determine the mechanical equivalent by means of the physical properties of various gases, and since for most gases these properties are determined with a degree of precision which cannot at present be exceeded, it would seem that in this way we should obtain a value superior to all the others in accuracy. The formula applied to air gives us the number 426, almost identical with the mean of Joule's experiments, if we take for the volume of the unit of weight, for the coefficient of expansion, and for specific heat for constant pressure, the values given by Regnault, and take for the specific heat for constant volume the best value as given by experiments upon the velocity of sound. The agreement of this calculation with the experiments of Joule upon friction is, in fact, most remarkable.

IX.

Unfortunately this agreement does not exist when we apply the formula to other gases. We obtain, however, 425—a very close value—for hydrogen, oxygen, and nitrogen, while for carbonic acid gas we obtain a value considerably different. Indeed we obtain for this gas two very different values, according as we take one or the other of the two determinations of Regnault for 0° and 100°.* For other gases the deviation is still greater. Whence come these deviations? A great part, without doubt, are due to the uncertainty as to the value of the specific heat for constant volume. We must, however, add that the formula is not equally reliable for all gases, since the disgregation work cannot be disregarded in all.

The laws of Mariotte and Gay-Lussac hold accurately for no gas ; they are only approximate expressions of the truth for those gases which are furthest from their points of liquefaction. It is only for these gases that the agreement of their mechanical and thermal properties allows us to assume that the influence of the molecular forces is zero. On the other hand, gases like carbonic acid, which we can easily liquefy, whose coefficient of expansion is five-tenths greater than air, and which changes very rapidly with the pressure ; gases, finally, which even under the pressure of the atmosphere do not follow the law of Mariotte ; for all such we have every reason to believe that a noticeable work of the molecular forces accompanies changes of volume.

If we apply to such a gas a formula which assumes the absence of all disgregation work, it simply shows that we do not understand the principles of which we make use. If we say, as has been said, that there are as many mechanical equivalents as there are gases, we indirectly declare the possibility of perpetual motion.

It would seem an immediate consequence of the above, that by repeating the experiments of Joule with carbonic acid and similar gases, and determining the amount of heat which disappears when they expand without performing outer work, we might obtain a measure of the disgregation work, and thus correct the above formula and express the true relations between the different properties of such gases. Without, however, entirely changing the experimental methods of Joule, there seems little hope of obtaining in this way any satisfactory results. In the experiment which has been described, the expanding gas is surrounded by water, and even under a pressure of 22 atmospheres the mass of the gas cannot compare with that of the water. It is easy to comprehend that if, for example, the mass of the water is only twenty times that of carbonic acid gas, and the specific heat of the water about five times as great, the absorption of a quantity of heat which would change the temperature of the gas one degree would alter that of the apparatus at most only $\frac{1}{100}$th of a degree. The phenomena in question, therefore, would be completely masked by the unavoidable errors of experiment. It is necessary to find some method of doing away with the liquid as a heat-measuring substance, and observing the change of temperature in a stream of gas, which, without performing outer work, experiences a considerable change in elastic force. Under such circumstances the entire heat disappearing is equivalent to the disgregation work accompanying the expansion. These conditions are actually complied with in an experimental method conceived by Sir William Thompson.* Our space does not allow us to describe it here. The application of this method to hydrogen, air, and carbonic acid has shown that the change of temperature for hydrogen is almost zero, that it is noticeable for air, and very considerable for carbonic acid, just as might have been expected from the experiments of Regnault. Hydrogen seems, indeed, the furthest removed of any gas from its point of liquefaction. Oxygen and nitrogen show a less perfect accord with the properties of a perfect gas. Carbonic acid gas, finally, deviates decidedly.

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* See Note 12.

It is, therefore, perfectly natural that in hydrogen the disgregation work should be very small—almost zero—also small in nitrogen and in the air, but still there—and that it should have a considerable value in carbonic acid. The results of experiment are not completely satisfactory, nor exact enough to furnish a reliable value for correction of our formula. They suffice, however, to furnish an explanation of the variation which has been found in the mechanical equivalent, as determined from various gases, and they show that it is allowable to use the formula without correction for air and hydrogen. We may consider it as tolerably certain that the exact value of *J* lies between 424 and 426, the results obtained from the consideration of these two gases; or still more, having reference to the uncertainty in the value of the specific heat by constant volume, between 420 and 430. We shall, however, continue, in what follows, to make use of the value 424.

I have devoted considerable space to this first application of the theory—much more than I can give to others of which I intend to speak. I do not wish, however, to lay any especial stress upon the study of the expansion and compression of gases; but I thought it well to show thus early that the mechanical theory of heat leads to results which agree with fact, and submits to calculation not only known phenomena, but also predicts new ones, and that this prediction is capable of numerical verification.

I have sought to excite in you the same impression which, without doubt, those of you who are familiar with the study of optics have already experienced, as, proceeding from the undulatory theory, they made their first applications of it to the phenomena of reflection and refraction. The simplicity with which this theory harmonizes known facts, the fruitfulness of the views it presents, the accuracy of its predictions, afford the most convincing proof that it closely expresses the truth, or, at least, opens a path which leads to the truth.

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* See Note 13.

I shall consider my object obtained if this first lecture shall have produced a somewhat similar conviction.

INTRODUCTION.

SECOND LECTURE.

CONTENTS.

I. Recapitulation of the first lecture.—Objects of the second lecture: Investigation of heat engines and application of the theory.

II. Comparison of steam and gas engines.—Opposing views held by physicists and mechanics as to the relative value of these two engines.—Statement of the considerations by which the relative disadvantages of the steam engine have been sought to be proved. These considerations met by Hirn's experiments.

III. General expression for the efficiency of a Stirling hot-air engine.—This expression does not indicate any superiority over the steam engine.

IV. Generalization and simplification of the expression for the efficiency of gas engines.—Absolute temperature, absolute zero of temperature.—Deduction of the second fundamental principle of the mechanical heat theory: A constant ratio exists between the quantity of heat transformed into work in a perfect heat engine, and the quantity of heat transferred from a hot to a colder body.—One real advantage of the gas engine.—Practical objections.—Advantages of engines with superheated steam.—Engines with two fluids.

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IX. Table of the chief determinations of the mechanical equivalent.—Remarks upon.—Applications of the new theory to chemistry.—Measurement of the work of chemical forces by means of the heat generated.—Mechanical explanation of some electro-chemical phenomena.—Mechanical significance of the measurement of electro-motive force.—Experiments of Regnault upon metal amalgams.

X. Applications to animal physiology.—Mayer's theory of respiration and muscular motion.—Experiments of Hirn and Béclard.

XI. Applications to botany.—Necessity of sunlight for vegetation.—Remarks upon the common origin of all motion on the earth.

XII. Upon the duration of the sun's heat.—Hypothesis of Mayer.—Calculations of W. Thomson.—Remarks upon the scope of the new theory.

XIII. The mechanical theory of heat reveals laws of phenomena, but does not disclose their mechanism.

XIV. Historical.—The forerunners of the theory; Daniel Bernoulli, Lavoisier and Laplace, Rumford, Davy, Young.—Special influence of Sadi Carnot and Clapeyron.—The discoverers of the new principle: Mayer, Colding, Joule.—Helmholtz and his treatise upon the conservation of force.—Works of Clausius, Rankine, Thomson, Zeuner.

LECTURE II.

We have now passed in brief review the phenomena, by the consideration of which science has attained to the recognition of the new principle of the equivalence of heat and work.

Starting from the laws of mechanics, we were at first brought face to face with an apparent contradiction between these laws and the usual theory of machines. To reconcile this contradiction, it was necessary to include the phenomenon of heat among the mechanical effects which occurred in the entire machine during its motion. The heat generated by friction was thus found to be equivalent to the difference between the work of the motive forces and of the resistances; the heat absorbed during the motion of the machine was shown to be the equivalent of the work done.

The correspondence of the numerical results in both cases gave us confidence in the correctness of our views, and allowed us to frame a very precise idea of the mechanical equivalent of heat. We have also recognized the contradiction to which we are led if we assume that the value of this equivalent can change; and we have still further convinced ourselves of the correctness of our new principles by applying them in various ways. Our first application was with reference to the change of volume or of condition of bodies by heat. For solid and liquid bodies we have done little more than point out difficulties, and briefly notice the method by which they may be met. We have treated gases more thoroughly. Experiment has shown us that for air and other gases far removed from their point of liquefaction, the disgregation work, or the work of the molecular forces which accompanies a change of volume, is either zero or very slight. This fact has permitted us to compare the amount of heat which must be imparted to a gas in order to obtain a certain amount of outer work, with that work itself. Thus we found a new determination of the mechanical equivalent, and at the same time deduced a necessary relation between the different mechanical and thermal properties of the same gas.

I have sought, in this development, to keep observation and theory side by side, and, in some degree, show you that every experiment was the realization of an idea; and finally, to make it as evident as possible how firmly all parts of our new theory are held together.

I shall now pursue an opposite method, and plunge at once into the midst of facts, or, so to speak, into the midst of practical industry, and I shall seek to deduce general physical laws from the study of special phenomena, such as are presented by the study of those machines which derive their motive power from the action of heat. The investigation of "heat engines"[*] will thus form the subject proper of this lecture. The remaining portion of it will seek to give you a review of those applications of the new theory which lie outside of the domain of physics, and especially of mechanics.

II.

There are two kinds of heat engines of special importance; the steam engine and the hot-air or caloric engine.[†] Upon hot-air engines much at one time was said and written. Great stress has been laid upon their improvement, and almost unlimited expectations have been formed of their mechanical efficiency.

A superficial knowledge of Joule's experiments upon gases soon spread, and it was for a time firmly believed that soon all the heat furnished by the fuel would he utilized. On the other hand, after Regnault's experiments upon the latent heat of vaporization, which seemed to show that only an inconsiderable fraction of the power of the heat absorbed could be utilized, many physicists formed an unfavorable impression of the steam engine. Thus a kind of conflict arose—I can hardly say between theory and experience, but between one view which apparently harmonized with theory, and between the ever-accumulating results of experience. In practice, gas engines have never been found of such economical value as to balance the difficulties which attend their introduction. Let it be the first object of our remarks, therefore, to point out the significance of this apparent conflict. For the sake of clearness, we shall take a numerical example, and shall therefore choose a steam engine working under a steam pressure of 5 atmospheres, or with a steam temperature of 152°, and shall first assume that there is no condenser. The steam enters the cylinder in the saturated condition at a temperature of 152°; the formation of every kilogram of steam requires, according to Regnault's experiments, a quantity of heat denoted by the number 653, diminished by the temperature, t, of the feed water. As it enters the cylinder, the steam raises the piston until the communication with the boiler is closed, when it expands a certain amount, and finally discharges into the air under the pressure of the atmosphere. If we assume that the steam remains saturated during the expansion, then the final temperature is 100°, and every kilogram of steam which leaves the cylinder takes with it $637 - t$ heat units, which it gives up in condensing into water of the temperature t. Out of the $653 - t$ heat units absorbed in the generation of the steam, only 16 disappear in the engine. These 16 units are transformed into work; all the rest is wasted in the atmosphere.

Thus if, for example, t is only 10°, only $\frac{16}{643}$ds, or less than $\frac{1}{40}$th, of the heat furnished to the boiler by the fuel is utilized. This fraction, which we may call the "efficiency," is increased somewhat by the application of the condenser, but it is always very small. If, for example, the condenser has a temperature of 40°, and the steam expands in the cylinder to such an extent that its pressure is reduced to that in the condenser, which never can be the case in practice, the quantity of heat which one kilogram of steam brings to the condenser is only

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[*] Under heat engines we include all those machines whose motive power is due to the disappearance or transformation of heat.

[†] Hot-air engines are known also as gas engines and caloric engines. We call any machine a hot-air engine whose action depends upon the heating and expansion or the cooling and contraction of any of the so-called "permanent" gases.

$$619 - 40 = 579 \text{ heat units.}$$

If, further, the condenser is fed with the boiler water itself, each kilogram of steam requires for its formation only

$$658 - 40 = 618 \text{ heat units.}$$

Hence 34 heat units are utilized, and the efficiency becomes $\frac{34}{618}$ths, or about $\frac{1}{18}$th.

The condenser is therefore of considerable advantage, but the heat utilized is still very small in comparison with the total heat imparted.

These are nearly the words of Regnault in his criticism of the steam engine, which has been extensively repeated. According to this, the most important motor of our civilization is but a very imperfect machine.

Now let us turn to experience. The treatise of Hirn, from which we have already taken several important results, gives us the data for a reply. We find there four satisfactory and consistent series of experiments upon the steam engine, which are nearly identical with those we have already alluded to. The temperature of the boiler, as a mean of the four experiments, was 146°, that of the condenser, 34°. Assuming perfect expansion, we have for the efficiency, in precisely the same way as before, $\frac{34}{617}$ths, which is nearer to $\frac{1}{18}$th than to $\frac{1}{19}$th. This should therefore be the limit which can never be exceeded, and which probably experiments can never show. But notwithstanding this, the singular fact remains that Hirn's engines gave much better results. The excess of the heat taken by the steam from the boiler, over that given out in the condenser, that is, the heat expended in producing work, was never less than $\frac{1}{10}$th of the total amount of heat; it was sometimes even $\frac{1}{6}$th, and in the average $\frac{1}{8}$th.

Here, then, is a direct contradiction. Upon the one hand, a theory approved by many physicists gives for the efficiency of an engine a value but little higher than $\frac{1}{19}$th; upon the other hand, experiments made upon machines in actual use, which therefore must be very far from perfect, and which must be fitted with special apparatus for determining the efficiency, give a result twice as great. The accuracy of the experiments is proved by the very close value of the mechanical equivalent which they have given. The error must therefore be sought in the theoretical conclusions.

Now we have assumed without any proof that the steam which after expansion leaves the cylinder and is discharged either into the air or into the condenser *is saturated*. This assumption enabled us to base our calculation upon the total heat of vaporization as determined by Regnault. The facts observed by Hirn contradict this entirely groundless assumption, and prove that the phenomena of expansion follow much more complicated laws, and that a much greater part of the heat is utilized. The steam therefore cannot remain saturated during expansion. Still less can it become heated above the point of saturation, and become at the end of expansion superheated, that is, possess a less elastic force than that which corresponds to its temperature when just saturated; for a given amount of superheated steam would give up to the air or to the condenser more heat than the same amount of saturated steam, and the coefficient of efficiency would therefore be less than that already computed.

There is only a third supposition possible, viz.: that the originally saturated steam *condenses* during its expansion in the cylinder, and a part of it becomes water. This supposition is, moreover, correct. We can point, in confirmation of it, to an almost daily occurrence in practice. Every one knows that water collects in the cylinder if it is not jacketed. Rankine has shown that the principal cause of this is the condensation during expansion, and not, as some have thought, the accidental introduction of water from the boiler. Hirn has given us a direct experimental proof. A copper cylinder, 2 meters long and 0.15 in diameter, was closed at both ends by thick glass plates. Two pipes, with cocks, were connected, the one with the boiler and the other with the air. First the air-cock was partly opened and the steam-cock fully opened. Steam thus entered from the boiler, drove out the air from the cylinder, and filled it with dry and perfectly saturated steam. The cylinder was then as transparent as if filled only with air. The air-cock was now fully opened. The steam escaped rapidly, expanded in doing so, and in a moment the cylinder, before so transparent, became perfectly opaque, and the condensation was visible.

I need not point out that this condensation increases the amount of heat which disappears in the machine, that is, which is turned into work. Every kilogram of steam which reaches the cylinder from the boiler requires for its production the amount of heat already given. But the heat which still remains in the steam when it enters the condenser or the air, is diminished by the latent heat of that amount of steam which during the expansion has been liquefied.

It is not wholly saturated steam which leaves the cylinder, but a mixture of steam and water, and the heat converted into work is no longer equal to the difference of the total heat of vaporization for two different temperatures, but is equal to this difference increased by a considerable fraction of the latent heat. Condensation during expansion is thus a physical property of steam, to which the steam engine owes a large part of its efficiency.*

III.

Now let us consider the hot-air engine, and see, if we can, how far those hopes are justified which accompanied its discovery. Without doubt, in such an engine we may convert all the heat into work, when our object is simply to raise a loaded piston and then allow it to sink again. Practice, however, demands a very different result; it demands a continual activity,—a periodic motion, which shall be incessantly repeated so long as heat is consumed. For example, it is required that the piston of a hot-air engine, after it has been raised to a certain height, shall sink again to its original position, and that this action shall be repeated indefinitely. But the air under the piston opposes the downward motion, and this resistance can only be overcome by the expenditure of a certain amount of work. While the air is thus compressed, it becomes heated, and this heat must be withdrawn in order to bring about the original condition.

If, therefore, during the first period, all the heat imparted can be converted into work, in the second, a portion of the work thus obtained is consumed in order to generate heat; only the remainder of the work is at our disposal.

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* See Note 14.

The question is, whether, when everything is taken into account, the hot-air engine possesses any peculiar advantages. Let us, as an example, take one of these engines whose theory is simplest, and which has been the most tried in practice; the engine of Robert Stirling, which dates back to the year 1816.

In this engine the air is first heated under constant volume, then it expands under constant temperature, it is then cooled to its original temperature while keeping its new volume, finally, it is compressed without change of temperature to its original volume. The expansion takes place under a much higher temperature, and hence under a much higher pressure, than the compression. The work performed in the first case is greater than that absorbed in the second, and the excess can be utilized.

Let us represent this entire cycle of changes by a geometrical construction. Let OA, *Fig.* 4, be the volume of one unit in weight of gas for the initial temperature t_0 and let the ordinate AT_0 be the corresponding pressure p_0. The air is first, without change of volume, raised from the temperature t_0 to the temperature t_1, which requires

$$c_v(t_1 - t_0)$$

units of heat, if c_v is the specific heat at constant volume. During this rise of temperature, the pressure rises from p_0 to p_1, represented in the figure by AT_1. But as the volume remains unchanged, no work is performed. The pressure upon the piston simply rises from p_0 to p_1, while the piston itself does not move. Now the load on the piston is gradually diminished, the air expands, *without changing in temperature*, from the volume v_0 to the volume v_1, represented by OB. The temperature remaining constant, the volume varies inversely as the pressure, and the curve T_1T_1, which is approximately an equilateral hyperbola, gives the law of change of volume with pressure; the last ordinate, BT_1, being the end pressure. Outer work is performed, which is represented by the area AT_1T_1B. But while the air is thus expanding, in order to preserve the temperature unchanged, heat must be added to the air, which heat is the equivalent of the outer work represented by the area AT_1T_1B, since the inner work in case of air is zero. In the third operation the temperature is brought back to t_0, without change of volume. The pressure accordingly falls from BT_1 to BT_0 without any expenditure of work, and therefore

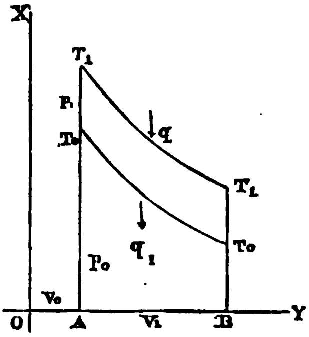

FIG. 4.

$$c_v(t_1 - t_0)$$

heat units must be abtracted, if, as is very probable, the specific heat of air for constant volume is independent of the density. In the fourth and last period the air is compressed, while its temperature, t_0, remains constant, until it has its original volume, V_0. Here work is performed upon the air, and heat must be abstracted in order to keep the temperature constant. The hyperbola T_0T_0 gives the relation between volume and pressure. The area ABT_0T_0 represents the work done in compression, which is equivalent to the heat q' abstracted.

The air receives, therefore, in the first two operations,

$$c_v(t_1 - t_0) + q$$

heat units, and outer work is performed by the gas, which is represented by the area AT_1T_1B. In the last two operations heat is abstracted from the gas equal to

$$c_v(t_1 - t_0) + q'$$

heat units, and a work has been performed upon it represented by the area ABT_0T_0. A quantity of heat, $q - q'$, has thus disappeared, and an equivalent work been performed, represented by the area $T_0T_1T_1T_0$. The heat utilized is $q - q'$, while the total consumption of heat is $c_v(t_1 - t_0) + q$, and the heat not utilized at all would seem to be $c_v(t_1 - t_0) + q'$.

A little consideration, however, will show that this last is not exactly the case, and that the heat not utilized, and which is therefore wasted, is q'.*

The quantity of heat $c_v(t_1 - t_0)$ which the air loses in the third period, while cooling from t_1 to t_0, without changing in volume, can be utilized in raising the temperature of another portion of gas, whose weight is unity, from t_0 to t_1. This portion will then be ready to expand, performing work without change of temperature, and when it in turn is cooled, the heat given up by it can raise the first portion from t_0 to t_1, and so on. By such an arrangement the heat $c_v(t_1 - t_0)$ may pass from one to the other of two equal portions of air which keep the machine in motion. Since we may conceive a perfect machine in which this transfer can take place without loss, this amount of heat is no part of the useful nor useless expenditure, it is at disposal in every cycle. It is different with the heat q' which the air parts with while being compressed under constant temperature. Since this must be absorbed by a cooling apparatus, whose temperature is t_0, it cannot again be used in order to raise the air above this temperature, nor to keep the temperature during expansion at t_1. It may, without doubt, be used in another machine in which the highest temperature does not exceed t_0, but, so far as the first is concerned, it is lost entirely. Hence we have

$$\frac{q - q'}{q}$$

as the ratio of the heat utilized to the whole heat used.

The quantities q and q' are easy to determine, since their mechanical equivalents are represented by the areas AT_1T_1B and AT_0T_0B. Thus if, as always, J is the mechanical equivalent of heat,

$$Jq = \text{area } AT_1T_1B,$$
$$Jq' = \text{area } AT_0T_0B,$$
$$\frac{q - q'}{q} = \frac{\text{area } AT_1T_1B - \text{area } AT_0T_0B}{\text{area } AT_1T_1B}.$$

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* See Note 15.

The determination of the hyperbolic areas is best performed by integration. We have, from known formulæ,

$$\text{area } AT_1T_1B = p_1v_0 \text{ log. nat. } \frac{v_1}{v_0},$$

$$\text{area } AT_0T_0B = p_0v_0 \text{ log. nat. } \frac{v_1}{v_0}.$$

But p_1 and p_0 are the pressures of the same mass of air, at the same volume, for the temperatures t_1 and t_0. We have, therefore, if α is the coefficient of expansion,

$$\frac{p_1}{p_0} = \frac{1 + \alpha t_1}{1 + \alpha t_0},$$

or, finally,

$$\frac{q - q'}{q} = \frac{p_1 - p_0}{p_1} = \frac{\alpha (t_1 - t_0)}{1 + \alpha t_1}.$$

This formula, of remarkable simplicity, gives at once the coefficient of efficiency of an engine of the kind discussed, provided that we only know the end temperatures between which the engine works. It is also evident that this coefficient is greater than is ever attainable in practice. If, now, we assume a gas engine, working between the same limits of temperature as Hirn's engine, we have, putting $t_1 = 146°$, $t_0 = 84°$, $\alpha = \frac{1}{273}$d, for the greatest efficiency possible, $\frac{62}{419}$ths, or a little less than $\frac{1}{7}$ths. This number is not so much greater than $\frac{1}{8}$th, which Hirn found for the steam engine, that one can conclude any considerable advantage of the one over the other. It would not be surprising if the practical imperfections of a gas engine, working between 146° and 84°, should reduce the useful work to $\frac{1}{8}$th of the total. The great importance ascribed at one time to gas engines does not appear, therefore, in any degree justified.

IV.

We may, however, go still further, and prove that, considered from an economical standpoint, *all* engines which work between the same temperatures give the same result. If we discuss any engine in the same way as we have already done for Stirling's, we shall see :

First, that in any given engine the ratio of the useful to the total expenditure of heat is a maximum, when no heat is used to change the temperature of the gases, or when, at least, this heat is a fixed amount, which never leaves the system, but is used over and over for the same purpose.

Second, that in such case the maximum value of the ratio is

$$\frac{\alpha (t_1 - t_0)}{1 + \alpha t_1},$$

where t_1 and t_0 are the highest and lowest temperatures* which occur in the cycle of changes in the machine. We may give to this expression a noteworthy significance by dividing numerator and denominator by the coefficient of expansion, α. It then becomes

$$\frac{t_1 - t_0}{\frac{1}{\alpha} + t_1},$$

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* See Note 16.

for which we may write

$$\frac{T_1 - T_0}{T_1}$$

if we let

$$T = \frac{1}{\alpha} + t,$$

that is, if we count the temperature from $-\frac{1}{\alpha}$, or from $-273°$ C., instead of zero. Now, what is this temperature, which, when measured from $-273°$ as the zero point, thus simplifies the expression for the economical coefficient? It is that temperature to which the gas is raised if $-273°$ is the zero of our thermometer, and if the gas is cooled under constant volume to $-273°$, its pressure would be, *supposing α to remain constant*, zero. This, then, is the temperature at which the molecules, while preserving their original distances apart, which they possess under ordinary temperatures, become motionless, that is, no longer impinge upon outer bodies, and therefore cease to exert that mechanical effect which we call pressure. In a word, this is the temperature for which the sum of the living forces is zero. But the expressions living force and heat are identical, and we can thus say that the temperature of $-273°$ C. *is the absolute zero of temperature.*

It has been attempted to determine this point in various ways, and at one time it was supposed that it was infinitely removed from ordinary temperatures. From this point of view we call the temperatures T_1 and T_0 "absolute temperatures." By the aid of this definition we may frame the following principle, which includes the theory of all hot-air engines:

"In every hot-air engine, in whatever manner it works, provided only no portion of the heat goes to useless increase of temperature of the air, the ratio of the useful expenditure of heat to the total is equal to the difference of the absolute temperatures between which the engine works, divided by the greatest of these temperatures." *

Does not the simplicity of this law produce the impression of a natural principle? Does it not appear probable that the expression $\frac{T_1 - T_0}{T_1}$ indicates always the ratio of the useful to the total expenditure of heat in a heat engine, whatever may be the changes in the machine, and whatever the bodies may be which are made use of for the transformation of the heat into work?

In fact, it is just as impossible that this ratio shall have two different values for two machines working between the same limits of temperature, as that the mechanical equivalent of heat should have two different values.

Let us consider, first, that in a heat engine the excess of the total heat expenditure over the useful is that portion of the heat derived from the fuel, which during the action of the machine is transferred to a colder body, and thus forever lost to the machine. If the engine is reversible, and this must necessarily be the case with all engines worked by expansion or change of condition, it will, when set in reverse action by an outer force, take heat from a colder body and transfer it to a hotter, and in this then collects all the heat generated while work is applied.

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* See Note 17.

The ratio of the heat thus generated to the total quantity of heat in the hotter body is exactly equal to the ratio of the useful consumption to the total in the usual action of the machine.

Let us now assume that the ratio in question has, in two different machines, two different values. It is not difficult to conceive these two machines so united that the one in which this ratio has the greatest value shall set the other in reverse action, and that the whole work developed in the first by the action of the heat is completely consumed in working the second. The action of these two machines, once started, would continue indefinitely without expenditure of heat or of work, since all the heat consumed in the first machine would reappear in the second.

Let H be the heat consumed in the one and reproduced in the other in the same time, R' and R'' the ratios of useful heat consumed to total in the first and second, and let, according to our supposition,

$$R' > R''.$$

While the first machine consumes the heat H for the preservation of its motion, it transfers from a source of heat, whose temperature is t_1, a quantity of heat, $\frac{H}{R'} - H$, to the condensing apparatus, whose temperature is t_0. In the same time the second machine reproduces the heat H, and transfers from the condensing apparatus to the source a quantity of heat, $\frac{H}{R''} - H$. The final result of the combination is, that from a cold to a hotter body the amount of heat, $\frac{H}{R''} - \frac{H}{R'}$, is transferred without the expenditure of any corresponding work. If this result is not a contradiction similar to the production of perpetual motion, it is, at any rate, a direct contradiction of the general laws which we have deduced concerning heat, and is sufficient to show that the hypothesis which led to it is not allowable.

The tendency of heat to pass from one body to another lies, so to speak, in the definition of the idea of "unequal temperature." The temperature of the body which gives out heat is the highest, and that of the body which receives it is the lowest. So long as theory cannot define with sharpness the conditions which we denote by the term "temperature," we can assign no decisive reason why there must be but one sequence of temperatures, and we may be inclined to admit that it is not impossible that bodies which do not interchange heat, and thus appear to have equal temperatures, if put in certain mutual relations, may conduct themselves as though of different temperatures.

All experience, however, gives a most decided negative to this supposition. It shows that equality or inequality of temperature is an absolute fact, independent of the experimental process by which it is made evident. If, for example, temperatures are recognized as equal by conduction, they are also equal by radiation. It is not possible to explain this law by even the most recent advances of theory.* It is sufficient that all the facts justify us in recognizing in this an absolute principle.

Fourier has based upon this principle his theory of radiant heat and of equilibrium of temperature, and although this theory may seem unsatisfactory in view of the discovery of the diversity of heat rays and of elective absorption, as shown by most bodies, and may have therefore seemed unsatisfactory to many, still all doubt must vanish in view of the wonderful discoveries to which, by a new application of the same principle, Kirchhoff has been conducted.

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* See Note 18.

We assume, however, nothing hypothetical, and we base ourselves upon the most certain facts of experience when we propound the following principles:

First, it is impossible for heat to pass from a cold to a warmer body without being, at the same time, accompanied by some phenomena which may be regarded as the cause of such transfer. Especially in no machine in which neither heat nor work is given out can any such transfer occur.

Second, it follows necessarily from this first law that the ratio of the useful to the total expenditure of heat, in a machine whose action depends upon change of volume or of condition of aggregation, is independent of the constitution of the body, and is determined solely by the extremes of temperature between which the machine works, provided that heat is not consumed in bringing about change of temperature. The formula

$$\frac{T_1 - T_0}{T_1},$$

which we have found directly for the hot-air engine, holds good therefore for every engine. It shows us at once that if any engine is superior economically to another, it is not because the bodies which serve to transfer heat and convert it into work possess this or that property. The only advantage which one body can present over another is in the wider limits of temperature rendered available.

From this point of view the superiority of the hot-air engine over the steam engine becomes evident. We cannot have the temperature of a steam boiler much above 150° or 160°, because the very rapid increase of pressure for higher temperatures would require extraordinary thickness. Since, on the other hand, it requires but little less than a rise of temperature of 273° to increase the pressure of a gas one atmosphere, we see what enormous temperature limits we may have in a hot-air engine, without greater strength than that required for an ordinary high-pressure steam engine.

We should have, then, greater economical advantages, were we not opposed by practical difficulties; as, for instance, the oxidation and the rapid deterioration of metal which always accompanies highly heated air. The use of superheated steam would seem to remove this objection, without greatly diminishing the peculiar advantages of the gas engine. Superheated steam is, in fact, a gas; its pressure near the point of saturation increases undoubtedly with rising temperature more rapidly than that of air; but all its thermal and mechanical properties coincide more with those of air the higher its temperature is raised.

Future progress would therefore seem in the direction of such an application, in which the peculiar advantages of the hot-air engine are combined with those of the steam engine.

Engines working with two kinds of steam, of which much has been heard, are an attempt to increase the mechanical efficiency of the steam engine by diminishing the lower limit of temperature. The water-steam which is condensed in the condenser is made to heat and vaporize a more volatile liquid, like ether or chloroform. This new steam works a second engine. It thus becomes possible to lower the temperature of the condenser below that which would exist for water-steam alone. The increase of motive power in such a construction is shown by the diminution of T_0 in our formula; but it is evident that this gain is not comparable with that which we may obtain in the steam engine with superheated steam, by increase of the upper limit T_1.

V.

There remains still a third kind of apparatus which we may include among heat engines, although it is apparently totally different from the hot-air and steam engine, viz., the electro-magnetic engine; and in spite of the small practical results thus far attained by it, these lectures would be incomplete without some discussion of its value and efficiency.

There is, moreover, from a purely scientific standpoint, hardly a subject more fruitful in interesting and novel views than the theory of the electro-magnetic engine, and I shall therefore devote at least as much space to it as to the comparison between the steam and hot-air engine, even although its practical value at present is less.

If we neglect differences of detail, we may divide all electro-magnetic engines into two classes; either oscillating or rotary. In the oscillating engine a fixed wire coil or an electro-magnet, as soon as the current passes, attracts another wire coil, electro-magnet, magnetized steel rod, or a piece of soft iron. As soon as this movable piece comes in contact with the fixed, by means of a circuit-breaker, the attraction is changed into repulsion, or it is neutralized by the attraction of another piece. The motion is thus reversed, and this action is repeated indefinitely. Such a motion can be utilized in a manner similar to that of a piston.

In the rotary engine the fixed and moving pieces are situated in the radii of two concentric circles or wheels. When the current passes, the movable wheel strives to take its position of stable equilibrium, but in the moment at which this is reached the circuit-breaker acts, the wheel is carried round by its momentum, and a continuous rotary motion is the result. This motion can be utilized like any other motion of the same kind produced by any mechanical force. In both cases the principle of construction and the origin of the force is the same. The action of the currents or of the magnets strives to bring about a condition of stable equilibrium, and some physical change in the system at the moment that this condition is satisfied continues the motion. What is the mechanical expression for this entire action?

Let us first consider the case in which the machine, in spite of the passage of the current, is held fast without motion. The electro-motive voltaic chain, or battery, and the engine, form, then, a fixed system, in which two kinds of processes are simultaneously going on. In the battery there is in a given time a certain amount of chemical action; at the same time, in all the conductors through which the current passes, heat is generated, and, so long as the machine is not in motion, this is all. In the battery we have chemical action;

atoms obey their affinities, pass from one condition to another in which their affinities are satisfied, and so into equilibrium. From the definition of mechanical work, it follows that, in such a series of changes, positive work is performed. In the system of conductors which the stream passes through, there is generated a certain amount of living force in the shape of heat. Necessarily there must be an equivalence between the work of the chemical forces and the heat simultaneously developed in the conductors and battery. A given amount of chemical action of a given kind must correspond to the generation of a constant amount of heat, whatever may be the constitution of the circuit and of the battery.

These theoretical conclusions are confirmed by a remarkable experiment by Joule and Favre. A large calorimeter, which was essentially simply an immense mercury thermometer, with two cavities in its bulb for the reception of bodies, was used by Favre, and the following determinations made: First, in one of the cavities was placed a simple galvanic element of zinc and platinum immersed in acid water, united by a very short copper wire. Thus was determined the amount of heat generated by one equivalent of zinc when decomposed, taking the equivalent at 66 grams. The mean of a number of good determinations showed that this amount of heat was sufficient to raise 37,360 grams of water one degree. Then the thick, short copper wire was replaced by a thin wire of considerable length, wound in a spiral. The decomposition of a given quantity of zinc then was found to give a much less quantity of heat, and the diminution was greater the longer and thinner the wire. The wire itself was notably heated, and when it was included in the other cavity, so that the total heat, both in the elements and circuit was determined, the sum was found to be precisely the same as in the first experiment. The decomposition of 66 grams of zinc, again, generated 37,360 heat units. Repeated in the most diverse ways, with conductors and elements of the most diverse kind, the same results were always obtained: so that in all cases in which the action of the current performed no outer work, the heat in the entire circuit and the chemical action were found to be perfectly equivalent.

If now the machine is in motion, living force is generated or work is performed outside of the circle, as, for example, the raising of a weight to a certain height. If the heat generated in the circuit should still remain the same, we should have at one time the work of the chemical forces in the battery equivalent to a certain quantity of heat, and, at another time, equivalent to the same heat increased by a certain mechanical work, which is plainly impossible. Accordingly, if by the action of the current in any system of spirals or electro-magnets, outer work is performed, there must be a diminution of the heat generated in the entire circuit by a given amount of chemical action, and this diminution must be the exact equivalent of the outer work performed. Experiment has confirmed this conclusion. In the second cavity of his calorimeter Favre replaced the conducting wire of the previous experiment, by a very small electro-magnetic engine, which, by means of a mechanism unnecessary here to describe, raised a weight. Under these new conditions the decomposition of 66 grams of zinc generated less than 37,360 heat units, and the observed difference stood in a constant relation to the work of the engine.*

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* See Note 19.

Every unit of heat was thus found to correspond to an outer work of 448 units. The difference between this number and the mechanical equivalent of heat, as determined by Joule, or as determined from the properties of gases, does not exceed the limits which may properly be ascribed to errors of observation.

VI.

In an electro-magnetic engine there is, therefore, a loss of heat as soon as mechanical work is performed, and hence it is with perfect propriety that we have classed such machines among heat engines. The mechanical power is due to a partial transformation of the heat caused by chemical action in the battery; just as, in the steam engine, it is due to a partial transformation of the heat caused by combustion of fuel under the boiler. In the one as in the other case, such transformation depends upon certain physical laws, which may be regarded as so many general consequences of the mechanical theory of heat. The study of the steam engine has revealed to us the condensation of steam when expanding; the study of the electro-magnetic engine makes evident to us the necessity of the phenomena of induction.

There is but one way in which we can comprehend how the motion of a machine can diminish the amount of heat generated in a conducting wire by a certain amount of chemical action. The generation of heat in a unit of time is proportional to the square of the intensity of the current, while the intensity itself is proportional to the amount of chemical action in the same time. It is evident that the heat generated by the decomposition of one equivalent of metal is directly proportional to the intensity of the current which causes this chemical action, or inversely proportional to the time of decomposition. If, thus, i is the intensity of current, and t the number of seconds required for the decomposition of one equivalent of metal, then, since the chemical action and intensity of current are proportional, the product $i\,t$ is equal to a constant, k. The heat generated by the decomposition of one equivalent of zinc is proportional to i^2t, or can be represented by ik, or by $\frac{k^2}{t}$. It is therefore necessary that the chemical action in a battery, the current from which works an electro-magnetic engine, must be lessened, and hence the intensity of the current diminished, by the motion of the engine. If a galvanometer is interposed in the current, its deviation during motion of the engine must be less than when the engine is at rest, and the difference will be greater, the greater the work of the machine corresponding to a given chemical action. This is completely confirmed by experiment. There can be no doubt as to the fundamental fact that the motion of an electro-magnetic engine diminishes the intensity of the current. What can be the cause of this diminution? Is it an increase of the resistance to the current? Is it a process similar to that which separates and puts in motion the two kinds of electricity in the battery?

An increase of resistance is impossible, because experiment has proved that the resistance of a conductor is the same, whether it is at rest or in motion. It is therefore necessary that, in a machine whose parts are relatively in motion, a current shall tend to give rise to an opposite current, or, using the customary expression, an electro-motive force shall be generated, opposed to that of the battery.

But whatever takes place in a machine in consequence of its motion, must take place also in any system of conductors and currents when in any sort of motion. If, therefore, a closed conductor is moved in the neighborhood of a magnet or current, the motion must cause in the conductor a current opposed to that which would have to pass through it, in order, by means of electro-magnetic forces, to continue such motion.

In this sentence you have already recognized one of the fundamental laws of induction, and it would not be difficult to prove, analogically at least, in similar manner, all those discoveries which have rendered so famous the name of Faraday. Induction currents, the existence of which seemed at first so marvelous to physicists, were observed by Ampère* ten years before Faraday, but without his daring to believe them. It was sought in vain to deduce them from the phenomena of static electricity. By means of the mechanical theory of heat they receive their true interpretation. The generation of induction currents are the means which nature employs in order to produce work in the electro-magnetic engine. The laws governing induction currents are such that the equation of living forces is fulfilled both for motion of the machine as well as for rest.

If we consider on one hand the known expression for the mutual action of two currents, and on the other the proportionality of the heat generated by the current to the square of the intensity, as given by experiment, and unite these two facts by the principle which we have deduced, we can determine generally both the direction and intensity of the induced current generated by the relative motion of a current and a closed conductor. We may in this way discover all the laws deduced by Neumann, in 1845, in an entirely different manner, upon which chiefly rests his scientific fame.

This remarkable relation between the mechanical theory of heat and the phenomena of induction was first made known by Helmholtz, in 1847.

VII.

Among the number of laws deduced in this manner by theory, and confirmed by experiment, is that of the proportionality of the induced current to the velocity of the motion producing it.† As the motion of an electro-magnet is accelerated, the electro-motive force of induction increases, and hence the current intensity of the battery diminishes. The absolute work which the machine can perform in a given time is therefore diminished, but in the same time the heat generated by the decomposition of a given quantity of zinc diminishes also; the fraction of the work of the chemical forces which is transformed into heat diminishes, and the portion equivalent to the work of the machine approaches unity the more the velocity increases. We may, therefore, by a suitable increase of the velocity, transform with any desired completeness the entire work of the chemical action, or, what is the same thing, the entire heat generated, into mechanical work.‡

Thus the electro-magnetic engine, which has so far proved in practice the most imperfect of all engines, is theoretically the most perfect and efficient. It only can utilize all the heat. It does not, however, follow that it is only necessary to conform to the requirements of theory, that is, to give the engine the greatest possible velocity, in order to make it practically available. The zinc and acids necessary for the battery are expensive, more so than that of the carbon used with them, and in spite of the theoretical superiority of the electro-magnetic engine, it is much more economical to consume this carbon as fuel in a steam or hot-air engine. This must be the case until we can obtain with less cost bodies which possess powerful chemical affinities, that is, substances which strive energetically to remain in chemical union, or to return into union. The solution of such a problem appears not much more probable than the discovery of deposits of native zinc, or of springs of sulphuric acid.

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* See Note 20, at the end of these lectures. † See Note 21. ‡ See Note 22.

VIII.

We have by no means exhausted all that may be learned from the electro-magnetic engine. We may arrive at results not less important than the preceding by assuming the ordinary action of the engine to be reversed, so that work is consumed instead of being produced.

If, for example, the current passes through the fixed spiral, and we unite the ends of the moving spirals by a conducting wire, so that one or more circuits are formed, the machine cannot be set in action without causing induction currents in these circuits.

These induced currents oppose a resistance to the motion of the machine, which increases the amount of that work necessary in order to work the machine with constant velocity. In the same time, the wires traversed by the induction current are heated, and the final result is the transformation of a certain amount of work into heat. The determination of these two quantities furnishes a new value for the mechanical equivalent, *J*.

It was with such experiments that Joule, in 1843, began his researches. He deduced a value for *J* of 452, deviating considerably, therefore, from the later and more accurate determinations. But, however considerable the difference may appear, we may safely assume that it can be entirely attributed to the difficulties of the determination and the imperfection of the apparatus.*

If we substitute for the wire spirals, composed of wire of greater or less length and fineness, in which the induction current is of small intensity, and therefore generates but little heat, a metallic plate of 0.01 meter in thickness, and of a diameter corresponding to the dimensions of the fixed electro-magnet, we shall find that the current is greatly increased and that a great amount of heat is generated. There must, therefore, be considerable expenditure of work.

This new form of the experiment is interesting on two accounts. From a theoretical point of view we notice that the principle of the equivalence of work and heat gives in this case a direct relation between the beginning and end of a series of effects, the intermediate portions of which are at present but little understood, which are very different in conductors of considerable dimensions from simple wires. In the second place, the heat generation is so considerable that it is possible to make it evident by instruments of suitable delicacy, and show it to a room full of persons.

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* See Note 23.

Some of you have, without doubt, recognized in these remarks the experiment of Foucault, which attracted so much attention. In order to derive from it all the information possible, it will be advantageous to give it two different forms.

First we give, by some simple mechanism, the metallic plate a great velocity, without allowing the current to pass through the conducting wire of the electro-magnet between whose poles the plate revolves. As soon as the desired velocity is attained, we let the current pass, and the plate, by the influence of the induced currents, is brought instantly to rest. The induced currents cease as soon as the plate comes to rest, but the heat remains. We may say that the final result of the experiment is that all the living forces which before belonged to the entire mass have been transferred to the molecules and become visible as heat. The sudden stoppage of the plate can be easily proved, and shows plainly the existence and intensity of the induced currents, but by reason of the magnitude of the number which expresses the mechanical equivalent, the heat developed is but slight and can only be made evident by the most sensitive instruments. In the second form of the experiment it is different. We let the stream pass through the electro-magnet, and then seek to put the plate in motion. The effort necessary for this is a visible sign of the resistance to be overcome. A few minutes of motion cause a rise of temperature of 50 to 60 degrees.

IX.

This experiment is the last which I shall borrow from physics. Before leaving the domain of this science let me direct your attention to this table, in which we have grouped those determinations of the mechanical equivalent which are the most reliable.

TABLE OF THE MECHANICAL EQUIVALENT.

Manner of Determination.	Theoretical Principles pointed out by	Experimental Data given by	Value of Equivalent.
General properties of air.....	Mayer Clausius	Regnault Molland von Beck	426
Friction.....................	Joule	Joule Favre	425 418
Work of steam engine........	Clausius	Hirn	418
Heat of induced currents.....	Joule	Joule	452
Electro-magnetic engines.....	Favre	Favre	443
Daniell battery..............	Bosscha	Weber Joule	420
Metal wire in circuit.........	Clausius	Quintus Icilius	400

The coincidence of these results is very satisfactory, and may be regarded as confirmation of the theory.

There are only two values which deviate greatly, the value 452, found by Joule in his experiments with induced currents, and the value 400, given by Quintus Icilius. I have already pointed out the imperfections of Joule's determination.

As to the value found by Icilius, it is sufficient to remark that his methods require the combination of a great number of accurate determinations which are independent of each other. It is, therefore, small matter of wonder, if his result deviates much from the number 425, which seems to be the mean of the other determinations.*

Let us now consider the application of the new theory in chemistry. In the three kinds of machines which we have considered, we see that the motive power is due to a consumption of heat. But from whence comes this heat, if not from the action of chemical forces? In the steam and hot-air engines we have heat generated by the chemical action of combustion, and while this heat calls into existence a series of physical processes, a part of it disappears and reappears in the shape of mechanical work.

In the electro-magnetic engine the transformation is direct. The heating effects of a certain amount of chemical action are diminished by the action of the opposed "induced" currents, by an amount exactly equal to the mechanical work performed. This difference, however, cannot conceal the fundamental identity of the three cases. In all three the motive force is either a direct or indirect transformation of chemical affinities.

These mysterious forces, which seem to elude all exact determination, thus come under the dominion of general mechanical laws, and are susceptible of numerical determinations. We cannot determine their actual intensities, that is, measure the accelerations which, in a given time, they impart to the atoms upon which they act; but their work in the composition or decomposition of any combination can now be determined with the same exactness as the work of falling water.

If, for example, one gram of hydrogen and eight grams of oxygen, of a certain temperature, are brought together under such circumstances as to cause their union, and if we then bring the nine grams of steam or water back to the original temperature, the amount of heat which must be imparted to outer bodies, multiplied by the mechanical equivalent, is the exact work of the chemical action, provided that the union is not accompanied by any outer work, that no living force is imparted to other bodies, or imparted to the bodies which take part in the chemical action. The case of an explosion which is accompanied by mechanical action is therefore excluded. You will readily recognize that this limitation is unavoidable, for, as we have seen in the electro-magnetic engine, a constant amount of chemical action can generate a different amount of heat, according as we have a simultaneous development of mechanical work or not.

I scarcely need to call your attention to the importance of this new point of view in thermo-chemical investigations. It forms at once a bond of union between chemistry and general mechanics. Nor is this one of those superficial and unfruitful remarks which are constantly uttered as to the universality of mechanical laws or the dependence of every phenomenon upon motion. We can give examples of chemical action which can only now be perfectly explained by mechanical considerations. Such examples are found in that part of chemistry which we may call electro-chemistry, and which is properly considered as equally belonging to chemistry and physics.

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* See Note 24.

You know that a current which passes through a compound conductor always decomposes it. You know, further, that every chemical action which occurs between two conducting bodies which form a closed circuit causes a current. Hence it seems evident that there must be decomposition when we put the poles of a battery in connection with two strips of platinum immersed in a compound conducting liquid. This conclusion is, however, inexact. The decomposition of water, for example, is impossible by means of a battery composed of zinc and platinum or copper, with water acidulated with sulphuric acid. Ordinarily, in order to increase the conductive power of a fluid, acid is added to it, but even this fails here. There is no decomposition, and no appreciable current flows through the apparatus.* These facts appeared for a long time incomprehensible, but it is easy to show that it is mechanically impossible under these circumstances to decompose the water. The negative work of the chemical affinities is, by such decomposition, greater than the positive work of the affinities in the battery.

We know that the decomposition of one equivalent of zinc in every dilute acid corresponds to 87,360 heat units. On the other hand, the combustion of one equivalent of hydrogen generates 84,460 heat units. It is clear that, in the battery with acid water, the negative work of the chemical affinities must be exactly equal and opposed to the positive work of the same affinities which in an apparatus serve for the combustion of the hydrogen. If, therefore, according to the laws of electro-chemistry, we assume that each equivalent of zinc decomposed in the battery causes the decomposition of one equivalent of water, or the formation of two equivalents of hydrogen, and if we take into account, also, the heat generated in the conductor by the current, we shall find in a system at rest a greater negative than positive work, and at the same time, in addition, a generation of heat, that is, of living force. This is a mechanical contradiction, the existence of which shows why decomposition cannot occur. For this explanation we are indebted to Favre.†

Undoubtedly this phenomenon appears essentially different from ordinary chemical processes. There is in the system a regular combination of substances acting upon each other, and also conductors which take no part in the chemical actions, and yet whose presence is absolutely necessary. All this does not seem to resemble very closely the reactions which go on in the test tube of the chemist. If, however, we recall the fact, which is to-day settled, that the action of acid hydrates upon metals is always a pure galvanic process, in which the metal, its impurities, and the acid form a galvanic chain, we shall probably be inclined to regard the difference as only accidental, and to see in this first application of mechanical considerations to electro-chemical processes the type of a series of applications which may in future extend over the whole domain of chemistry.‡

Just as electro-chemical phenomena find their explanation in the consideration of the heat effects of combinations, so the theory of electric currents allows in many cases of other determinations, in the place of calorimetric measurements, which may be made much easier by means of the galvanometer and the rheostat. From Ohm's laws, together with the laws of electric heating, we deduce that the total heat generated by the chemical action involved in the decomposition of one equivalent of zinc is proportional to that number which we call the electro-motive force. This at least is always the case when there is no disturbing action by reason of a gas developed upon the surface of a metal in the battery or in the circuit.

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* See Note 25. † See Note 26. ‡ See Note 27.

This relation, first clearly announced by Helmholtz in 1847, appears to have been discovered by Joule in 1841. It gives to the determination of electromotive forces especial interest, and has led Regnault to interesting conclusions as to the constitutions of metallic amalgams.*

We have long known that the electro-motive force of a galvanic battery is considerably increased when we substitute amalgamated for pure zinc, but thus far have been able to give for this peculiar result only still more peculiar explanations. Regnault remarked that amalgamated zinc generated more heat, in combining with oxygen and an acid, than pure zinc, and that, consequently, when the mercury is separated from the zinc, heat is developed. The opposite process—the formation of the amalgam—must therefore produce cold. The electro-motive force, on the other hand, is diminished when we substitute amalgamated for pure cadmium. The amalgamation of cadmium must therefore produce heat. Both these conclusions are perfectly confirmed by experiment. These phenomena find their explanation in the almost perfect identity in chemical properties of zinc and cadmium, and their great difference in latent heat. The two metals possess probably almost the same affinities for quicksilver, and their union with this substance must therefore involve almost equal quantities of heat. When the amalgam is decomposed, we have then, as the caloric result, only the difference between the heat developed by chemical action, and that absorbed during decomposition. We thus see how the zinc can produce cold and the cadmium heat, since the first metal requires for liquefaction about twice as much heat as the second. These considerations apply to all metallic amalgams, and are in accordance with experiment. (See the remarks of Regnault in the Comptes rendus, 1860, vol. li., p. 778.)

X.

But not only machines owe their moving force to the work of chemical affinities. The motive power of man and animals is due to the same cause. Breathing and all the chemical reactions which take place between the outer atmosphere and the body serve not only to preserve a constant temperature and to remove waste materials. Breathing is also the source of the capability which a living being possesses of moving exterior bodies, or, by means of some outer point of resistance, of moving itself.

However complex these chemical reactions may be, individually, their final result corresponds to the natural tendency of the affinities. This is a constant formation of water and carbonic acid at the expense of the hydrogen and carbon which exist, either in the body or in the food, in a combination in which their affinities for oxygen are not completely satisfied. The work of the chemical processes in breathing is thus essentially positive. If the animal is at rest, this work is the equivalent of the amount of heat which is constantly developed in order to replace the loss of heat due to radiation, contact with the air, and perspiration.* If the animal is in motion, a portion of the work of the chemical affinities is equivalent to the outer work performed, and only the remainder is converted into heat. Therefore, the same amount of chemical action causes in an organism at rest a greater amount of heat than when the organism is in motion.

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* See Note 28.

These ideas, which were first expressed by Julius Robert Mayer, in 1845, have, in fact, caused in general physiology an advance similar to that which was due to the discoveries of Lavoisier and Sennebier upon respiration, at the end of the last century. They have not remained purely theoretical; two different series of experiments have given very remarkable confirmation. The first is due to Hirn. It consisted in confining a man in a closed room, who for some time remained at rest. He then performed, for some time, work in a tread-mill, and in both cases the heat and chemical action of respiration were observed. Each time the heat generated and the carbonic acid exhaled were measured. The ratio of the first to the second was less during motion than during rest. A given amount of chemical action, therefore, developed less heat while work was performed than during rest. The difference, for each individual, was closely proportional to the work performed. The conditions of the experiment are, however, too complex, and the changes in the body too difficult of measurement, in order to determine in this way, as Hirn endeavored to do, the value of the mechanical equivalent of heat.

Béclard attacked the question in another way, and by means of an experiment which any one who possesses a good thermometer can repeat, showed that heat, in the organism, is transformed into work. By the simple application of such an instrument to the muscles of the arm, he showed that the heat generated by muscular contraction was always less when the muscles, during contraction, performed outer work, such as the raising of a weight. He showed, further, that the heat, on the other hand, is increased when the muscles support a weight, which, falling under the action of gravity, performs outer positive work.

The results of these two series of experiments are the most valuable by which experimental physiology has in recent times been enriched. It is, moreover, clear that they are in nowise contradicted by the daily experience that every bodily exertion is accompanied by heat. The contraction of a muscle increases undoubtedly the heat generated in an organism in a given time, but it increases also the consumption by respiration, so that, even without direct proof, we might conclude the necessity of nourishment as a consequence of work.

The investigations of Hirn and Béclard simply show that, in accordance with the theory of Mayer, the consumption increases in a greater ratio than the generation of heat. Every animal, every being endowed with voluntary motion, can thus be regarded as a heat engine. Every motion is but a partial transformation into work of heat, furnished by combustion of fuel in the shape of food, etc., perfectly comparable to that transformation which occurs in the electro-magnetic engine. If a living being can apparently increase at will the sum of the living forces surrounding him at any moment, it is only under the condition that the sum of the living forces of the heat generated by chemical actions in his own organism shall be diminished by a precisely equivalent amount. We may, in fact, say that he possesses only the power of directing the living forces which are being constantly generated within him by the action of chemical affinities; and in order to show the true nature of this power I cannot do better than to borrow from Mayer the comparison of the will in a sentient animal to the man at the helm of a steamship, which he indeed guides, but without being in any sense the physical cause of the motion. "The motion of the vessel," says Mayer, "obeys the will of the pilot and the engine-driver—the spiritual influence without which the ship could not move, or would be destroyed on the first reef. The pilot guides, but he does not move. For motion, the force in the coal is necessary, and without this the ship must remain at rest—dead—no matter how strong the will of the pilot."

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* See Note 29.

XI.

An essentially different field is opened to us by the vegetable kingdom. In the higher plants, at least, the final result of the life processes is opposed to the chemical affinities. Under conditions which continually tend to convert substances into carbonic acid and water, still the higher plants are constantly increasing the quantity of these substances already existing. The work of the affinities within them is therefore a negative one, and our, as yet, complete ignorance as to the mechanism of plant life need not prevent our giving complete assent to this conclusion, for it is, after all, only formulating that which goes on in every forest and on every meadow. Apparently without sustenance, and year after year, wood and grasses are produced, and removed by man. But this continual triumph of the vegetable kingdom over chemical affinities can only be sustained by an equivalent consumption of living force or of heat. Hence, for all vegetation, the direct or indirect action of the sun is an absolute necessity. Only infusorial plants and parasites are exceptions.* Neither the special refrangibility of those rays which are considered as especially favorable to vegetation,† nor the weakness of their thermometric action, distinguishes these rays essentially from those which are called "heat rays." That which the plant absorbs from the sun is heat—that is, living force; a vibrating motion of atoms distinguished only by period and amplitude of vibration from those which act upon the thermometer. By the consumption of this living force the amount of combustible material is increased. In the combustion of the products of vegetation we simply recover this living force, which opposed the chemical affinities and prevented combination. Thus, to a transformation of the sun's heat we owe the fuel we burn and the vegetable food which sustains the energies of man and beast. To a similar transformation we owe all the mineral fuel which sustains our industries. When we remember, further, that it is the sun which makes the wind to blow, which evaporates water, and causes rain and sustains rivers, you will recognize that not only the motion of the tides, but every motion upon this earth, has its origin directly or indirectly in the sun.

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* See Note 30.

† See Note 31. Verdet wrote the above lines under the erroneous impression that the more refrangible part of the spectrum was that which furnished the living force to the plant. The Note 31 gives those experiments and views which are now generally accepted.

XII.

This beautiful natural harmony turns our attention to the centre of our system, and leads us to the consideration of the astronomical applications of the new theory.

You are all familiar with the hypothesis of Buffon as to the origin of the sun's heat. According to him, the unceasing fall of comets upon the surface of the sun furnishes the materials of combustion. The more we have learned about the motions of comets, and the more we have departed from the idea that the sun is a furnaçe similar to our.artificial sources of heat, the more has Buffon's hypothesis been forgotten. The mechanical theory of heat has revived it again and rejuvenated it. Mayer first called attention to the fact that a body arriving at the sun would lose, at the moment of its impact, the enormous quantity of living force due to the action of gravity, and that this loss of living force must cause a development of heat. It is sufficient, therefore, to account for all the heat which the sun gives out into space, if its mass is continually growing by the addition of comets, aerolites, and other cosmical bodies.

William Thompson, who has developed and followed up this idea of Mayer with equal sagacity and boldness, believes that these bodies, which by their fall heat the sun, probably come from that immense cloud which surrounds it, to which astronomers give the name of "zodiacal light." Assuming this, he was able to calculate the mass which must yearly fall upon the sun, in order to compensate the loss of heat which follows, from the researches of Pouillet upon the thermometrical effects of the sun's rays. If this mass has the mean density of the sun, it would form a layer of only twenty meters in thickness. This thickness is considerably less, if we assume with Watterson that the mass which the sun attracts streams in from all quarters of space. In either case there is an inconsiderable increase of diameter, which would elude the closest scrutiny for many years. Even according to Thompson's estimate, it would require no less than four hundred years in order that the angle subtended at the sun should be increased by one-tenth of a second.

But another consequence of the hypothesis allows of an easier test by experiment. The sun turns upon its axis in about 25 days. Every foreign substance which unites with it diminishes its velocity of rotation. Thompson has calculated the thickness of the layers gradually deposited upon the sun's surface, and found that the retardation would amount to one hour in 58 years. Unfortunately, we cannot at present determine the time of rotation within an hour. It is very difficult of determination, as it must be found by observation of sun spots, which have also a proper motion of their own, as well as sharing the motion of the sun. The influence of this proper motion can only be eliminated by long series of observations. This second confirmation therefore is at present impossible, and would appear to be so for a long time to come. But it is not, like the first, deferred to an infinite time.

The fundamental idea of the mechanical theory of heat has been combined with the hypothesis of Laplace as to the origin of the solar system, and we thus obtain another explanation of the heat of the sun and planets. The endeavor has even been made to deduce in this manner the age of the sun. I do not require that you shall follow me in speculations of this kind, which may perhaps appear as extremely hypothetical, or for which the test of experience appears too distant; but I would point out how far the scope of the new theory extends.* In this connection it has been said that science is on the road to the discovery of new laws, as fundamental and as accurate as those made known by Newton in his generation. You may perhaps be inclined to see nothing chimerical in this view.

XIII.

I cannot, however, allow you to believe that these laws are yet discovered. Now that I have indicated all that the new theory teaches us, I must also call your attention to what it compels us to neglect. The principle of the equivalence of heat and work is only one form of the equation of living forces. A special advantage connected with the application of this equation is, that it allows us to express relations between different conditions of the same system which are independent of intermediate conditions. Its disadvantage is, that it shows us nothing as to these intermediate conditions. This is, strictly, the true character of the new theory. It teaches us the "why" and the "how much," but it does not answer the "how." Thus we know indeed that steam, in expanding, transforms a part of the heat which it contains into work or living force; we know that induced currents are necessary in order that both motive power and heat may be furnished; but in both cases the process itself, the play of the elementary forces, is unknown. It is much to have determined the true nature of a problem, and to have confined within fixed limits the field opened out by hypothesis. The application of the mechanical theory of heat to gases has led directly to the discovery of a theory of their constitution, which at least expresses very satisfactorily the known facts. We may justly hope that this will not be the only example, and that the new theory, after showing the necessary connection between phenomena, will also aid us to penetrate into the inmost secrets of nature.

XIV.

The importance which we must now attribute to the new theory renders it necessary as well as desirable that I should give a short history of it, in which I shall endeavor to do justice to the principal discoverers. This is so much the more necessary, inasmuch as I have thus far given only its ideas in logical order, without reference to the historical sequence of the discoveries.

We can distinguish two periods in this science. In the first, which reaches up to the year 1842, similar ideas of the mechanical heat theory were held by different authors, but the facts thus explained were soon regarded from different points of view, and attempts were made to refer them to general laws. The real principle, however, was not discovered, and all these attempts remained unfruitful and without essential influence upon the progress of science. The work of this period bore, nevertheless, its fruits in due time, and, as often happens with great discoveries, was revived again about the year 1842, when several geniuses expressed the new ideas with sharpness and clearness. Shortly after the beginning of that period of rapid progress which always attends the discovery of a true principle, a few years sufficed to erect the shapely structure, which I have endeavored hastily to show you over.

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* See Note 22.

The first name among the list of those whom we can call the forerunners of the mechanical theory of heat, is the famous one of Daniel Bernoulli. The Hydrodynamics of this great geometer and physicist, which for more than a century remained neglected and forgotten, contains the theory of the constitution of gases to which I have already referred. His contemporaries probably saw in it only the effects of the old Cartesian hypothesis, and not until recent times has it been recognized that here was the germ of a new science.

In the year 1780, somewhat more than forty years after the publication of the Hydrodynamics, Lavoisier and Laplace, while speaking, in their treatise upon heat,* of the two hypotheses which can be formed of this physical agent, expressed themselves as follows: "Other physicists believe that heat is only the result of imperceptible vibrations of matter. . . . . In any system heat is the living force of the imperceptible vibrations of the molecules of a body; it is the sum of the products of the masses of each molecule into the square of the velocities. . . . . We do not assume to decide between the two hypotheses. Many phenomena seem to sustain the last; as, for example, the heat generated by the friction of two bodies; but there are others which seem better explained by the first. It may be that both hold good." But after this plain and clear definition we find nowhere in the treatise any attempt to compare the living forces of heat with ordinary living force, such as the rotation or motion of the center of gravity of a body. They never compare heat with anything else than itself, and it contributes accordingly little to the value of their remarks whether heat is regarded as an indestructible matter or as a quantity of living force.

Indeed they go further, and regard as proved a principle which is in direct contradiction to that of the transformation of heat into work. "All changes of heat," they say, "whether actual or only apparent, which any system of bodies undergoes while changing its condition, are repeated in reverse order when the system is brought back to its original condition." If they had added, *provided such change of condition is not accompanied by outer work,* the mechanical theory of heat would have been founded. But without such proviso this assertion of Lavoisier and Laplace is an error, which is daily confuted by the steam or electro-magnetic engine.

It is impossible to say how the views of Lavoisier upon this question would have changed had he lived longer. We conclude from his treatise upon chemistry, that up to the year 1789 he had not completely given up the theory that heat consists in a motion of molecules.

It is indeed true, that, probably in deference to the general opinion, he spoke of gases as if they were composed of a union of certain bases with caloric. But he continually made limitations of which we find no trace in the writings of his scholars, and it was not without some hesitation that he placed light and heat at the head of the list of simple bodies.

As to Laplace, his views underwent rapid change. In all which he wrote during his connection with Lavoisier he appeared as the ardent advocate of the materiality of heat. His weighty authority alone procured belief for the theory, which rested upon not the slightest proof.

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* Mémoire sur la chaleur. Mémoires de l'Académie des Sciences, 1780, p. 357.

Toward the end of the last century, about the years 1798 and 1799, two experiments were made which sufficed to show the untenability of the theory espoused by the author of the "Mécanique Célèste." These were the famous experiments of Rumford and Davy upon heat generated by friction. Rumford, at the foundry at Munich, measured as exactly as he was able the heat generated in boring a cannon, and in order to leave no doubt as to its origin, he determined the specific heat of the bronze and of the borings. There appeared no perceptible difference between the two, and thus the only reasonable explanation which could be offered by the material theory of heat was negatived decisively.

It had, in fact, been assumed that in the pulverized bodies the specific heat was much less than in the same bodies solid, and it followed indeed from this assumption that the pulverization of a body by the friction must be accompanied by heat. But it was forgotten that, in such case, friction itself must create heat when there is no change in the rubbing surfaces. The experiment of Rumford showed, moreover, the incorrectness of any such assumption.

The experiment of Davy, about a year later, showed, if possible, still more conclusively the error of the old theory. Two pieces of ice, rubbed together, melted very rapidly, and formed a liquid whose specific heat was more than double that of the ice. Davy also used every precaution in order to show that the generation of heat was not accompanied by any noticeable absorption in any part of the apparatus.

Among the contemporaries of Rumford and Davy, Young appears to have been the only one to fully realize the scope of these experiments. In his lectures on natural philosophy, published in 1807, he placed them among his immortal discoveries as to the nature of light, and he all but reached the true principle of the mechanical heat theory. He was the first to cast doubt upon the principle of Lavoisier and Laplace, to which I have alluded. "Probably not in a single case," he says, in his lecture upon the nature of heat, "is the heat absorbed exactly equal to the heat given up in the reverse process." In this simple doubt lies concealed the essential principle of the mechanical theory of heat. Young, indeed, admitted the probability of the equivalence of the absorbed and generated heat, but the simple expression of doubt upon an axiom of this character in the year 1807 is noteworthy.*

Unfortunately, this was the period when the law of double refraction was looked upon as an argument in favor of the emission theory; the same period in which the elegant treatises of Fresnel remained forgotten. Even when, in the year 1824, the original genius of Sadi Carnot, awakened by the industrial revolution inaugurated by the steam engine, sought to unfold the general laws of heat, he accepted without question the materiality, and therefore the indestructibility, of heat as his starting-point.† It may perhaps surprise you when I add, that in spite of this fundamental error, the names of Sadi Carnot and of his learned commentator, Clapeyron, occupy distinguished places in the history of the science. To Sadi Carnot we owe the methods of discussion of which the mechanical theory of heat makes use. In his writings we find the first examples of the cycle process—of that series of changes by which a body passes in a determinate manner from one condition to another, and then in another determinate manner returns to its initial condition.

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* Lectures on Natural Philosophy, vol. i., p. 651, edition of 1867.

† Mémoire sur la puissance motrice de la chaleur. Journal de l'École polytechnique, vol. xiv., p. 170, 1834. Poggendorff's Annalen, vol. 59, p. 446.

Clapeyron has cleared up the obscurity of Carnot's treatise, and showed how to treat analytically and represent geometrically this new and fruitful method of treatment. These two have to a certain extent created the logic of the science. As true principles were discovered, they only needed to be subjected to the forms of this logic, and it is easily conceivable that without the work of Carnot and of Clapeyron the advance of the new theory would have been much less rapid.

Finally, we close this first portion of our historical sketch by recalling that Seguin, in a work published in 1839,* of more political than physical interest, has given views, as to the steam engine, closely related to those by which, in our first lecture, we have sought to render plain to you the transformation of heat into work.†

I come now to those labors by which, since the year 1842, the science has been built up. These labors are more especially those of three men, who without connection with one another, without even knowing each other, at the same time and in almost the same manner arrived at the same results. The priority in sequence of publication belongs undoubtedly to the German physician, Julius Robert Mayer, whose name we have already had frequent occasion to mention, and it is interesting to note that it was through observation of facts occurring in his medical practice that he was led to recognize a necessary equivalence between heat and work. The changes in the color of arterial and venous blood turned his attention to the theory of respiration.‡ He recognized at once the breath as the origin of the motive power of animals. The comparison of animals to heat engines led him gradually to the discovery of the important principle with which his name is forever associated. This, at least, is the account which he himself has given us in his writings of the development of his ideas.

We find, moreover, in these writings, the first determination of the mechanical equivalent of heat, deduced, in perfect accordance with principle, from the properties of gas, but incorrect in value because the true values of the coefficient of expansion and the specific heat of air were then very imperfectly determined. Mayer's papers, "Die organische Bewegung in ihrem Zusammenhange mit dem Stoffwechsel," and "Beiträge zur Dynamik des Himmels," which last appeared in 1848, contained also physiological and astronomical applications which show that he clearly appreciated the scope of his discovery.

About the time of the first publication of Mayer, a series of articles by Colding, an engineer at Copenhagen, were presented to the Royal Academy of Sciences at Copenhagen, which contained ideas upon the power of steam and hot-air engines very similar to those of Mayer, as well as an experimental determination of the mechanical equivalent of heat by friction, which does not seem to have been very exact. This is sufficient ground for including his name among those of the founders of the new theory. But it is readily conceived that the contributions of this physicist, written in a language but little known, and first printed several years after their receipt, had but little influence upon the development of the science.

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* Études sur l'influence des chemins de fer, p. 180. Paris, 1839.

† See Note 33.

‡ See Note 34.

The third discoverer of whom I have to speak, Joule, is perhaps the one who has contributed the most toward the proof of the new principles and their final reception. His first work appeared in 1843, and is undoubtedly later than the first works of Mayer and Colding. It contains experiments upon the heat generated by induced currents, and appears to have excited at first but little notice. His experiments in 1845 upon the heat effects of expansion and contraction of gases, were the first to give him the fame of introducing new ideas into science. His experiments upon friction gave the first reliable determination of the mechanical equivalent of heat. His views upon gases gave the first, and, until now, only complete explanation of a phenomenon whose laws could be laid down by theory without disclosing its mechanism.

Immediately after these three names we must place that of Helmholtz, who, in 1847, in his article, "The Conservation of Force," first united the new ideas into a complete structure, and drew from them fruitful and important applications to induction phenomena, electro-chemistry, thermo-electric currents, etc.

The development proper of the mechanical theory of heat as a science, the clear and methodical presentation of methods of investigation and discussion, and, finally, their application to machines, is due to four savans, whose names are the last that I shall mention; Clausius, Macquorn Rankine, William Thompson, and Gustav Zeuner. Their most important investigations extend from 1849 to the present day.

Many other workers might be mentioned. I have already had occasion to notice several in the course of these lectures; I will not seek to extend the list, but shall content myself with the names of those investigators who have laid the foundation stones of the edifice, upon whose completion so many have for the last thirty years labored with such signal success.

NOTES

AND

ADDITIONS TO THE LECTURES.

CONTENTS.

1. Upon perpetual motion.—2. Upon the origin of the motive power in the steam engine under the hypothesis of the materiality of heat.—3. Some experiments of Hirn which apparently contradict theory.—4. Upon a theorem of Coriolis.—5. Upon the law of expansion of gases.—6. Upon the inner work in crystals and some liquids.—7. Upon an incorrect determination of the mechanical equivalent of heat.—8. Upon bodies which contract under the action of heat.—9. Calorimetric measurements in which the outer work has not been considered.—10. Theory of the constitution of gases.—11. How gases and vapors perform outer work.—12. Upon the value of the mechanical equivalent given by carbolic acid.—13. Principle of the experiments of Thomson and Joule upon the heat phenomena of moving gas.—14. Upon the condensation which accompanies the expansion of steam.—15. Upon the regenerator in gas engines.—16. Determination of the economical coefficient for Ericsson's engine and for the engine without regenerator.—17. Upon gas engines in which the temperature sinks to the absolute zero.—18. Upon the necessity of the tendency of heat to pass from a warmer to a colder body.—19. Upon the rôle played by friction in the electro-chemical investigations of Favre.—20. The discovery of induction phenomena.—21. The deduction of induction phenomena from theory.—22. Upon the complete transformation of heat into work by the electro-magnetic engine.—23. Determination of the mechanical equivalent of heat by the electro-magnetic engine (Joule).—24. Upon the nature of the electro-magnetic and electro-dynamic forces.—25. Electrolytic convection.—26. Upon the polarization of the electrodes.—27. The decomposition of zinc in dilute acids.—28. Upon the application of the measurement of electro-motive forces to thermo-chemical investigations.—29. The influence of the friction of the blood upon animal heat.—30. Upon vegetation which takes place in the absence of light.—31. The absorption spectrum of chlorophyl and the influence of the color of light upon the growth of plants.—32. Remarks of Mayer upon the ebb and flow of the tides.—33. Upon a demonstration of Seguin relating to the steam engine.—34. The dependence of the color of venous blood upon the temperature.

Addition.—The entropy of the world tends towards a maximum.

NOTE 1.—(Page 5.)

THE PROBLEM OF PERPETUAL MOTION.

In accordance with usual custom, I have shown the impossibility of perpetual motion to be a consequence of the fundamental principles of mechanics, as well as of the manner in which the forces of nature act.

We may, however, also recognize in this an independent and apparently self-existent principle, which, at bottom, expresses nothing else than the necessity of a definite relation between cause and effect.

Considered thus, the principle of the impossibility of perpetual motion may be used to prove that all natural forces must act in the line joining any two mutually interacting material points, and that these mutual actions are functions of the distances apart.

This is the method adopted by Helmholtz in his famous Treatise "Die Erhaltung Kraft" (Berlin, 1847)—a method which may seem to many the best. Helmholtz says:

"Let us consider, first, a material point of the mass m, which moves under the influence of the forces of any number of bodies composing a fixed system A, then we can determine for every moment the position and velocity of this point. Let the time t be the primitive variable, and, dependent upon it, let the ordinates of m, with reference to a co-ordinate system fixed with respect to the system A, be x, y, and z, the tangential velocity be q, the three components of the same

$$u = \frac{dx}{dt}, \qquad v = \frac{dy}{dt}, \qquad w = \frac{dz}{dt},$$

and finally the components of the acting forces

$$X = m\frac{dx}{dt}, \qquad Y = m\frac{dy}{dt}, \qquad Z = m\frac{dz}{dt}.$$

Our principle requires that $\frac{1}{2}mq^2$, and hence q^2, shall be always the same when m has the same position with respect to A, and therefore not only a function of t, but also a function of the co-ordinates x, y, z, alone, that is,

$$d\,(q^2) = \frac{\delta\,(q^2)}{\delta x}\,dx + \frac{\delta\,(q^2)}{\delta y}\,dy + \frac{\delta\,(q^2)}{\delta z}dz. \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Since

$$q^2 = u^2 + v^2 + w^2$$

$$d\,(q^2) = 2udu + 2vdv + 2wdw.$$

If here, instead of u, we put $\frac{dx}{dt}$, instead of du, $\frac{Xdt}{m}$, and also for v and w similar values, we have

$$d\,(q^2) = \frac{2X}{m}\,dx + \frac{2Y}{m}\,dy + \frac{2Z}{m}\,dz. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Since equations (1) and (2) must be simultaneous for every dx, dy, dz, we have

$$\frac{\delta\,(q^2)}{\delta x} = \frac{2X}{m}, \qquad \frac{\delta\,(q^2)}{\delta y} = \frac{2Y}{m}, \qquad \frac{\delta\,(q^2)}{\delta z} = \frac{2Z}{m}.$$

If, however, q^2 is a function of x, y, z, it follows that X, Y, Z, that is, the direction and intensity of the acting force must be also only functions of the position of *m* with respect to *A*.

If we conceive instead of the system *A*, a single material point *a*, it follows from the above that the direction and intensity of the force which *a* exerts upon *m* is determined only by the relative position of *m* with respect to *a*. Since now the position of *m* is determined by the distance between *m* and *a*, in this case the direction and intensity of the force functions must be this distance *a*. If we conceive the co-ordinates referred to any system whose origin is at *a*, we must have

$$md(q)^2 = 2Xdx + 2Ydy + 2Zdz = 0 \quad . \; . \; . \; . \; . \; . \; . \quad (8)$$

$$d(r^2) = 2xdx + 2ydy + 2zdz = 0$$

$$dz = -\frac{xdx + ydy}{z}.$$

This value, in (8) gives

$$\left(X - \frac{x}{z}Z\right)dx + \left(Y - \frac{y}{z}Z\right)dy = 0,$$

hence $$X = \frac{x}{z}Z \quad \text{and} \quad Y = \frac{y}{z}Z,$$

that is, the resultants must be directed towards the origin.

Hence in a system which is subject generally to the law of the conservation of living force, the simple forces of the material points must be central forces.

NOTE 2.—(Page 14.)

THE ORIGIN OF THE MOTIVE POWER IN THE STEAM ENGINE UPON THE HYPOTHESIS OF THE MATERIALITY OF HEAT.

Sadi Carnot, assuming the materiality of heat, has given an explanation of the phenomena in the steam engine, which, although it does not agree with reality, is not so evidently erroneous as the hypotheses which have been framed in order, by the same hypothesis, to account for the heat generated by friction.

According to him, the imponderable fluid whose presence in bodies gives rise to those various effects which are called "heat," has an inherent tendency to pass from a hot body to a colder one, just as heavy bodies tend to fall from a high place to a lower; or rather, there is a similar tendency due to the action of the heat molecules on each other and the actions upon them of the ponderable molecules. Thus, the forces which act upon the heat molecules, furnish positive work whenever there is a transfer of heat from a hot body to a cold one, which cannot indeed be a priori determined, but which is, however, comparable to the work of gravity in a waterfall.

This is the true motive work in the engine. The heat from the boiler to the condenser experiences a kind of fall (this is Carnot's expression), and the work furnished by the engine is the equivalent of this mechanical process, just as the work of a water-wheel is the equivalent of the fall of the stream.

These views have in them nothing at variance with common sense, or which contradicts the general view of the phenomena; but it is evident that the assumption of the materiality of heat involves that of its indestructibility, and hence, in the case of the steam engine, gives rise to the following dilemma, the solution of which must be demanded from experience—either heat is something material, and then the steam must transfer as much heat to the condenser as it takes from the boiler—or heat is a motion of some kind, and then a part of the heat during the action of the engine must disappear, and thus give rise to outer work.

We have seen what answer experience has given.

NOTE 3.—(Page 14.)

SOME EXPERIMENTS BY HIRN WHICH APPARENTLY CONTRADICT THEORY.

The investigations of Hirn were undertaken in consequence of a prize offered by the Physical Society of Berlin for a numerical determination of the true value of the mechanical equivalent of heat. In the report of Clausius to the society, he calls attention to the error of Hirn's views upon the steam engine, and gives a correct explanation of the experiments.

Hirn did not agree with the views of Clausius, and, although when his treatise was published, he gave the report of the learned physicist in full, he sought to defend the correctness of his first calculations, and endeavored to justify them by two different methods of experiment—viz., by measuring the heat used by a steam engine without expansion, and by investigating the heat phenomena which accompany the efflux of a gas under high pressure into a vacuum.

It may not, perhaps, be without profit to show what the value of these new arguments amounts to.

In the following, we give the words in which Clausius refers to the incorrectness of Hirn's views :

"It can be easily shown how this error of Hirn arises.* He says, in justification of his assumption, 'when steam condenses under the same pressure at which it was generated, it gives up during condensation as much heat as must have been imparted in its generation.' This principle is indeed correct, but it has no application to the steam engine.

"When, in an engine working without expansion, the steam has entirely filled the cylinder back of the piston, and then the communication with the condenser is opened, only at first does the steam flow under full pressure into the condenser, and then the pressure gradually decreases as the steam still in the cylinder expands. By this expansion the steam still in the cylinder is considerably cooled, and, if not superheated or heated from without, will be partly condensed, even while still in the cylinder. In order to comply with the conditions implied in the above principle, the piston, during the efflux, should return with just such velocity as to keep the steam still in the cylinder always at full pressure. But then the back pressure would have to be as great as the driving pressure was, and no work could be obtained. If the author had extended his experiments to engines without expansion, he would undoubtedly have found for these also that the amount of heat given up is less than that received."

These last words, without doubt, led Hirn to make an experimental investigation of a steam engine without expansion.†

He does not appear, however, in this new investigation to have succeeded in overcoming all its difficulties, as he himself says, "the physicist may meet in experimental science with insurmountable obstacles."

He says, indeed, that he has established that, in an engine without expansion, the heat expenditure is either zero or can be neglected, but together with the experiments which give this strange result, he gives the data of another from which even still stranger conclusions may be drawn.

In an engine in which expansion occurred only through the fifth part of the stroke, not only was work performed, but also heat generated.

That the new methods of experiment which lead to such conclusions are to be preferred to those used by Hirn in his first experiments, is more than doubtful.

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* Fortschritte der Physik, 1855, Bd. xi., p. 21.

† Recherches sur l'équivalent mécanique de la chaleur, par Gustave Adolphe Hirn, Paris, 1858, p. 179.

The clear and decisive criticism of Clausius holds in full force.

Hirn opposes to Clausius also the following experiment: Into a receiver of black lead, surrounded by cold water, he allowed a jet of steam under high pressure to enter, the temperature of which was measured by a thermometer just before the steam reached the orifice.

He then collected the water condensed in a given time, and from the rise of temperature of the calorimeter, applying the necessary corrections, found the heat given up during condensation.

In this way a greater number was always found than that given by the expression

$$p\,[606.5 + 0.305t + 0.4805\,(T - t) - \tau],$$

which gives the heat contained in the steam at the moment at which it arrives at the orifice—where p is the weight of the steam, T its actual temperature, t the temperature at which, under the actual pressure, it would be saturated, and τ the temperature of the condensed water, assuming, according to Regnault's experiments, that 0.4805 is the specific heat of the steam.

Similar experiments, in which he used the condenser of a steam engine as a calorimeter, gave the same result.

Hirn concluded that saturated or superheated steam which condenses in a cooling vessel in which the pressure is less than the initial pressure of the steam generates heat.

The fact is remarkable and interesting, but easily to be accounted for.

The steam which leaves the orifice of efflux has a very great velocity, the liquid which results from the condensation is at rest. In the transformation from the gaseous to the liquid condition, then, a large amount of living force disappears, and there is, therefore, according to the new principles, a generation of heat.

It is, indeed, true that the outer work performed on the steam during its condensation is less than that which it performed during its generation, and this diminishes the heat generated during condensation, but there is no exact compensation. If, therefore, the steam which enters the calorimetric apparatus is saturated steam of five atmospheres, we must impart to each unit of weight of water of the temperature τ, in order to generate it, $651 - \tau$ heat units. A part, q', of this heat goes to increase the living force of the molecules. A second part, q'', corresponds to the change of aggregation or disgregation work. A third part, q''', is the equivalent of the outer work.

The last part, q''', can be taken equal to nearly 44 heat units, if we take for the absolute density of saturated steam of one atmosphere the value 1/[illegible], theoretically determined by Clausius,* and if we neglect the very small difference between the volume of water at τ degrees and at zero.

On the other hand, the recent labors of Minary and Resal† enable us to determine the weight of steam which will flow in five minutes from a boiler under five atmospheres' pressure, through an orifice of 0.007 meter diameter, when this orifice is at the end of a pipe of 0.15 meter diameter. This steam weight is 10.6 kilograms.

Hence we can easily find, with the preceding value for the density, and taking for the coefficient of contraction 0.44 (a value given by the experimenter) that the velocity of efflux is about 600 meters per second, and hence that every kilogram of steam which issued in Hirn's experiment carried with it a living force of about 180,000 meter-kilograms, equivalent to about 400 heat units. We see, therefore, that even without outer work there is more than compensation, and that the disappearance of the living force is more than sufficient to explain the phenomena observed by Hirn. Even a considerable error in the coefficient of contraction would not affect the conclusion.

It is worth remarking that the living force which the steam possesses when it leaves the orifice, is itself a transformation of the heat which the boiler possessed, and that hence the steam, at the moment it leaves the orifice, can no longer be in the same condition it had when in the boiler at a distance from the orifice.

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* The theoretical values of Clausius (Abhandlungen, Bd. i., p. 72) are confirmed by the experiments of Fairbairn and Tate. (Proc. of the Royal Soc., 1860, in Phil. Mag., 4 ser. vol. xxi., p. 230, and Comptes Rendus, vol. lii., p. 706.)

† Annales des mines, vol. xviii., p. 653.

NOTE 4.—(Page 16.)

UPON A THEOREM BY CORIOLIS.

The following theorem, given by Coriolis in his classic work upon the calculation of the delivery of engines,[*] is, in a certain sense, an illustration of the general law which we have sought to enunciate:

> "The sum of the living forces of a system of molecules, whatever may be the kind of motion, may be divided into three parts:
>
> "1. The living force of all the molecules when concentrated at the center of gravity of the system.
>
> "2. The sum of the living forces of these molecules, when we assume that they constitute, in the same relative positions in which they occur, a body of invariable form to which is imparted the mean motion about the center of gravity.
>
> "3. The sum of the living forces of the molecules, by reason of the relative velocities which they possess with reference to co-ordinate planes which partake of the mean motion of rotation."

In the equation of works we have usually to take account only of the two first portions, that is, the living forces due to the rectilinear and rotary motion of the body. The third portion is usually submitted to calculation only when the vibrations are sensible or apparent, as in the case of sound vibrations.

The fundamental idea of the new theory is to seek this third part in the heat.

It is, moreover, evident that the action of mechanical forces in most cases will give rise to all three kinds of living forces, and that we have just as little reason to neglect the changes of the living force of heat as of the outwardly visible living forces. We may even add that the transformation of the outwardly visible living forces into the living force of heat takes place incessantly in nature before our eyes, and that it is chiefly in this way that the vibrations of a system about a stable position of equilibrium are extinguished.

NOTE 5.—(Page 17.)

THE LAW OF EXPANSION OF GASES.

The law of expansion of gases was held by all physicists, down to Magnus and Regnault, as exact. It is generally known as the law of Gay-Lussac.

It is, in my opinion, more correct to call it the law of Charles. The essential part of this law, viz., the approximate agreement of the expansion of different gases, and hence the proportionality of all these expansions to the temperature, as determined by a thermometer, which is itself formed by a gas, was proved by Charles in the simplest manner.

The reservoir of a kind of mercury barometer was filled with gas. The apparatus was exposed successively to the action of two different temperatures (the ordinary temperature of exterior objects and that of boiling water), and the rise of the mercury in the barometer tube observed. Charles found that this rise for air, oxygen, nitrogen, hydrogen, and carbonic acid gas was the same, and no more was needed to establish the fact that the coefficient of expansion of these different gases is nearly the same, even if, in this manner, the exact value of the common coefficient could not be determined.[†]

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[*] Coriolis, Traité de la mécanique des corps solides et du calcul de l'effet des machines, 2d ed., p. 92.

[†] The experiments of Charles are mentioned by Gay-Lussac himself in his article upon the expansion of gases, Ann. de Chim., vol. xliii., p. 157.

Gay-Lussac has added to this simply a determination of the coefficient of expansion, which was inexact by about $\frac{1}{30}$th.

We may even say that the advance of science was in some degree retarded, in that he regarded as an absolute law that which was only an approximate expression.

According to Charles, the compound gases do not expand as much as those just named. It is not known exactly to what gases Charles thus refers, but it is probable to the same as those experimented upon by Gay-Lussac, viz., sulphurous acid and hydrochloric acid gas, for which he gave the same coefficient as for air.

We know now that the coefficient of expansion for sulphurous acid is $\frac{1}{15}$th greater than for air. In this important point, then, Charles was more accurate than Gay-Lussac, and however imperfect his method of experimenting may be considered, it is not open to the charge of not being able to distinguish differences in the quantity to be measured of $\frac{1}{15}$th.

NOTE 6.—(Page 18.)

THE DISGREGATION WORK IN CRYSTALS AND SOME LIQUIDS.

In liquids and non-crystalline solids it is possible that for a simple rise of temperature only, when there is no change of volume, there is no disgregation work.

It is, without doubt, essentially different for crystalline solids, at least for those belonging to the tesseral system The unequal expansion in different directions, caused in these bodies by the action of heat, does not allow the assumption that, when the expansion is prevented by a sufficient increase of pressure, there is no change in the arrangement of the molecules.

If, for example, a crystal of kalkspar is heated, and at the same time compressed in such manner that its volume remains constant, the crystal tends to elongate in the direction of its principal axis, and to contract in a direction perpendicular to this. It is certain that even if there is no change of volume there is a change of shape, and hence disgregation work. Even if, by a suitable distribution of pressure and tension upon the surface, not only change of volume but also of shape is prevented, still there may be a change in the relative direction of the molecules, if not in the relative position of the center of gravity.

This, at least, seems extremely probable from the change of optical properties in different directions, caused by the action of heat upon the crystal, and which does not seem accounted for by simple inequality of expansions.

It is to be expected that, even in a liquid, every change of temperature must be accompanied by a perceptible disgregation work, even when the volume does not change, when the liquid is near the point of solidification, and when, therefore, the lawless arrangement of molecules, characteristic of the liquid condition, tends toward a regular arrangement, if not of the entire mass, at least of its different parts.

We see, thus, how careful we must be before we assume that the disgregation work, under given conditions, is zero. The invariability of the mean distances of the molecules is by no means a guaranty of this.

Thus, for example, water cooled below the temperature corresponding to its maximum density, has, for the same pressure, the same volume at two different temperatures four degrees apart. The outer work between these two conditions is zero, but nothing justifies us in assuming the disgregation work as zero also. We can scarcely comprehend the anomaly of maximum density otherwise than by assuming that the relative direction of the molecules approaches law and definite arrangement the more the freezing-point is approached, and that, when for two different temperatures the volume is the same, but the arrangement of molecules different, the transition from the one temperature to the other is accompanied by perceptible disgregation work.

NOTE 7.—(Page 18.)

AN INCORRECT DETERMINATION OF THE MECHANICAL EQUIVALENT OF HEAT.

To several physicists, among them Kupffer and Masson, the following considerations have appeared permissible, and the values thus obtained for the mechanical equivalent do not appear to deviate much from the true value.

Let P be a tension, which, when applied uniformly to the surface of the unit of volume of a body, will cause an expansion, Δ, equal to that caused by a rise of temperature of 1 degree.

The work of this force which must be applied in order to cause the extension required is $P\Delta$.

On the other hand, we must impart to the body, in order to make it expand through the same distance, a quantity of heat equal to the product of the specific heat for constant pressure, c_p, into the weight of the unit of volume, that is, the density D.

If the work $P\Delta$ were the mechanical equivalent of this amount of heat, we should have the relation

$$P\Delta = Jc_pD,$$

which, according to Kupffer, is confirmed by experiment.[*]

But little attention is required to comprehend in what respect this method of treatment is defective. The heat, c_pD, consists of three parts: 1, The increase in the sum of the living forces; 2, the mechanical equivalent of the disgregation work; 3, the equivalent of the outer work.

This third part would be zero if the expansion took place in a vacuum. Under ordinary conditions it takes place under atmospheric pressure, and hence this third part can be neglected as very small compared to the second.

It is essentially different with the first part. This we cannot neglect without implicitly assuming that the specific heat for constant volume is insignificant in comparison to that for constant pressure. We cannot, therefore, regard the disgregation work as the mechanical equivalent of the entire quantity c_pD. It is, moreover, very doubtful whether the expression $P\Delta$ is the precise value of the disgregation work, for $P\Delta$ is the work of the forces which, by their mechanical action, cause an expansion of Δ, under the assumption *that the temperature of the body remains constant.* If, also, there is nowhere a development of perceptible velocity, it is only under similar circumstances the equivalent of the disgregation work. Nothing justifies us in putting this work equal to that in the body, when, by the action of heat, it expands under change of temperature. These two works are, indeed, of the same character, and change in the same way, when we pass from one body to another, but it is at least doubtful that they are identical.

All that we can say generally, is, that the resistance to tension is a certain indication of the intensity of the molecular forces, and that a considerable part of the heat which is imparted to a body is employed in overcoming these forces. The specific heat and the resistance to tension, or the coefficient of elasticity which measures it, change in the same way, for bodies of the same kind, as metals.

The same rather superficial law may be extended also to the latent heat of liquefaction, and thus it happens that Person has been led to find a numerical relation between the coefficient of elasticity and the liquefaction heat of various metals, which may be regarded as approximately indicated by experiment. It is, however, impossible to deduce any such relation in strict accordance with theory.

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[*] This is about the way in which Masson gives the view of Kupffer in his Treatise "Ueber die Beziehung der physikalischen Eigenschaften der Körper" (Ann. de Chim. et de Phys., 3 serie, vol. liii., p. 256). It is probable that this interpretation of the idea of the learned Director of the Physical Observatory at St. Petersburg is correct, but we cannot answer for it, as in the original text, instead of the sharp and clear expression "work," we find always the words "mechanical effect," which have no definite signification in the usual vocabulary of mathematics. (Bulletin de la classe des sciences physiques et mathématiques de l'Académie de St. Petersbourg, vol. x., and Pogg. Annalen, vol. lxxxvi., p. 310.)

It is probable that the formula of Kupffer has about the same value as that of Person, and that it is the approximate expression of a relation which theory is incapable of deducing.

We have indeed not proved that this formula is false, but simply that it cannot be deduced from any *à priori* considerations. If we consider the general fact in the special sense that the coefficient of elasticity and the specific heat changed together, it is just as allowable, and about as valuable, to seek this relation by experiment, as to find any other relations by comparison with other cases.

Certainly, such a comparison can have no such legitimate value as Kupffer attributes to it. In order to give the weight P as a function of the coefficient of elasticity, Kupffer uses an old formula of Poisson, which every one now recognizes as inexact, and indeed, most probably, not similarly inexact for all bodies. Hence, a factor which Kupffer regards in his calculations as constant, varies for different metals, and since this change is not yet known for the metals Kupffer has considered, it is not possible to introduce the necessary corrections and subject the empirical value of his formula to rigid test.

NOTE 8.—(Page 20.)

BODIES WHICH CONTRACT WHEN HEATED.

It is almost unnecessary to remark, that in such exceptional cases as the melting of ice and change of volume of water below 4°, in which there is a diminution of volume under the action of heat, the discussion is reversed. We consider a period during which the body expands while cooling, and hence performs outer work L while giving up heat Q. During another period, let the body, by the application of outer work L', and while receiving heat Q', be brought back to its original condition. If L' is less than L, we obtain an outer work $L-L'$, for which there must be an equivalent absorption of heat; Q' must be greater than Q, and we have

$$L - L' = J(Q' - Q).$$

The case of bodies which, within certain limits of temperature, contract under the action of heat, is well suited to direct attention to the views given in the preceding note. If we limit the comparison of the outer work with that heat which, in the one and the same transformation, is imparted or abstracted from the body in order to change it from one condition to another, we are led to the peculiar conclusion that the generation of heat, as well as its disappearance, can give rise to work.

Nothing is more suited to make apparent how necessary it is to take into account the work of the molecular forces. If, by some local disturbance, by contact with a piece of ice, or even by a particle of dust, we cause a mass of water at zero to crystallize, the molecular forces thus called into play by this accidental disturbance of equilibrium place the molecules in those positions which constitute a solid body, and the positive work during this process of change has for its equivalent both the heat generated and the outer work performed by expansion. If, inversely, we melt the ice, the heat imparted must be the equivalent of the excess of the disgregation work over the outer. In ordinary cases, on the other hand, the heat imparted during melting and withdrawn during solidification is the equivalent of the sum, and not of the difference of the disgregation work and outer work.

If a strip of vulcanized rubber is elongated by tension, we have a rise of temperature, while the temperature of a metal wire, under the same circumstances, is lowered. This is due to the fact that heat expands the metal, but contracts the rubber. This is a point which Joule has completely cleared up.*

NOTE 9.—(Page 20.)

UPON CALORIMETRIC MEASUREMENTS IN WHICH NO ACCOUNT HAS BEEN TAKEN OF THE OUTER WORK.

The necessity of taking account, in all phenomena depending upon the action of heat, of the outer work, would seem to justify the fear that a large part of our calorimetric measurements are liable to be affected by a fundamental error, as they were made at a time when the principle of the mechanical theory of heat was scarcely suspected. A little consideration, however, will serve to show that such fear is groundless. Strictly speaking, we must undoubtedly admit that specific heat and latent heat depend always upon the outer pressure under which bodies expand or change their aggregate condition. But, under ordinary circumstances, the outer work is so small for solids and liquids, that such dependence gives rise to a correction so slight that it is imperceptible even to the most sensitive methods of measurement.

For gases, the influence of such correction is so great that account has always been taken of it, and it has always been held as indispensable to give, for example, the pressure of a gas under which the specific heat has been determined. Only in the case of vapors have errors been committed.

Every investigation upon the latent heat of vaporization, in which an outer work is not performed upon the steam when condensing equal to that performed by it during its formation, is essentially erroneous, and can give no reliable result.

Regnault has, therefore, very properly, in his experiments upon the latent heat of vaporization of water, maintained in all parts of his apparatus a uniform pressure. The new theory by no means invalidates the value of the results obtained by this distinguished physicist, but it rather adds to their weight, and uses them to attain new results. It does, however, deprive numerous investigations, in which this precaution is neglected, of all claim to reliability.

NOTE 10.—(Page 21.)

THEORY OF THE CONSTITUTION OF GASES.

If we suppose, in a confined space, a large number of molecules separated by such intervals that their mutual actions may be neglected, and assume that these molecules are at rest, it is evident that they can exert no influence on each other, and that a portion of these molecules can undergo any change of state, without affecting in the least the condition of the others. Upon bodies which confine the system, there can be no such action as pressure. Individual molecules may indeed be so near the bounding body as to act upon it; but, by reason of the assumption of the mean distance of the molecules apart, the number will be very small in comparison to the number of molecules which must act together in order to cause the pressure of a liquid upon a solid or upon another liquid.

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* Joule, Phil. Mag., vol. xvi., p. 227, 1857; Ann. de Chim. et de Phys., vol. lii., p. 226; Thomson, Phil. Mag., 1857, vol. viii., p. 504. See also Tyndall, Heat as a Mode of Motion, 2d Ed., p. 115; also Villari, Pogg. Ann., vol. cxliv., p. 274; Schmulewitsch, Vierteljahresschrift der naturforsch. Gesellschaft in Zürich, Jahrgang xi., Heft 3, and Pogg. Ann., vol. cxliv., p. 280.

This property was first discovered by Gough, with non-vulcanized rubber, in 1806. Nicholson's Journ., vol. xiii., p. 305; Gehlen's Journ., vol. ix., p. 217.

Later Reusch observed similar phenomena with gutta-percha. Pogg. Ann., vol. cxliv., p. 315.

Govi ascribes the heating under tension to numberless gas bubbles.

Certainly, nothing can resemble less a gas than this incoherent collection, which can hardly be called even a system. We have, nevertheless, seen in the text that it is not easy to do without the assumption that in gases the distances apart of the molecules is incomparably greater than for any other bodies. If, however, we ascribe motion to these molecules, the state of things is changed, and the known properties of a perfect gas are necessary consequences of such an assumption.

In consequence of their motion, the molecules will impinge upon each other and upon the bounding surface. In a short time there will be a mean condition, the chief properties of which can be easily recognized. By reason of the size of the inter-molecular spaces, almost all the molecules must move at any moment, as if influenced by no deviating forces; that is, in straight lines, and with a uniform velocity common to all the molecules in the final condition, but different for different molecules. Those which accidentally approach each other at any moment act upon each other, and mutually influence each other's paths and velocities. But these changes last but a short time, after which the molecules recede and return to the general conditions of the system. Individual molecules may also impinge centrally or obliquely; but, since both the masses and velocities of individual molecules are by hypothesis equal, the direction of the velocities may be altered by impact, but not their amounts. We see, therefore, that, in order to find the action which the system exerts upon the confining boundaries, we can assume as the actual condition one in which all the molecules move incessantly in straight lines in all conceivable directions without striking.

If the boundaries are perfectly elastic, every impinging molecule will be thrown back, the direction of its motion changed, but its velocity unchanged, so that the total condition of the system remains invariable. Let us assume this condition as fulfilled, and seek what force must be exerted upon a boundary of given surface; what pressure, for example, must be applied, in order to keep it immovable. This force must be capable of reversing the normal component of the velocity of every molecule impinging in a given time, or, what amounts to the same thing, of imparting a normal velocity, in the opposite direction, of double the intensity of the component.

This must evidently be proportional to the uniform velocity of the molecules and their masses. It must also be proportional to the number of molecules impinging in a given time, that is, to the number of molecules in a unit of volume, and, further, proportional a second time to the velocity; for the time which any molecule requires to traverse the space between two boundaries is inversely proportional to the velocity, and hence the number of impacts which any molecule makes in a given time is proportional to the velocity.

The pressure, therefore, which must be exerted is proportional to the mass and number of molecules in the unit of volume, and to the square of the velocity.

The proportionality between pressure and number of molecules is nothing more than the proportionality between pressure and density expressed by Mariotte's law.

The proportionality to the mass and square of the velocity is also easily interpreted.

If we accept the usual views as to the nature of heat, we can regard the velocity of the molecules as an indication of the temperature of the gas, which changes in the same degree as the temperature itself. We have thus a theoretical definition of equality of temperature. We say that two gases possess the same temperatures if, when united under the same pressure, they do not affect each other's condition. If we assume that, for two different gases under the same conditions, there are in equal volumes an equal number of molecules, then the temperatures are alike when the product of the mass of a molecule by the square of the velocity is the same in both.

The equality of the living force of the molecules includes, therefore, the equality of temperature.

In other words we may say, that the living force of the molecules is a function of the temperature, which is the same for all gases. The proportionality of the pressure with the living force indicates, therefore, that in all gases the relation between pressure and temperature is the same. From this identity, together with Mariotte's law, we easily deduce the agreement in coefficient of expansion. If, as is customary, we measure the temperature by the air-thermometer, we know that, if the temperature indicated is t, and the coefficient of expansion is α, the pressure for constant volume is proportional to the expression

$$\frac{1}{\alpha} + t \quad \text{or} \quad 273 + t.$$

The living force of the molecules is, therefore, proportional to the temperature measured by a thermometer for which the zero point is at − 273° C. At this temperature of − 273° the living force of the molecules is zero, or we may say that at this temperature the gas contains no more heat—the absolute zero of temperature is then reached, and the gas ceases to be a gas, and becomes that incoherent mass of atoms, independent and immovable, which we have just spoken of.

If, finally, we assume, with all chemists, that, under the same pressure, all simple gases contain in equal volumes the same number of molecules, the changes of temperature are proportional to the changes in the living force of each molecule. We see then, that in order to heat equal volumes of different gases the same number of degrees, the same amount of heat is necessary. This conclusion is direct when rise of temperature occurs without change of volume; it follows, also, when there is a change of volume, when we consider the formulæ on page 23.

Thus, the characteristic properties of perfect gases find a simple and natural explanation. The idea of a "perfect gas" is itself sharply defined, and it is easy to conceive of imperfect gases, which do not strictly follow Mariotte's law, whose coefficient of expansion varies with the pressure, and which, for equal volumes, do not possess the same heat capacity, such as air and oxygen. In the system of individual molecules moving rapidly in all directions, which we have considered, we have assumed that at any given moment the number of molecules, whose motion is not rectilinear and uniform, is inconsiderable in comparison with those whose motion satisfies these two conditions; or, what amounts to the same thing, that for each molecule the duration of the period of disturbance is vanishingly small compared to the period in which the motion is uniform. If, now, the ratio of these two periods, while indeed still very small, is not vanishingly so, the preceding considerations do not hold strictly; and our conclusions no longer represent with precision the properties of the system, but are more or less approximate expressions of these properties. It is also evident that the more we diminish the distances apart of the molecules, that is, the more we condense the gas, the less reason we have to presume perfectly uniform motions, and the more, therefore, is the deviation from the condition of a perfect gas. This perfect condition is, in fact, an ideal state toward which gases approach as their state of rarefaction increases, but which they can never exactly attain.[*]

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[*] The theory given in this note is by no means new. It was indicated by Daniel Bernoulli in his Hydrodynamica, in 1738.

After being forgotten by the world it was revived again, about 1822, by Herapath. Only in recent times has it received its present shape by Joule, Krönig, and Clausius. Clausius has treated it in the most general manner, and added to its completeness by taking into account, in addition to the rectilinear motion of the molecules, their inner motions, motions of rotation, and the probable motions of imponderable fluids. In a presentation designed to be elementary in character, no account can be taken of such a treatment.

It may suffice to refer for further information to the original treatises of Joule, Krönig, and Clausius. Joule, Phil. Mag. 4 Ser., vol. xiv., p. 211; Krönig, Pogg. Ann. vol. xcix., p. 315; Clausius, Pogg. Ann. vol. c., p. 353; and Abhandlungen, vol. ii., p. 229.

NOTE 11.—(Page 23.)

HOW GASES AND VAPORS PERFORM OUTER WORK.

The theory given in Note 10 does not deduce the pressure of gases from the direct action of a repulsive force, but refers it to incessant impacts. It is thus possible to conceive how a change of volume can be accompanied by no inner work, although all gases seem to possess the tendency to expand and to resist compression. When the volume of a gas changes, the number of molecules in a given space changes also, and when a change of temperature occurs, the velocity of the molecules changes. But so long as the mean interval between the molecules does not exceed certain limits, their mutual actions are imperceptible, as well after as before the change of volume, and hence require no work. The mechanism of the relation which exists between outer work and heat absorbed or generated, is not hard to conceive. When we compress a gas, we apply a force to a movable piston which is greater than that necessary to reverse all the normal components of all the molecules which in a given time impinge upon the piston.

The velocity of all the molecules is thus directly or indirectly increased, and the work of the outer pressure is equivalent to the increase in the sum of the living forces of the molecules, that is, to the heat generated.

The reverse is the case for expansion. The molecules impart to the piston, upon which there is no sufficient force, according to the laws of impact, a portion of their living force, and this impartation of living force is, from our standpoint, an absorption of heat or a production of work.

Similar considerations apply to vapors and their work in the engines they operate. There is thus an imparting of living force when the piston is raised, and a reappearance of living force when it sinks. If work is performed by the engine, it is sufficient that there is no equality in the two cases. The living force which fails to reappear is the equivalent of the work done. Thus disappears the apparent contradiction in the fact that a system may produce work, while the inner works are zero.

NOTE 12.—(Page 24.)

THE VALUE OF THE MECHANICAL EQUIVALENT OF HEAT AS GIVEN BY CARBONIC ACID.

In determining the mechanical equivalent of heat from the properties of carbonic acid, the specific heat at constant pressure was taken at 0.2168, as given by Regnault in April, 1853. According as we take for the ratio of the two specific heats, 1.2867 as given by Masson, or 1.3382, given by Dulong, we obtain 402 or 355. But the number 0.2168 expresses only the mean specific heat between 0 and 210°C., and this mean specific heat is not the exact value for a given temperature.

From the experiments of Regnault (given in Vol. 26 of the Mémoires de l'Academie des Sciences), carbonic acid has at the temperatures 0 and 100 the specific heat 0.1870 and 0.2645. If we insert these values in the formulæ on page 23, we have for J the values

410 and 357, or 465 and 406,

according as we take for $\frac{C_p}{C_v}$ the value given by Masson or by Dulong.

NOTE 13.—(Page 25.)

PRINCIPLE OF THE METHOD OF INVESTIGATION OF THOMSON AND JOULE OF THE HEAT PHENOMENA IN GASES IN MOTION.

The method of investigation used by William Thomson and Joule consists in allowing a stream of gas to pass through a porous diaphragm, from which it issues with considerably reduced pressure. The friction absorbs nearly all the velocity due to the expansion. A sensitive thermometer gives the temperature of the gas before and after efflux.

It was so arranged for air, carbonic acid, and hydrogen, that simple expansion, even when accompanied by no outer work, caused a slight change of temperature, nearly proportional to the pressure, and which depended on the initial temperature. Thus has been determined the relation between the inner work and the outer, when a gas expands under pressure. If we assume the expansion as very small, and if the temperature is nearly 15°, the ratio of these values for air is $\frac{1}{417}$; for carbonic acid, $\frac{1}{77}$; for hydrogen, completely imperceptible.

The formula of page 23, which gives the mechanical equivalent of heat from the work performed by slight expansion and the heat absorbed, is therefore applicable without error to hydrogen. For air there is a slight error, which, however, is less than the error which may arise from the inaccuracy of the value for the specific heat for constant volume. For carbonic acid, finally, the left side of the equation must be increased by $\frac{1}{77}$th.*

It is certainly too early to seek in this manner to obtain a satisfactory agreement in the values of the mechanical equivalent of heat for various gases.

The density, the coefficient of expansion, the specific heat for constant volume, are very exactly known from Regnault's experiments, for air, hydrogen, and carbonic acid. But there still remains considerable uncertainty in the values attributed to the specific heats for constant volume. This evades direct determination, and must be deduced from observations upon the velocity of sound, or from heat phenomena which are caused by changes of volume; and, in the present state of such experiments, we can scarcely assume that, except in the case of air, it has been determined with accuracy. It follows, moreover, from the formula and the known values of C_p and C_v, that every error which is made in the value of C_v, in the case of air, causes a double error in the resulting value of *J*, and, in the case of carbonic acid, a threefold error.†

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* In a treatise especially intended to diminish the difference between the values of the mechanical equivalent as given by the formula, Baumgartner has taken the ratio of the inner to the outer work, according to Thomson and Joule, equal to $\frac{1}{590}$ for hydrogen, $\frac{1}{179}$ for air, and $\frac{1}{37}$ for carbonic acid.

These values are certainly given by Thomson and Joule, but they relate to the case in which the pressure falls from 4.7 atmospheres to one atmosphere. It is a great error to apply them as corrections to a formula deduced for such a slight change of pressure as that which accompanies a change of volume of the amount of the coefficient of expansion. (Sitzungsberichte der K. K. Akademie der Wissenschaften, Vienna, vol. xxxviii., p. 344.)

† If we assume the density of the air about $\frac{1}{770}$, its specific heat under constant pressure to be known, its coefficient of expansion at about $\frac{1}{273}$, and the ratio of the specific heats at about $\frac{1}{405}$, we find in the value of *J* deduced from the formula an error of $\frac{1}{50}$, or about 8 units.

For carbonic acid the difference in the value of C_v, given by the experiments of Dulong and Masson, is so great that no dependence can be placed upon calculation. We see, therefore, that it is not yet time to discuss this correction, and all that we can say with safety is, that the coincidence of the results for air and hydrogen renders it certain that the value of the equivalent must lie between 420 and 430.

NOTE 14.—(Page 32.)

UPON THE CONDENSATION OF STEAM IN EXPANDING.

The condensation of saturated steam during expansion was shown on theoretical grounds by Rankine and Clausius independently in 1850. Theory gives a necessary relation between the latent heat of vaporization of water, its specific heat, and the amount of heat which must be imparted to the unit of weight of steam, when simultaneously heated and compressed, so that it shall remain saturated. Since all the quantities which enter the equation, with the exception of the third, are given by Regnault's experiments, this may be found, and in this way it has been found to be negative. We must, therefore, *abstract* heat from steam which is compressed and heated, in order to keep it saturated; and, on the other hand, heat must be imparted to steam which expands and cools, in order to keep it from condensing.

When the expansion occurs without addition of heat from without, all the steam cannot therefore remain in the pure saturated state, and in order that a part of it may remain saturated, another part must condense, and thus furnish the necessary heat.

NOTE 15.—(Page 34.)

THE REGENERATOR IN HOT-AIR ENGINES.

It may appear as if the same reasons which cause the heat quantity q' to be lost to the engine, also contradicted the possibility of the unlimited usefulness of the quantity $c_v(t_1 - t_0)$.

In fact, we can scarcely see any other way of reducing the temperature from t_1 to t_0, than by contact with a cold body, which receives heat as the gas cools, but which, at the end, has the same temperature—t_0—as the gas. Under such circumstances, indeed, the quantity of heat $c_v(t_1 - t_0)$ would be contained in a body of the temperature t_0, and could not therefore be used for heating a new charge of gas, so that it would also be lost as well as the heat q'. This difficulty has been met by Stirling in a very ingenious manner. The gas is cooled in the engine from t_1 to t_0, by passing through a porous conducting body, such as a net-work of wires, to the different layers of which it imparts its heat. If this body is at first at the temperature t_0, its different layers will be raised by contact with the departing gas to some temperature higher than t_0, but somewhat less than t_1, except the last layer, which, if the body has sufficient thickness, will have the temperature t_0. When, then, a second charge of gas of the temperature t_0 enters, it will be gradually heated by contact with the successive layers, and will enter the cylinder with a higher temperature than t_0, so that in order to raise this charge up to t_1 will require less heat than the first charge.

After this charge has acted in the engine, and passes out, it finds all the layers, with the exception of the last, at a higher temperature than t_0, and will raise them all to a higher temperature than the first did. Thus, a third charge will enter the cylinder hotter than the second, and upon issuing, will leave the layers still hotter than the second did, and the temperature of the first layer still nearer therefore the temperature t_1 of the issuing charge. The amount of heat which must be imparted by the fire in order to raise each successive charge of air from t_0 to t°_1, will decrease.

Theoretically, the engine approaches constantly the condition in the text, in which the heat $c_v(t_1 - t_0)$ is constantly given up by the air to the regenerator, and then given back by the latter to the fresh charge, thus remaining always in the engine.

In practice, a certain fraction of this heat must be lost and made good by the fire at every stroke. Experiment shows that the heat of this fraction may be less than $\frac{1}{10}$th.

The porous body which is thus used is called the "regenerator." It has been constructed in various ways. A system of glass tubes has been used, also metal wires, and wire net-work. Glass and similar substances are too bad conductors, and answer the purpose very imperfectly. Metal wires and net-work are better, but they are quickly oxidized by the action of the hot air.

This purely practical difficulty is one of the chief hindrances to the extended application of the hot-air engine.

NOTE 16.—(Page 35.)

DETERMINATION OF THE EFFICIENCY OF THE ERICSSON'S ENGINE AND THE ENGINE WITHOUT REGENERATOR.

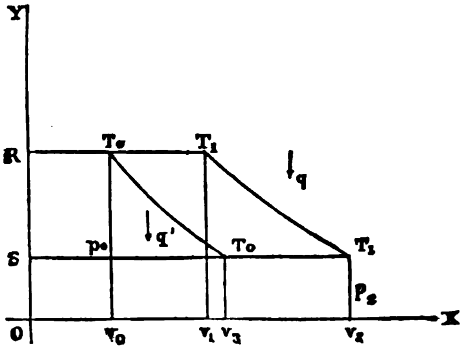

FIG. 5.

Let us take as an example an Ericsson hot-air engine. In this the air is first heated under constant pressure, then allowed to expand and cool, then still further cooled under constant pressure, and finally by compression brought back to its original condition. As in Stirling's engine, we can represent these successive operations graphically. Let $O\,v_0$ be the volume v_0 of the unit of weight of air, of the temperature t_0; and pressure p_0. Thus $v_0\,T_0$ is the pressure. The air is first heated under this constant pressure p_0, from the temperature t_0 to the temperature t_1, which requires the amount of heat $c_p(t_1 - t_0)$. Let $O\,v_1$ be the volume v_1 at the end of this operation.

Now the air expands from the volume v_1 to $Ov_2 = v_2$, while the temperature t_1 remains unchanged. The ordinates to the hyperbola $T_1\,T_1$, give at every instant of expansion the pressure of the air. Let the final pressure be p_2. Next, the air is cooled under this constant pressure p_2, until the temperature is again t_0. It is finally compressed, while the temperature remains unchanged, till the pressure is again p_0. The hyperbola $T_0\,T_0$ gives the pressure at any point during the compression. The area $T_0\,T_1\,T_1\,T_0$ is evidently the geometrical representation of the outer work. We can easily find this by prolonging the lines $T_1\,T_0$ to intersections R and S with the axis of Y, and finding the difference of the hyperbolic areas $R\,S\,T_1\,T_1$ and $R\,S\,T_0\,T_0$. We thus find

$$\text{Area} \quad T_0\,T_1\,T_1\,T_0 = (v_1 - v_0)p_0 \log \text{nat} \frac{p_0}{p_2}.$$

The heat utilized is therefore equal to the quotient of this expression when divided by the mechanical equivalent of heat. As to the total heat imparted and the heat not utilized we have, when we denote by q the heat imparted during the second operation, and by q' that abstracted during the fourth,

$$c_p\,(t_1 - t_0) + q$$
$$c_p(t_1 - t_0) + q'.$$

We can, therefore, just as in Stirling's engine, by means of a regenerator, keep the heat $c_p(t_1 - t_0)$ in the engine.

Finally, the amounts of heat q and q' are the heat equivalents of the work represented by the hyperbolic areas $v_1\, T_1\, T_1\, v_2$ and $v_0\, T_0\, T_0\, v_3$. These are

$$p_0\, v_1 \log \text{nat} \frac{p_0}{p_2}, \text{ and } p_0\, v_0 \log \text{nat} \frac{p_0}{p_3}.$$

The ratio of the useful work to the total is hence

$$\frac{v_1 - v_0}{v_1},$$

or again

$$\frac{\alpha\,(t_1 - t_0)}{1 + \alpha t_1}.$$

Let us consider, finally, a third kind of engine, which, indeed, is not practically realized, but which is theoretically the most perfect, since it does not require a regenerator. The air first expands, while its temperature is kept constantly T_1 by the addition of heat. The hyperbolic arc $T_1\, T_1$ gives the relation between pressure and volume during the expansion.

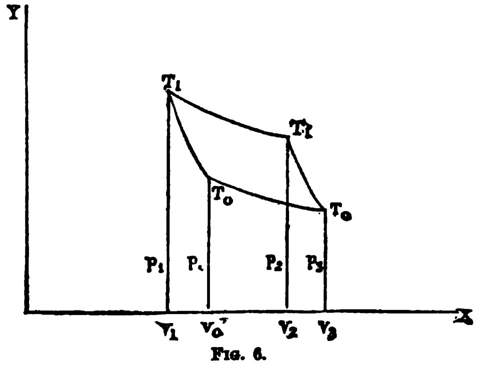

FIG. 6.

Let the initial pressure be p_1 and the final p_2. We now let the air still further expand, but without adding or abstracting heat, so that the temperature gradually sinks to t_0, and the pressure varies as the ordinates to the curve $T_1\, T_0$, which must approach the axis of x more rapidly than $T_1\, T_1$. Let p_3 be the final pressure, and t_0 the corresponding temperature. In the third period the air is compressed, while, at the same time, heat is abstracted in such a manner that the temperature is kept constant, so that the hyperbolic arc $T_0\, T_0$ gives the relation between pressure and volume. This compression is carried up to the point where the pressure is p_0. Finally, the air is still further compressed, but without imparting or abstracting heat, until the pressure is again p_1, and the temperature t_1.

We see at once the analogy with the preceding cases. In the first operation the heat imparted is

$$\frac{1}{J}\, p_1\, v_1 \log \text{nat} \frac{p_1}{p_2},$$

and in the third, that abstracted is

$$\frac{1}{J}\, p_0\, v_0 \log \text{nat} \frac{p_0}{p_3}.$$

Moreover, when α is the coefficient of expansion, we have, for the relation between pressure and temperature in the second and fourth periods,*

$$\left(\frac{1+\alpha t_1}{1+\alpha t_0}\right)^{\frac{c_p - c_v}{c_p}} = \frac{p_2}{p_3},$$

and

$$\left(\frac{1+\alpha t_1}{1+\alpha t_0}\right)^{\frac{c_p - c_v}{c_p}} = \frac{p_1}{p_0}.$$

Hence,
$$\frac{p_2}{p_3} = \frac{p_1}{p_0}, \quad \text{or} \quad \frac{p_1}{p_2} = \frac{p_0}{p_3}.$$

We have, therefore, for the ratio of the heat utilized to the total expenditure of heat

$$\frac{p_1 v_1 - p_0 v_0}{p_1 v_1}, \quad \text{or} \quad \frac{\alpha(t_1 - t_0)}{1 + \alpha t_1}.$$

NOTE 17.—(Page 36.)

HOT-AIR ENGINES IN WHICH THE TEMPERATURE FALLS TO THE ABSOLUTE ZERO OF TEMPERATURE.

It follows from the general formula that if it were possible, in a hot-air engine which satisfies the above conditions, to reduce the temperature down to the absolute zero, the efficiency would be unity. It is not difficult to see the reason. In Stirling's engine, for example, if the third operation—viz., that in which the gas is compressed under withdrawal of heat—takes place under the temperature of absolute zero, the gas possesses at this temperature no pressure. No work, therefore, is necessary to compress it, and the total work performed in the second period will be disposable. In Ericsson's engine the gas, in order to have, at a temperature indefinitely near the absolute zero, a noticeable pressure, must have an indefinitely small volume. The work in the fourth operation will be infinitely small, and we have at disposal the total work in the second period. In the engine, finally, without regenerator, if the third operation, as in Stirling's engine, occurs at the temperature of absolute zero, it requires also no expenditure of mechanical work.

It may not be without profit to consider for a moment how it can be possible that, at the temperature of absolute zero, a gas can be compressed without requiring work to compress it.

Let us consider a system of molecules which are in absolute rest, and so far distant from each other that their mutual actions can be disregarded. If, now, this system is compressed by means of a piston, the piston will impart a certain velocity to the molecules which it meets; but, since by hypothesis the temperature is kept at absolute zero, these velocities remain infinitely small. We require upon the piston, therefore, only a force which imparts to a finite number of molecules in a finite time an infinitely small velocity, that is, an infinitely small force.

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* This is the law of "adiabatic" expansion or "Poisson's law." See Poisson, Traité de Mécanique, vol. v., chap. 4; Weisbach, vol. ii., art. 376; also, page 168, chap. v., of the present volume.

NOTE 18.—(Page 37.)

THE NECESSARY TENDENCY OF HEAT TO PASS FROM A WARMER TO A COLDER BODY.

In a system composed only of a perfect and simple gas, the tendency of heat to pass from a warmer to a colder body is a necessary consequence of the laws of impact of elastic bodies. We have seen, in a preceding note (No. 10), that in such bodies the temperature is proportional to the living force of the individual molecules, when this temperature is reckoned from the absolute zero, that is, from — 273° C. It is at once evident, that when different perfect gases unite, those molecules which possess the greatest living force will give up by impact a part of their living force to those which have less; or, in other words, heat is always and of necessity imparted by the molecules of the warmer gas to those of the colder.

When, therefore, we say that in such systems, when subjected to any cycle of changes in which the final and initial conditions are the same, heat can in no case pass from a cold to a warmer body, we simply express a truth as clearly proven as the impossibility of perpetual motion.

This hardly holds for other cases. But we may, however, presume that the general laws of heat, equilibrium, and motion, are nothing more than pure mechanical principles, and although we choose a gas as the subject of comparison, we may still thus form some idea as to what properly constitutes equilibrium and difference of temperature for other bodies. If a solid or liquid body is of equal temperature with a gas, the molecules must have such a state of motion that, so long as the center of gravity of the body is not changed, the gas molecules which come in contact with the body neither receive nor part with living force. It follows, then, that if two solid bodies are in temperature equilibrium with the same gas, their temperature will not alter, *i. e.*, the motions of their molecules will not change, when they are brought into direct contact with each other. That which holds for this case, holds good also when the ether, upon which all radiation phenomena depend, is the medium of communication.

NOTE 19.—(Page 40.)

THE INFLUENCE OF FRICTION IN THE ELECTRO-THERMAL INVESTIGATIONS OF FAVRE.

It is not necessary, in these experiments, to take account of the influence of friction, but we compare directly the observed diminution of heat with the useful work of the engine. The friction of the engine undoubtedly develops heat, and this heat acts in the calorimeter just as well as that developed by the passage of the current. But the development of heat by friction means in reality the performance of work and generation of living force, and this double production produces an equivalent diminution of the heat developed in the circuit. Thus the friction increases, on the one hand, the heat in the calorimeter, and on the other, diminishes it by a precisely equal amount. We need not, therefore, consider it at all. The only correction is due to the friction of the rollers, outside of the calorimeter, by aid of which a weight is raised. Experiment has confirmed the strict compensation of the two opposite effects of friction. Whether the engine is at rest, or whether in action, without raising the weight, we have always in the calorimeter the same amount of heat.

NOTE 20.—(Page 42.)

THE DISCOVERY OF INDUCTION PHENOMENA.

The experiment referred to is given in the Annales de Chimie et de Physique, 2 Serie, vol. xxi., p. 47. An annular plate of copper was hung by a silk thread in the plane of a circular frame, upon which was wound a number of turns of insulated copper wire. A powerful iron magnet was placed with one pole in the circle and the other without.

As soon as the current passed, the circle was attracted or repelled by the electro-magnet; but the duration of the action, as in all similar induction phenomena, was very short. This fact, probably, prevented Ampère from feeling confidence in his experiment, for he failed to draw from it the least conclusion, and nothing further is said of it till Faraday published his discovery. This is the more surprising as Ampère, at the time when, in association with de la Rive, he undertook this experiment (1822, at Genf), sought, in so many words at least, "to produce an electric current by the action of another current." These are his words ten years later.

NOTE 21.—(Page 42.)

DEDUCTION OF THE LAWS OF INDUCTION FROM THEORY.

We consider a battery consisting of any number of equal or unequal elements. According to the laws of Faraday, the amounts of chemical action which are developed in the same time in the different elements are mutually equivalent.

If, therefore, L', L'', L''', etc., are the works of the chemical forces in the different elements during the time required in each for the decomposition of one equivalent of metal, the total amount of heat developed in the battery and conductors, assumed at rest, is

$$\frac{L' + L'' + L''' + \cdots}{J}$$

or,

$$\frac{\Sigma L}{J}.$$

On the other hand, Joule's experiments have shown that the amount of heat developed in a unit of time in a conductor, is proportional to the resistance and to the square of the intensity of the current. Let R be the total resistance of the conducting wire and battery; Y the intensity of current, then the heat developed in a unit of time in the battery and conductors is proportional to Y^2R or to $Y\Sigma A$, if ΣA denotes the sum of the electro-motive forces, and, according to Ohm's law

$$Y = \frac{\Sigma A}{R}.$$

If θ is the time necessary to decompose one equivalent of metal in each element, the amount of heat which we have just represented by ΣL will be proportional to $J\theta \, \Sigma A$, or, simply, to ΣA, when we take as unit that intensity of current which corresponds to the decomposition of one equivalent of metal in the unit of time. We shall therefore have, when we have properly chosen the unit of electro-motive force,

$$\frac{\Sigma L}{J} = \Sigma A.$$

Let us now assume that the circuit as a whole, or that portions of it, under the influence of outer points of magnetic attraction, or of the mutual action of the various elements, moves. Then the work of the chemical forces is equivalent to the heat developed, and to the work of the electro-magnetic or electrodynamic forces. We denote by Udt that part of this work which is performed in the indefinitely small time dt.

Let, further, i be the corresponding intensity of the current, expressed in the assumed unit. Then idt is the fraction of one equivalent of metal decomposed in each element in the time dt. Finally, let Qdt be the total amount of heat developed. Then, according to what has been said,

$$idt\frac{\Sigma L}{J} = Qdt + \frac{Udt}{J}.$$

Combining Ohm's law with Joule's, we see that Qdt is always proportional to the product of idt by the sum of the electro-motive forces. It is impossible that this sum should remain ΣA. It is necessary that by the action of motion it shall become less. In other words, to the living forces whose sum is represented by ΣA, we have an opposing force F, which must satisfy the condition

$$i\frac{\Sigma L}{J} = i(\Sigma A - F) + \frac{U}{J}.$$

We shall now investigate separately the two cases which we have distinguished.

As soon as the circuit (battery included) moves as a whole, and without changing its shape, under the influence of outer force centers, the elementary work Udt is proportional to the energy C of these force centers, to the intensity i of the stream, and to a function, φ, which depends upon the relative position of the circuit and these force centers at any given moment, upon the kind of motion, and upon the distance vdt passed through by any element. We have, therefore,

$$\frac{\Sigma L}{J} = \Sigma A - F + \frac{C\varphi v}{J},$$

or,

$$F = \frac{C\varphi v}{J}.$$

The factor v is the velocity at a given moment. We see therefore that the electro-motive force of induction is proportional to the velocity of displacement, and to the expression $C\varphi$, which, when multiplied by vdt, gives the elementary work of the outer forces upon a circuit traversed by a current whose intensity is unity.

When the elements of the circuit change, by reason of their mutual action, we can represent the elementary work of their mutual actions by $i^2\psi vdt$, where ψ is a similar function to φ. Therefore,

$$\frac{\Sigma L}{J} = \Sigma A - F + \frac{i\psi v}{J},$$

or,

$$F = \frac{i\psi v}{J}.$$

The electro-motive force in this case is proportional to the intensity of the stream and to the velocity of the relative motion.

In the general case of change of form, and total or partial displacement by the influence of outer forces, the electro-motive force of induction is the sum of the two preceding expressions.

NOTE 22.—(Page 42.)

THE COMPLETE TRANSFORMATION OF HEAT INTO WORK, BY THE ELECTRO-MAGNETIC ENGINE.

Let us consider an electro-magnetic engine. We assume first that only the immovable pieces are traversed by the current, and that the movable pieces are permanent magnets.* We assume such an engine, which, under the influence of outer resistance, has attained its condition of normal activity, so that in successive periods the rotation is identical. This condition does not include, strictly speaking, uniformity of motion; but, in a well-constructed machine, in which the intensity of the mutual action of the magnets and wire spirals varies from one moment of the rotation to another but little, the rotation can be considered as essentially uniform. If we call V the velocity of this rotation, the electro-motive force of induction is KV, where K is a constant coefficient which depends upon the strength of the moving magnets and the arrangement of the machine. Hence, if we denote by A the sum of the electro-motive forces, by R the resistance, and by i the intensity, we have

$$i = \frac{A - KV}{R}.$$

The heat corresponding to the decomposition of one equivalent of metal in each element is, therefore,

$$A - KV.$$

In rest, it would be A. The heat transformed into work is then KV. The ratio $\frac{KV}{A}$ of these two quantities increases with the velocity, and approaches unity as the electro-motive force $A - KV$ and the intensity approaches zero. If the movable and immovable parts are traversed by the same current, the electro-motive force of induction is expressed by hVi, so that

$$i = \frac{A - hVi}{R}, \quad \text{or,} \quad i = \frac{A}{R + hV}.$$

The amount of heat developed per unit of time in the circuit is, therefore,

$$\left(\frac{A}{R + hV}\right)^2 R, \quad \text{or,} \quad iA\frac{R}{R + hV}.$$

During the time θ, which is required for the decomposition of one equivalent of metal in each element, the heat developed is

$$i\theta A\frac{R}{R + hV}, \quad \text{or,} \quad A\frac{R}{R + hV},$$

since we assume (see preceding note) that $i\theta$ is equal to unity. In a state of rest, this quantity is A. The heat transformed into work is then

$$A\frac{hV}{R + hV},$$

which approaches A the greater V becomes.

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* Fromment has often constructed machines of this kind. The theory of those in which the immovable pieces are magnets and the movable ones wire coils, does not differ essentially from the present presentation.

NOTE 23.—(Page 43.)

DETERMINATION OF THE MECHANICAL EQUIVALENT OF HEAT BY ELECTRO-MAGNETS.—(JOULE.)

Joule caused, by means of a weight, a movable electro-magnet to turn between the poles of an immovable electro-magnet of great power. He determined first the weight which was necessary in order to give to the apparatus a constant velocity under the influence of friction, the current of both electro-magnets being open. Then the conducting wire of the fixed electro-magnet was connected with the battery, and that of the movable closed with a short thick wire, and the weight determined which had to be added to maintain the same constant velocity, as also the heat developed in the movable current.

This last part of the experiment appears to have left much to be desired. The movable electro-magnet was placed in a glass vessel filled with water, and the rise of temperature of this compound system directly observed, in order to find the heat generated. Two constant sources of error must tend to cause this determination to give too small values. First, it is extremely doubtful whether there is simultaneously a common temperature in the water and the soft iron and the insulated copper wire which form the movable system. Moreover, the long cylindrical shape of the system favors the cooling by radiation and contact with the air. This last is also increased by the rotary motion. Whatever care is taken in applying corrections, it can hardly be avoided estimating too low the heat developed by the given expenditure of work, and hence obtaining too large a value for the mechanical equivalent. It is, therefore, not surprising, that the value deduced from these experiments is about $\frac{1}{12}$th greater than the probable value. In some special experiments, the difference is even still greater.

NOTE 24.—(Page 45.)

THE NATURE OF ELECTRO-MAGNETIC AND ELECTRO-DYNAMIC FORCES.

It may be objected that we have made use in our lectures of the principle of the impossibility of perpetual motion as an absolute truth. It may be said that we apparently forget that there are natural forces, such as electro-magnetic and electro-dynamic, which do not depend alone upon mass and distance; that there are forces with whose help therefore we can cause in certain cases rotary motion, the velocity of which may be indefinitely accelerated. We had the intention of noticing this objection in the second lecture, when speaking of the electro-magnetic engine; but it appeared better, on consideration, to reserve it for a note.

Let us consider first the electro-magnetic forces. Experiment shows that magnets act upon currents, and inversely. All the effects of these actions can be referred to a system of forces which affect the different elements of the current, which depend not only upon the distances, but also upon certain angles, and which do not act in the straight lines connecting the current elements and the magnetic centers.

For a closed circuit of invariable form, this system of forces can be replaced by an equivalent, which is apparently entirely different from the preceding, and consists of forces which satisfy the ordinary conditions of action of natural forces. In this case, the difficulty disappears at once of itself. This substitution is however no longer possible when the circuit is not closed, or when, in other words, the closed conductors traversed by the current consist of several independent portions. The motion of each of these parts depends solely upon forces which act upon its various elements, and it is clear that this motion, under certain circumstances, is one of rotation, which, without the influence of friction, resistance of the air, and similar resistances, would be infinitely accelerated.

Ampère has repeatedly declared that here is an actual exception to the general laws of mechanics. There is no treatise or presentation of any completeness, upon electro-magnetism, in which this is not illustrated. There is even no elementary presentation in which the fact of indefinitely accelerated rotation is not in various ways experimentally shown. But error is committed, and any presentation must be very imperfect, if anything real is seen in this apparent exception. We will consider one of the simplest experiments of this character, which is to be found in all regular courses in Physics. A small rectilinear horizontal current turns about a vertical axis through one of its ends, under the action of a vertical magnet, situated in the prolongation of the axis.

It requires but little attention to recognize that, at the end of each revolution, the velocity is somewhat greater than at first, at least as long as, under the influence of the resistances, the maximum is not attained. Perpetual motion seems therefore attained, since, at the end and at the beginning of each revolution, the position of the current and the magnet is the same. But does this coincidence of position include the condition that nothing is changed in the total system of mutually interacting bodies? This system consists not only of the current and the movable magnet, but also of the battery which sets the electric current in motion, and the conductors which unite the battery with the two ends of the movable current. We will not speak of the special phenomena which occur at the point of contact of movable and immovable parts. The battery is the seat of incessant transformation of chemical actions. Is it therefore strange that such transformation should cause a continuous increase of the velocity of rotation of a movable wire? The actual mechanism by which the phenomena are caused is unknown, but nothing necessitates us to admit that the action of the real elementary forces does not follow the general laws of action of natural forces. The assumed elementary forces, to which we are necessarily led when we limit our consideration to the magnet and movable current, are functions of the angle, and perpendicular to the plane of the magnet and current. These forces are not in the least analogous to the elementary forces which govern the motions of the stars or the fall of bodies. They are pure mathematical symbols, which represent not the reality, but only the last stages to which, thus far, analysis of the phenomena has led us. We can say the same of the electro-dynamic forces, and of the famous formula by which Ampère has represented what he calls the opposite action of two current elements. This formula is an experimental law, which, in its unlimited fruitfulness, indeed, exhausts every possible variation of the phenomena; but which possesses no reality outside of the circle of phenomena for which it forms the general bond. If, for example, it were possible to place two current elements, independently of any voltaic circuit, in the same physical condition as when they form an actual part of such a circuit, it proves nothing, that, in accordance with Ampère's laws, they must approach or recede from each other. All that we can assert is that these laws represent the phenomena in all cases open to experiment. We can see in them only the translation of the secret mechanism by which the phenomena are produced, and nothing prevents the admission that the actual forces involved in such mechanism are simple functions of the distances, and act in the line connecting any two mutually acting points.

This was, moreover, Ampère's own view of his discoveries. If he seldom referred to it, even sometimes apparently rejected it for the opposite, it was only not to offend the scientific views of his contemporaries, who without it had difficulty enough to appreciate his experiments, and who would have rejected his hypotheses without proof. But, in the remarks which he has added to the presentation of his theory (read at the official session of the Academy, April 8, 1822), he has expressed himself in a manner which allows no doubt as to his convictions.

"I remarked," he says, "1. That the attractions and repulsions, whose existence between portions of the conducting wires I had recognized, could not arise in the same way as that of ordinary electricity, by reason of unequal distribution of the two fluids which mutually attract each other, and every part of which, of the same kind, is repelled, since all the hitherto known properties of the conducting wires show that neither the one nor the other of the two fluids occurs in larger quantities, in a body which serves as conductor of an electrical current, than when the same bodies are in their natural condition.*

"2. I remarked that it is difficult not to conclude, therefore, that these attractions and repulsions are caused by the extremely rapid motion of the two electric fluids which traverse the conductor, by reason of almost instantaneous decomposition and composition in opposite directions; a motion assumed by all physicists since Volta, and which the theory given by this renowned savant of the admirable instruments constructed by him, substantiates.

"3. If we ascribe the attractions and repulsions of the conducting wires to this cause, we cannot avoid, if we explain the ordinary electrical phenomena in the customary manner, admitting, further, that the motions of the two electricities in the wires are propagated in every direction in the neutral fluid formed by this union, with which, necessarily, all space must be filled; so that when the motions thus arising in the surrounding medium, caused by two small current portions, mutually coincide, there is a tendency to approach, which is, in fact, the case when we observe attraction; and that when the two motions are opposed, the two current portions tend to repel each other, as experiment also shows.

"4. If we consider these attractions and repulsions as actually caused by these reasons, the law that a small portion of the electric current can be replaced by two others, which stand to it in the same relation as two forces to their resultant, is a necessary consequence of this assumption; since velocities are composed like forces, and since the motion which the small portion of a current, represented in intensity and direction by the resultant, imparts to the fluid which fills space, is necessarily equal to that which is caused in the same fluid by the union of the two small current portions which, in similar manner, are represented by the two components.

"At the time when I was occupied by these ideas, Fresnel communicated to me his elegant researches upon light, from which he deduced the laws which determine all the conditions of optical phenomena.

"I was surprised at the agreement between his views and those to which I had been led by the consideration of electro-dynamic attractions and repulsions.

"He showed, from the accordance of these phenomena, that the ethereal fluid of space, which cannot be regarded as the result of the union of the two electricities, must be nearly incompressible, and must permeate all bodies, as gas flows through a net, and that the motions caused in this fluid must be propagated by a kind of friction, which enables the moving layers to set in motion others. Hence it was natural to suppose that the flowing electric current in a conducting wire caused the surrounding neutral fluid to take part in its motion, and in part rubbed on it, so that a reaction of this fluid on the current was caused. This reaction can cause no tendency to a displacement of the wire, so long as the difference of velocities on all sides of the wire is the same. There will be, however, a tendency to move the wire as soon as a second current exists, and, indeed, either toward the side on which this difference of velocity, and hence of reaction, is less—that is, the side upon which another electric current tends to move the fluid in the same direction—or toward the opposite side, upon which this difference is greater, because there another electric current exists which tends to move the fluid in the opposite direction, according as the two mutually acting currents flow in the same, or in opposite directions.

"These views certainly make clear the attraction between similarly flowing, and the repulsion between oppositely flowing currents, in accordance with experiment; but I have not forgotten the fact that, as it is not possible to calculate all the effects of the motion of fluids, they are too general to serve as the foundation for a law whose correctness can be confirmed by direct and exact experiment. This is the reason why I have confined myself to representing it simply as a fact based upon observation."

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* We know now that free electricity exists upon the surface of conductors through which a current flows. The distribution of this electricity is, however, such that it has no influence upon the electro-dynamic phenomena. Moreover, by the composition of forces, which are only functions of the distances, we could never obtain resultants which are functions of the angles.

Thus far Ampère. It is interesting to see how the renowned author of the "Théorie des phénomènes electrodynamiques" recognized back of the problem solved by him, another still deeper and more difficult, the solution of which he left to the future.*

NOTE 25.—(Page 46.)

ELECTROLYTIC CONVECTION.

When a current is made to perform decomposition of water, the chemical heat-equivalent of the decomposition must be abstracted from the heat-equivalent of the electrolytic process in the galvanic battery. Since, now, for the decomposition of one equivalent of metal in each element of the battery, one equivalent of water is decomposed in the voltameter, a decomposition of water can only occur when the electrolytic processes in the battery develop more heat than can be generated by the reunion of the oxyhydrogen gas developed in the voltameter. About 1¾ Daniell's cells are necessary to give continuous decomposition of water.†

With a single Daniell cell, therefore, no decomposition of water is possible.

The conditions are essentially different when both electrodes and the liquid of the voltameter are completely saturated with hydrogen or with oxygen.

There is then a transmission of electricity from one electrode to another, either through a non-electrolytic conduction of the water, or through a process which Helmholtz calls electrolytic convection. This peculiar process consists in the fact that, under such conditions, an electrolytic decomposition of the water and a separation of hydrogen and oxygen can take place. If thus, for example, the voltameter is completely saturated with hydrogen, the oxygen combines immediately, upon its generation, with the condensed hydrogen upon the surface of the platinum. Then the negative work of the water decomposition is compensated by the positive work of the water formation on the one electrode. In this case the water decomposition is connected with no essential consumption of heat. The condition of such a process is, therefore, that on one electrode more, and upon the other correspondingly less, hydrogen occurs. The entire process is thus limited to a different distribution of the gas contained in the liquid.

Thus Helmholtz found that a current, which was able to decompose in 24 hours 60 milligrams of silver, could pass for a day, without diminution of its strength, through such a voltameter saturated with hydrogen, without causing more than a just appreciable polarization.

Especially under very low pressure, for more rapid development of hydrogen, the hydrogen separated in gaseous state.

With such a voltameter, the pair of platinum plates being laden with hydrogen gas, a development of hydrogen may be caused by one Daniell's cell.

This phenomenon was earlier noticed by Poggendorff. It finds its explanation in Helmholtz's experiments, and does not stand, as we see, in contradiction to the principle of equivalence.

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* The complete solution of the problem has been essayed, in recent times, by Helmholtz in his paper "Ueber die Bewegungsgleichungen der Electricitaet für ruhende, leitende Körper," Crell's Journal, vol. lxxi., p. 57, and by Carl Neumann, in his work "Theorie der elektrischen Kräfte," 1873.

† Thomson—On the Mechanical Theory of Electrolysis, Phil. Mag., 1851. He gives the quantity at 1.318 Daniell cells.

NOTE 26.—(Page 46.)

UPON THE POLARIZATION OF THE ELECTRODES.

We may, by the aid of the same mechanical consideration, deduce the necessity of another phenomenon, viz., that of the polarization of the electrodes.

When the circuit is completely metallic and remains immovable, the heat developed in a given time represents the total work of the chemical forces. When the circuit also contains a compound liquid, the heat developed in the cells by the same amount of chemical action must be less, since it represents only the excess of the positive work in the voltaic cells over the negative work in the decomposing apparatus. It is, therefore, necessary that this heat shall be less than that which is obtained when the liquid is replaced by a metallic conductor of the same resistance. This can, however, only be the case when the liquid changes the current intensity in some other manner than by the introduction of its resistance.

Since, now, we know that there are no means of diminishing the intensity of a current other than increasing the resistance of the conductor, or diminishing the electro-motive force, we see that the introduction of such a liquid must have, as an immediate and necessary consequence, a diminution of the total electro-motive force; that is, the development of an electro-motive counter-force.

Upon this rests directly the polarization of the electrodes. In consequence of this polarization, the current of a single cell of the ordinary battery reduces, by the introduction of a voltameter with acid water, to zero, and hence the decomposition of water under these circumstances is impossible. When the liquid, during its decomposition by the action of one of the chemical elements originated by the decomposition, is again formed upon the corresponding electrode, the work of the chemical forces is actually zero, and we know that then no polarization can occur.

NOTE 27.—(Page 46.)

THE DECOMPOSITION OF ZINC IN DILUTE ACIDS.

It has been long observed that when commercial zinc is dissolved in acid water, the generation of hydrogen does not take place at all points of the metal, but at certain special points, which appear thus to be different from the others.

De la Rive has observed that these points are fewer with distilled zinc, and that the development of hydrogen takes place more slowly than for ordinary zinc. Finally, Almeida has found, after he had succeeded in producing perfectly pure zinc by galvanic process, that this metal resisted perfectly the action of dilute sulphuric acid. In both cases the pure zinc assumed the properties of the ordinary metal when some other metal was added, so that the acid came in contact with a surface not homogeneous in character.

NOTE 28.—(Page 47.)

UPON THE APPLICATION OF THE MEASUREMENT OF ELECTRO-MOTIVE FORCES TO THERMO-CHEMICAL INVESTIGATIONS.

In the first part of Note 22 we have said that the heat developed in a given time by a current in its total circuit, is proportional to the product of the intensity and the sum of the electro-motive forces. If we consider different circuits, each of which consists only of a single pair and metallic conductors, the amounts of heat developed in a unit of time in these different circuits are to each other as the products of the intensity and electro-motive forces of each pair. Since, however, the intensity is proportional to the number (whole or fractional) of metal equivalents decomposed, it follows that the heat developed by the decomposition of one equivalent of metal in the different elements, is directly as the electro-motive force itself. We can, therefore, replace calorimetric measurements by measurements of the electro-motive forces, provided that we know in a few cases, by direct experiment, for a certain amount of heat developed, the corresponding electro-motive force.

The practical advantage of this method is apparent, but its application involves some difficulties. In all those cases in which the chemical action which causes the current is accompanied by a development of gas, the electro-motive force varies with the intensity of the current. But, by local heat phenomena occurring at those points at which the gas is generated, the case may happen that the total heat production is constant. We cannot therefore speak, without specifying further, of any proportionality between the two quantities. Many observations made with care and skill, because no account was taken of these circumstances, have lost the greater part of their value.

NOTE 29.—(Page 48.)

THE INFLUENCE OF THE FRICTION OF THE BLOOD UPON THE ANIMAL HEAT.

These views hold good in spite of the interior motions in organisms, and in spite of the resistances which these encounter. There is no reason for taking account of that part of these resistances due to the action of the outer forces——as, for example, gravity—so long as there is no displacement of the center of gravity of a body. The interior circulation of fluids, the movements of the muscles resulting, the elastic reactions of the vessels, cannot give rise to any work of gravity.

As to the inner resistances, these are the frictions which must develop just as much heat as the muscular force, which maintains the motion of the liquids in spite of friction, consumes. We see then how useless is the investigation of the influence of the friction of the blood in the vessels upon the heat of animals, which some physiologists have made. In order to overcome this friction, the action of the heart is necessary. In order to maintain this action, a portion of the heat furnished by the interior combustion in the organism is necessary. This loss of heat is, however, completely replaced by the heat generated by the friction of the blood in the total circulatory system. There is thus only another distribution of the heat, while its total amount remains unchanged.

So long as the animal remains at rest, we are perfectly justified in comparing this total heat quantity with the sum of the chemical actions arising from respiration.*

NOTE 30.—(Page 49.)

UPON VEGETATION WHICH IS CARRIED ON WITHOUT THE INFLUENCE OF LIGHT.

When the influence of light is withdrawn from the higher plants, two cases may occur: either they may act like inanimate bodies, absorbing oxygen from the air and allowing the water and carbonic acid in the soil to filter through their organism, then bleaching, and if often increasing in their dimensions, still the proportion of combustible substances seeming rather to diminish than to increase; or a part of their tissues is destroyed by more or less rapid oxidation, and experiences far-reaching changes, which nevertheless do not require the action of any outer forces; these can be regarded as oxidations brought about by the natural activity of the affinities, as, for example, in the germination of seed.

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* See Hirn, Remarques sur le rôle réel que joue le frottement des muscles dans le phénomène de la calorification des êtres vivants à sang chaud ou à sang froid. Cosmos, 1862, vol. xxi., p. 257.

The question now arises whether the same is true of the lower plants, whose life is almost entirely independent of the influence of light, or if not, in what way is this influence replaced, and how is it possible that they grow, and that their vegetation is accompanied by a negative work of the affinities.

In its present state, experimental physiology gives no reliable answer to these questions. In order to answer them, we must first have exact comparative analyses of lower plants which have completed their development, and we must investigate chemically the materials by the use of which they have developed. In most cases these materials are decomposing organic bodies, and it is possible that the simple elements composing every organism, such as carbon, hydrogen, oxygen, nitrogen, occur in these in the same proportions as in the plants themselves, but in a different grouping.

The vegetable life can, therefore, only be a series of equivalent transformations, which demand no expenditure of work furnished by outer forces.

If, on the other hand, experiment shows that in the tissues of the lower plants, without any action of light, carbon and hydrogen exist in relatively higher proportions than in the organic substances upon which they live, we may, it seems to me, account for it somewhat as follows. Almost always during the development of such plants, the organic bodies which serve as nourishment are decomposed, and pass gradually into a condition in which they tend to follow the natural activity of the affinities. There is thus evidently a positive work of the affinities, and hence a production of heat. Is it not possible that a part of this heat is made use of by the plant itself, and causes phenomena which correspond to a negative work of the affinities? In this way the action of the sun's rays may be replaced. It seems as if an observation of Pasteur gave a certain probability to this view. Pasteur has shown that the formation of acid in alcohol is brought about by the oxygen, which the countless organisms living upon the surface of the liquid condense. When these plants are not present, the oxygen of the air is not capable of oxidizing the alcohol; when the oxygen is absent, these plants cannot live.

The oxidation does not appear to proceed, however, from any special activity of the plants, but seems only to depend upon their presence and the property which they possess in a considerable degree of condensing gases upon the surface. It is not therefore reasoning in a circle, when we assume that the oxidation of the alcohol is a necessary condition for the acid-forming vegetation. It would even be quite natural to suppose that the heat developed by this oxidation, and which is so considerable that no thermometer is required to detect it, is in part made use of for the production of such phenomena of vegetable life, which are opposed to the tendency of the affinities.

In the life processes of the barm fungus there seems something similar. The sugar is decomposed during the fermentation into alcohol and carbonic acid. The alcohol possesses a considerably less heat of combustion than the quantity of sugar which is necessary for its formation. There is, therefore, work or living force performed in this decomposition. A part of this living force is applied in calling forth the chemical processes which are involved in the formation of the cells of the fungus. Another part is directly transformed into heat. On the one side we have, during fermentation, a process which corresponds to the natural tendency of the affinities; on the other, the satisfaction of the chemical force of attraction is the source of the positive work, upon which the fungus draws in order to form new cells, or perform negative work.

We see, therefore, that by fermentation, likewise, cells are formed, plant organisms increase, and not inconsiderable quantities of heat are developed without any outer force, such as light, as the cause.*

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* See A. Mayer, Pogg. Ann., vol. cxlii, p. 293.

NOTE 31.—(Page 49.)

THE ABSORPTION SPECTRUM OF CHLOROPHYLL, AND THE INFLUENCE OF COLORED LIGHT UPON THE GROWTH OF PLANTS.

As long as the action of the sun's rays upon plants was not recognized as the cause of those processes which continually go on in the growth of vegetation, it must have seemed very puzzling from whence the enormous amounts of living force could originate, which are accumulated ready for work, in plants. Now we know that these processes, which are opposed to the natural tendency of the chemical affinities (with very few exceptions, noticed in Note 30) take place only under the action of light, and, indeed, of light upon those parts of plants which contain chlorophyll.

The green chlorophyll-containing plants take from the air the carbon which they contain. The carbonic acid continually absorbed by the leaves is decomposed by the action of light in those cells containing chlorophyll, and the superfluous oxygen is given out.

The oxygen developed by the plant may serve as a measure of the decomposition, if it is really, as is generally assumed, completely, or under various circumstances in equal degree, separated by the respiratory organs of the plant.

Since this decomposition of the carbonic acid takes place only in the cells containing chlorophyll, and in these only under the action of light, it is suggestive to investigate the optical character of this coloring matter.

If we deprive a plant of its chlorophyll by treatment with water, alcohol, or ether, and examine the spectrum of the light which passes through a fresh concentrated solution, it appears that different parts of the sun's light are more or less completely absorbed.

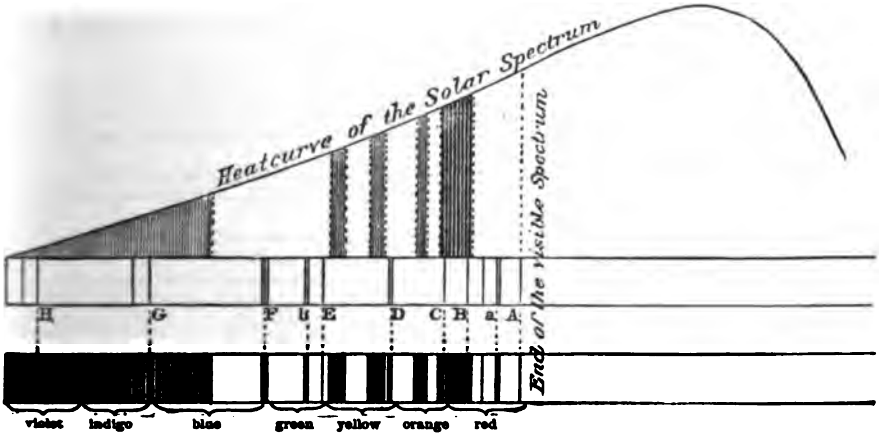

Fig. 7.

The extreme red remains completely unchanged,* but immediately behind

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* The influence of chlorophyll upon the ultra red portion of the spectrum has not, so far as I know, been investigated. It is to be desired that it may soon be done.

Fraunhofer's line *B*, Fig. 7, we have a black absorption strip which Hagenbach denotes by *I*. This strip is pretty sharply defined, and extends beyond line *C*. About at its middle there is a light portion. A second absorption strip (*II* of Hagenbach) occurs nearly in the middle between *C* and *D*; a third (*III*), a little behind *D*; a fourth (*IV*), in the green, just before *E*. These strips are, however, much less dark than the first in the red.

From the middle, between *F* and *G*, on, almost the rest of the entire spectrum is uniformly absorbed. The Figure shows the absorption spectrum of chlorophyll, and indicates the position of Fraunhofer's lines.

We see from the Figure that the yellow red rays, certain parts of the yellow, and, in less degree, the green and the indigo blue and violet portions of the spectrum, are almost completely cut off by the chlorophyll.

The absorption spectrum of the solid chlorophyll, as found in the leaf, agrees in number and arrangement of the strips with that of the solution,[*] and that of the leaves of living plants does not deviate.

Slight differences in the absorption phenomena in actual leaf organs and in solutions, are explained, as Melde suspected and Gerland has proved, by the diminution of the light by the other contents of the cells and the tissues of the leaves.

Only the absorbed rays can cause the processes of decomposition and assimilation in plants, and those rays not absorbed are of comparatively indifferent effect.

If we inquire now which of these rays has the greatest effect upon chemical processes, we must evidently seek them among those having the greatest mechanical energy.

This energy has no connection with the subjective sensation of light intensity, for our eyes may indeed decide whether a given red is lighter or darker than the red from another source, but they cannot compare the light rays of two different pure colors of the spectrum.

The best measure of the mechanical energy of a given color is, as Lommel has remarked, the heat effect, when we assume that the entire energy of a ray absorbed by a soot-covered thermopile is completely transformed into heat.

If this is the case, then the heat curve of the solar spectrum is that which expresses the energy of the various rays.

The heat effect of the violet and blue rays is very slight, and that of the red and ultra red very large.

In the Figure the curved line is the heat curve of the solar spectrum.

If we compare it with the absorption spectrum of chlorophyll, we see that the yellow rays especially must furnish the energy required for the assimilation of the carbonic acid, for these are the most completely absorbed, and possess the greatest mechanical energy.

From the preceding we can tell beforehand what influence the different parts of the spectrum—that is, the color of the acting light—will have upon the activity of the chlorophyll.

If we bring green plants into such portions of the solar spectrum[†] as are not absorbed by the chlorophyll, there can be no decomposition of carbonic acid. If we allow all that light to act, corresponding to absorption strip *I*, there is a very active decomposition of carbonic acid and development of oxygen.

The activity of assimilation must then be little less than one-half of that for free exposure, if parts of the ultra red portion of the spectrum are also absorbed by chlorophyll.

The amount of heat obtained by the absorption of strip *I* is about equal to the total heat obtained by the absorption of the entire portion of the spectrum from between *F* and *G*.

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[*] Lommel (Pogg. Ann. vol. cxliii., p. 579) is indeed not entirely of this opinion, but Gerland's objections (Pogg. Ann. vol. cxliii., p. 605) to Lommel's views appear to me well taken. Also, we find here, that J. Müller's doubt, whether the spectrum of green leaves agrees with that of chlorophyll, rests only upon observations made under unfavorable conditions.

[†] In order to obtain sufficient intensity, a concave mirror must be used instead of the plane mirror of the heliostat, and care must be taken, by the use of rock-salt prisms and lenses, that as little heat as possible may be absorbed by the apparatus.

Smaller maxima of carbonic acid decomposition must occur when the plant is brought into strips *II*, *III*, *IV*.

In that part of the spectrum from between *F* and *G* on, there will be feeble decomposition.

If, by suitable means, we separated all the colors from between *F* and *G* to the red end of the spectrum, and allow the others to act upon the plant, the assimilation, assuming that the ultra red rays are of no effect, will be only half as great as for free exposure.

These assumptions, deduced from a comparison of the absorption spectrum of chlorophyll with the heat spectrum of the sun, are confirmed by experiment.

Draper has found the carbonic acid decomposition greatest in the yellow red of the actual spectrum, and his result has been confirmed later by the experiments of Sachs, Prillieux, A. Mayer, Pfeffer, and Baranetzky.

Most of these investigators have worked with colored glasses and solutions, and not with the actual spectrum, and their results therefore cannot be at once made use of.

If useful results are expected with colored glasses and solutions, we must not only determine, as was done by Sachs, the absorption spectrum of the glass or solution, but also, by special photometric measurements, what portions of the parts of the spectrum absorbed by the chlorophyll are still effective in the spectrum of the glass or solution.

When this is done, the results will undoubtedly be in agreement with theory.

Whether by the fluorescent properties of chlorophyll any change will be caused, cannot be decided without further information; but it does not, in view of the theoretical investigations of Lommel, appear probable.

NOTE 32.—(Page 51.)

VIEWS OF MAYER UPON THE PHENOMENON OF THE TIDES.

It may not be superfluous to add a few words upon an interesting astronomical application of the theory, the first suggestion of which is due to Mayer.

We know that, by reason of the combined action of the sun and moon, two waves originate at opposite points upon the sea and traverse the earth, causing the phenomenon of the tide. When the tidal wave meets the coast and shores, it causes currents and counter-currents, which cannot exist without friction and development of heat.

We have thus upon the surface of our planet a generation of heat. But the total living force (energy) of the planet cannot be increased by the mutual action of its different parts, and hence this apparent creation of heat can only be a transformation of other living force (kinetic energy) into the living force of heat (caloric energy).

The ebb and flow of the tides diminish, therefore, incessantly the living force (kinetic energy) of the earth. Probably, both the velocity of rotation and the rectilinear rotation diminish together; that is, the length of the sidereal day increases, and the major axis of the earth's orbit diminishes.

Similarly to the tides upon the surface of the ocean, Falb * has assumed a corresponding tide upon the surface of the fluid interior of the earth, and deduces from this assumption the regular return of earthquakes and volcanic eruptions.

If the solid earth crust and the fluid interior are without any intermediate space between them, such a tide could not form, but the tendency to its formation would be indicated by an increase of pressure of the fluid contents against the crust.

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* Falb, Grundzüge zu eine Theorie der Erdbeben und Vulcanansbrüche. Graz, 1871.

If, however, such tides actually occur in the earth's interior, they can cause effects similar to those of the tides on the surface.

It is, indeed, true that the change in length of the day, and the diminution of the axis of the earth's orbit, are so small that they would be imperceptible even in the course of centuries.* Still, from a theoretical standpoint, these conclusions are none the less interesting.

Such tides have certainly occurred upon the fluid masses of the planets, while they were still in a fluid condition.

It may be that such tides, caused by the action of the earth upon the moon, have deprived it of its axial rotation, while it was still in the fluid condition, and this may be the reason why now it always presents to us the same side.

NOTE 33.—(Page 54.)

UPON A REMARK OF SEGUIN CONCERNING THE STEAM ENGINE.

In order to prove that during the action of a steam engine heat is necessarily used, Seguin remarks that if all the heat taken from the boiler were found in the condenser, this amount of heat would be sufficient to repeat the same action indefinitely, provided that it were possible to concentrate the heat contained in the condenser water, so that with it the fifteenth part of its mass could be heated to 100°, and then converted into saturated steam at this temperature; which is in entire agreement with theory.

We could thus obtain, by means of a finite amount of heat, an indefinite continuance of motion, which is neither probable nor in accordance with sound logic.

This argument is not completely satisfactory, because the concentration of heat assumed by Seguin implies an equivalent expenditure of work or heat. According to this argument, a body must be brought to a temperature of 100° by heat taken from another body at 40°.

We have seen in the preceding presentation under what conditions this is possible.

POSTSCRIPT OF VERDET TO THE NOTES, JULY 16, 1862.

While in press, I received the "L'exposition analytique et expérimentale de la théorie mécanique de la chaleur," by Hirn, forwarded by the author to the Academy at the session of July 7, 1862. In this work Hirn recognizes the error of his earlier conclusions, and gives the explanation of the peculiar results furnished by his experiments upon engines without expansion. Our criticisms upon this savant (page 14) are, therefore, rendered inapplicable.

NOTE 34.—(Page 54.)

THE DEPENDENCE OF THE COLOR OF VENOUS BLOOD UPON THE TEMPERATURE.

For a fuller understanding, it may be well to add some information as to the anatomical and physiological relations which exist in the higher animal organisms.

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* Mayer estimates that the length of the day would be increased by the action of the tides one-sixteenth of a second in 2,500 years.

The blood-vessels are of three classes.

1. The arteries, which carry the blood from the heart to all parts of the body.
2. The veins, which carry the blood back again to the heart.
3. The capillaries, which unite the extreme branches of the arteries and the veins.

In these blood-vessels the blood circulates incessantly, so long as life exists. The apparatus which causes the motion of the blood is the heart. It is divided into two parts, the right and left, by an impervious partition. Each half consists of a ventricle and auricle. Each ventricle is connected by valves with its auricle. The sides of the heart, especially the left, are formed of powerful muscles. From the left ventricle, at each contraction (systole), a portion of the bright blood in it is forced out into the aorta, and thence into all the other arteries. Since the arteries are already filled with blood, they are expanded by the entrance of the new blood. When the pressure from the heart ceases, and they contract (diastole), valves at the entrance of the aorta prevent the return of blood to the heart. The contraction of the elastic walls of the arteries forces the blood forward, through their ramifications, into the capillaries, and from these into the veins. The blood thus proceeding from the heart is called arterial, and is charged with oxygen.

In the capillaries, which permeate all the tissues, this oxygen combines with the carbon and hydrogen of the food. The heat generated by this and other chemical processes is one source of the animal heat of living organisms.

By this combustion carbonic acid is formed, which is held in solution in the blood, and gives it a dark color.

The dark colored blood, thus deprived of its oxygen and permeated with carbonic acid, is called venous blood. The veins carry this blood, and with it many products of digestion, which are likewise taken up by it in the capillaries, to the heart. It enters there the right auricle, and has thus made the "greater circuit." When the contraction ceases and the heart expands, the blood enters from the auricles the ventricles, and thus the venous blood passes from the right auricle into the right ventricle. From this it is expelled by the next contraction, and forced into the blood-vessels of the lungs. These branch out in the lungs into extremely fine capillary tubes, which surround like a network the countless ramifications of the air passages.

Through the very thin sides which separate the blood from the air passages, there is an interchange of gases. The carbonic acid in the blood is given out, and the oxygen of the air is absorbed.

The blood thus retakes its bright red color, and becomes again arterial.

From the capillary vessels of the lungs, this arterial blood passes by four veins back to the heart, and enters the left auricle. From this, at the next diastole, it passes into the left ventricle, and begins at the next systole its course anew.

The passage of the blood from the right ventricle, through the capillary system of the lungs back to the left auricle, we call the lesser circuit.

The human body is thus comparable to a steam engine. The nourishment is the fuel, from the combustion of which arises the increased temperature and power of both. In both cases the oxygen necessary for combustion is taken from the air. To the boiler correspond the capillary vessels. As chimney for the discharge of the carbonic acid formed by combustion, and at the same time as grate through which the air enters, we have the lungs and wind-pipe.

As already noticed, the heat generated in the body by combustion of the nourishment, serves a double purpose. A portion is transformed by the muscles into work; another serves to maintain the temperature of the body constant, that is, to supply the losses by radiation, conduction, perspiration, etc.

Many observations have shown that the temperature of the human body is independent of the outer temperature, and in a healthy condition about 37° C.

When the outer temperature is low, the loss of heat must then be greater, and more heat must be produced to cover this loss than when the outer temperature is higher. For low outer temperature, therefore, more nourishment must be burned, and in regions where the outer temperature is higher, less.

When more nourishment is burned, more oxygen must be inspired, and more carbonic acid formed. Hence in cold climates the venous blood must contain more carbonic acid than in the warm, and the arterial more oxygen.

The difference in color between venous and arterial blood depends upon the greater amount of carbonic acid in the one, and of oxygen in the other. There must, therefore, be a relation between the difference of color and the amount of nourishment consumed, or between this difference of color and the animal heat.

Hence the color of the venous blood must vary with the outer temperature.

In cold climates the venous blood is darker than in warmer. Hence it was that Mayer found in Java the venous blood much redder than in Europe.

This observation furnished him the starting-point for his discovery of the fundamental principle of the mechanical equivalent of heat.*

"THE ENTROPY OF THE WORLD TENDS TOWARD A MAXIMUM."†

In Chapter IV. of the second Lecture we have called attention to an important law: the amount of heat which in a perfect heat engine is transformed into work, and the amount which is simultaneously transferred from a hot body (the furnace) to a colder body (the condenser), stand in a constant relation.

This principle is expressed by the formula

$$q - q' = \alpha \frac{(t_1 - t_0)}{1 + \alpha t_1},$$

or, denoting $1 + \alpha t_1$ by T_1, the absolute temperature,

$$\frac{q - q'}{q} = \frac{T_1 - T_0}{T_1}.$$

Here q is the total heat expenditure of the warmer body, of the boiler, whose temperature is T_1, and q' is that part of this heat which is transferred to the colder body, to the condenser, whose absolute temperature is T_0.

Subtracting both members of the above equation from 1, we have

$$\frac{q'}{q} = \frac{T_0}{T_1}, \quad \text{or,} \quad \frac{q'}{T_0} = \frac{q}{T_1};$$

which may be written

$$\frac{q'}{T_0} - \frac{q}{T_1} = 0 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1).$$

This principle admits of considerable extension, and holds good in somewhat changed form, not only for the processes accomplished in heat engines, but for all processes by which heat is transformed into work, or, inversely, work into heat.

Since the change of heat into work, and likewise the transfer of heat of high temperature into heat of low temperature, is a transformation, this principle is called by Clausius, the law of the equivalence of transformations, and is regarded as the second general law of thermodynamics.

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* See Mayer, Mechanik der Wärme, 1867, p. 239.

† See Clausius, ueber den zweiten Hauptsatz der mechanischen Wärmetheorie, Braunschweig, 1867.

It holds good, as we see, only for cycle processes which are reversible, as in that of the perfect heat engine.

If the cycle process is more complex, the equation above is more generally

$$\Sigma \frac{q}{T} = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

There are, however, transformations of other forms of force into each other, and all follow this law.

Thus, for example, heat changes the arrangement of the molecules, while overcoming outer forces and the action of the molecular forces; while, therefore, it performs work.

When the action of the heat is sufficiently powerful, solid bodies become liquid, and liquids are changed into gases; therefore the state of aggregation is changed. Clausius calls this action of the heat "disgregation," and expresses the phenomenon by the words, "heat increases the disgregation of bodies."

An increase of disgregation corresponds, then, to a change of heat into work, and a decrease to a transformation of work into heat. There must, therefore, exist between the decrease of disgregation and such transformation a causal relation.

That the consideration of this disgregation conducts us to the preceding equation, may be seen from the following example:

Let us consider a quantity of gas which has the temperature t, volume v, and pressure p. Since, under such conditions, the mean distances of the molecules is determinate, the disgregation, which measures the distribution of the gas particles, is determinate. We denote this by Z. If we now allow the gas to expand, or compress it, under constant outer pressure, without change of temperature, the work performed or expended is $p\,(v_1 - v)$, where v_1 is the new volume.

The corresponding amount of heat q, absorbed or set free, is

$$q = \frac{p\,(v_1 - v)}{J} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

Let the disgregation in this new condition be Z_1, then the change of disgregation is

$$Z_1 - Z.$$

Let us now consider an equal amount of the same gas which has for another temperature t_1, the same volume v, and hence another pressure p_1.

The pressures p and p_1, according to the laws of the expansion of gases, are in the relation

$$\frac{p}{p_1} = \frac{1 + \alpha t}{1 + \alpha t_1}.$$

The disgregation of the new gas mass is evidently the same, viz., Z, since the mean distance of the molecules is the same.

If this gas takes the new volume v_1, the change of disgregation is necessarily the same, viz., $Z_1 - Z$.

The work obtained or expended is however evidently different, viz.,

$$p_1\,(v_1 - v),$$

and the heat q_1, which is absorbed or set free, if the temperature of the gas does not change, is

$$q_1 = \frac{p_1\,(v_1 - v)}{J} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

From equations (2) and (3) we have

$$\frac{q_1}{p_1} = \frac{v_1 - v}{J}, \quad \text{and} \quad \frac{q}{p} = \frac{v_1 - v}{J}.$$

The left sides are evidently alike. If we take account also of the equation

$$\frac{p}{p_1} = \frac{1 + \alpha t}{1 + \alpha t_1} = \frac{T}{T_1},$$

we have, when T and T_1 are the absolute temperatures,

$$\frac{q_1}{T_1} = \frac{q}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4),$$

an equation which corresponds perfectly with (1).

We see, therefore, that the amounts of heat necessary to cause the same change of disgregation are inversely as the absolute temperatures at which these amounts of heat are transformed.

The equivalent value of the heat corresponding to a determinate change of disgregation is, therefore, obtained by dividing the heat necessary for this change by the absolute temperature.

These two examples, viz., the theory of machines already alluded to, and the law of disgregation change here laid down, may suffice, if not to prove rigidly, at least to make intelligible the second law of thermo-dynamics.

We have now, in the course of our considerations, become acquainted with three kinds of changes, viz., the change of heat into work, or, inversely, the change of heat at a higher temperature into heat at a lower, and, finally, disgregation changes.

Every transformation of one kind corresponds always to a certain amount of another, and we can therefore say that, in every process, a change of one sort answers always to a corresponding change of another.

Entirely analogous relations can be stated for every transformation of one kind of force into another.

If we assume, temporarily, a unit for two equivalent transformations, we can set them equal.

Since every equation of the form

$$v_1 = v_2$$

can be put in the form

$$v_1 - v_2 = 0,$$

this principle can be expressed as follows:

In every process the algebraic sum of the transformations is zero.

It is necessary, however, to call attention to the limitation which in the one case is expressly made, and in the other is fulfilled, viz., that the process in question must be reversible.

Keeping this limitation, we have the second law of thermo-dynamics in the form as given by Clausius.

"In every process, however complicated, in which one or more bodies undergo reversible changes, the algebraic sum of all the transformations must be zero."

The second law is, therefore, well called the law of the equivalence of transformations, while the first is that of the equivalence of work and heat.

We shall next seek to make it evident, by further examples, that it is really necessary to introduce the limitation that the second law holds only for reversible transformations.

In the example already noticed, in which we consider the change of disgregation of a gas, it was always assumed that the pressure remained unchanged, or, better, that the outer pressure differed from the tension of the gas only by an infinitely small amount. Under this assumption it is possible to again compress the gas by the same outer pressure, and bring it back to its original condition.

The gas then passes through all the changes which it experienced during expansion, but in reverse order.

The gas, however, may experience the same changes of volume and disgregation in another manner.

If, thus, we connect the vessel containing the gas, whose volume is v, with another whose volume is $v_1 - v$, which is exhausted of air, and suddenly open the communication, the gas will enter the empty vessel until there is the same pressure in both. The volume of the gas is now v_1; the change of disgregation is the same as in the previous case.

From the experiments of Joule we know that during such a change of volume there is neither change of temperature nor work performed by the gas.

But the gas cannot be compressed back to its original condition without an expenditure of work and production of heat. The process is not reversible.

If the gas is compressed, its disgregation therefore diminished, we must have work transformed into heat; but, as we have seen, the disgregation may be increased without an equivalent transformation of work into heat or heat into work.

If, now, we call the transformation of work into heat and increase of disgregation positive, and the change of heat into work and decrease of disgregation negative transformations, we see that decrease of disgregation, that is, a negative change, cannot occur without a simultaneous positive transformation; but, on the other hand, increase of disgregation, or a positive change, *can* sometimes occur without a negative transformation.

Let us consider now other modes of transformation. When heat is transformed into work there is always a simultaneous increase of disgregation, or, as in the cycle processes of engines, heat passes from a hot to a colder body. If we call the transfer from the hot to the cold body positive, and the transformation of heat into work negative, we can say, since there is no example in which this negative transformation occurs without a corresponding positive one, that the negative transformation of heat into work is necessarily connected with a simultaneous positive transformation.

The positive change of work into heat can, however, as many examples show, occur without a corresponding simultaneous negative transformation. Thus, for example, in friction, resistance of air, and, in short, most prejudicial resistances, there is a change of work into heat, without simultaneous changes of disgregation, transfer of heat from higher to lower temperature, etc., necessarily occurring.

Here also, therefore, negative transformation of heat into work cannot occur without a simultaneous positive transformation, but positive transformation of work into heat can.

The third mode of transformation considered—viz., the transfer of heat from one body to another, or the change of a quantity of heat Q at the temperature T, into the quantity Q at the temperature T'—also confirms this law.

As is known, there is a natural tendency of heat to pass from a warmer to a colder body, and this process occurs in radiation and conduction without simultaneously giving rise to another.

On the other hand, a process opposed to this natural tendency of heat, the transfer of heat from a colder to a warmer body, can only occur when there is a simultaneous change of work into heat,* or an increase of disgregation.†

If we call, as already indicated, the transfer of heat from hot to cold body positive, and from cold to hot negative, we may conclude from all the cases discussed, that

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* We may recall the example on page 15 of a steam engine compelled by an outer force to reverse its ordinary action.

† As illustration, a hot gas of temperature A'' must be compressed by the expansion of a colder solid body which is heated from A to A', when both A and A' are less than A''.

Negative transformations can only occur when compensated by positive, but positive may occur without negative. Uncompensated transformations can therefore only be positive.

This principle allows of an interesting natural application. We have seen that there is a general tendency in nature to increase of disgregation, to transform work into heat, and to level heat differences. This is indeed but the result of the principle that uncompensated transformations can only be positive.

This tendency is denominated the "dissipation of energy."

The case of a natural transformation which is perfectly reversible is a limiting case which seldom or never occurs. It is, hence, the tendency of the positive transformations to accumulate. The heat of a body which can no longer be transferred to a colder body must remain heat, and can serve no longer for production of work, can no longer be transformed into other forms of action.

The amount of this untransformable heat must, hence, always increase, since it is continually added to by the uncompensated positive transformations.

The consequence is that the world tends toward a final condition in which all its forms of energy will be transformed into heat of uniform temperature, which can no more be transformed.

When this condition is attained all nature will be, and must remain, *dead*.

Thomson has, with rare acuteness, drawn the boldest consequences from these conclusions,[*] and expressed them as follows :

"1. There is in nature a universal tendency to the dissipation of mechanical energy.

"2. A restoration of mechanical energy (negative change) without more than an equivalent of dissipation, is impossible by inanimate material processes, and will probably never be attained by organized matter, whether endowed with vegetable life or with consciousness and will.

"3. Within a finite past period the earth must have been uninhabitable, and within some finite future period the earth must become again uninhabitable for men, beasts, and plants as now constituted. It may be then that processes may have existed or will exist, which are contrary to those natural laws which at present rule the world."

The boldness and scope of the conclusions drawn by Thomson from the simple equations which express the second law of thermo-dynamics, must challenge admiration.

The merit of the first discovery of these equations, however, belongs to Carnot and Clausius. Carnot discovered the second law in its essentials, though the form in which he expressed it was incorrect, since he proceeded from the false assumption of the indestructibility of heat.

Later, Clausius so modified Carnot's principle, that it no longer contradicted the first law and the principle of the conservation of force, and expressed it in correct form.[†] He showed that it followed from the inherent tendency of heat to equalize existing temperature differences.

From the second law, however, it follows that transformation of heat into work can never occur without compensation by a transfer of heat from a warmer to a colder body. But already, at that time, known phenomena taught that heat could pass from a warmer to a colder body without compensation, and that work could be transformed into heat without compensation, as in friction.

Later, Clausius gave in another form its most general expression to the law which we have already expressed, by saying that uncompensated changes can only be positive.[‡]

Clausius denoted the algebraic sum of all those changes which must take place, in order to bring a body into the condition in which it is at present, by the name "entropy."

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[*] Thomson, Proceed. of the Royal Soc. of Edinburgh, April, 1852, and Phil. Mag., 4 Series, vol. iv., p. 804, "On a Universal Tendency in Nature to the Dissipation of Mechanical Energy."

[†] Clausius, Pogg. Ann. Bd. 79, 1850. Abhandlungen Bd. 1, p. 50. "Ueber die bewegende Kraft der Wärme," etc.

[‡] Clausius, "Ueber den zweiten Hauptsatz der mechanischen Wärmetheorie," Braunschweig, 1867, p. 17.

Since now the sum of the positive changes can never be less than the sum of the simultaneous negative change, but in general is greater, it follows that the entropy of the world is continually increasing.

Clausius has formulated this in the words—"*Die Entropie der Welt strebt einem maximum zu.*"

"THE ENTROPY OF THE WORLD TENDS TOWARD A MAXIMUM."

THERMODYNAMICS.

PART FIRST.

GENERAL PRINCIPLES.—AIR AND HOT-AIR ENGINES.

PART FIRST.

CHAPTER I.

GENERATION OF HEAT BY MECHANICAL WORK AND THE REVERSE. —DETERMINATION OF THE MECHANICAL EQUIVALENT OF HEAT BY EXPERIMENT.

Heat Generated by Mechanical Action.—A great number of the phenomena of daily life prove to us that heat can be generated by mechanical work. If, for example, we strike a piece of metal repeatedly with a hammer, the metal becomes heated. The living force of the hammer, which is here destroyed at every stroke, appears thus to be transformed into heat. If we rub two pieces of dry wood together with sufficient rapidity, and during a sufficiently long time, they may even be set on fire. Here, again, we have heat generated by mechanical work. Especially well known is the generation of heat by axle friction when the axle is not well lubricated. Here, again, the work necessary for overcoming the friction generates heat.

These and many other facts have led to the question whether mechanical work and heat are not equivalent, or, in other words, whether by the expenditure of a certain amount of work we can generate a certain fixed quantity of heat.

Approximate Determination of the Heat Generated by Friction.—Count Rumford seems to have been the first to endeavor to answer this question by experiment, and, indeed, seems to have been among the first to express the idea that heat is nothing more than a motion of the molecules of a body.

In the foundry at Munich, of which he was the superintendent, he caused a blunt drill, of about 10,000 pounds in weight, set in motion by horses, to work upon the bottom of a cannon. The cannon was inclosed in a wooden box containing 26.6

pounds of water. At the end of 2½ hours the water was heated from 0° to 100°, or was caused to boil.

Although he made use of two horses for the experiment, he was of the opinion that the work could have been performed by one. Hence the mechanical work of one horse for 2½ hours sufficed to heat 26.6 pounds of water through 100°.

Since, now, one horse can raise in one hour 1,980,000 pounds one foot high, or can perform a mechanical work of 1,980,000 foot lbs., in 2½ hours it would perform 1,980,000 × 2½ = 4,950,000 foot lbs. This work heats 26.6 lbs. of water through 100°, or 2,660 lbs. through 1°. Therefore, to heat one pound of water one degree requires

$$\frac{4{,}950{,}000}{2{,}660} = 1{,}721 \text{ foot lbs.}$$

Let us reduce this result to French measures, which are almost universally used now in science, and which we shall always use hereafter unless the contrary is specially stated.

One pound equals 0.4536 kilogram, and one foot equals 0.3048 meter; hence 1 foot lb. = 0.4536 × 0.3048 = 0.13826 meter-kilograms, and 1,721 foot lbs. = 1,721 × 0.13826 = 237.945 meter-kilograms.

This result is evidently too large,* as we have taken no account of the heat lost by radiation and conduction. Thus it is plain that by the same mechanical work we could have heated a greater quantity of water, or that for the same amount of water less work would have been necessary if all the heat generated had gone to raise the temperature.

Experiment of Davy.—After Rumford we find Sir Humphry Davy announcing clearly that heat is not a matter transmitted from one body to another, as was held by most physicists; but that it consisted, most probably, in a rapid motion of the particles of a body.

He showed that two pieces of ice, when rubbed together, were converted into water, although no heat was imparted to them by exterior bodies. But we know that ice requires for melting a large amount of heat; for instance, to convert one pound of ice at 0° into water at 0° requires no less heat than to raise the same quantity of water from 0° to about 80°. This considerable amount of heat must therefore, in the above experiment, have been generated by the friction of the pieces of ice, and hence Davy concluded that heat must be a kind of motion.

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* [Note that this result corresponds to one *pound* of water heated one degree. One *kilogram*, or 2.2 lbs., will require 2.2 as much work, or considerably more than 424 meter-kilograms.]

In order to confirm still further the truth of this view, he made the following experiment: He placed under the receiver of an air-pump a clock-work, resting upon a piece of ice, in the upper surface of which there was a cavity filled with water. The clock-work caused a toothed wheel to rub against a surface of metal coated with wax. After the exhaustion of the air the clock-work was set in motion, and the wax was melted, while the water in the ice cavity remained fluid. The melting of the wax, therefore, was not due to heat obtained from the water nor from the clock-work; for, in the first case, the water would have been at least partially frozen, and in the other, heat would have been imparted also to the ice, and a portion of it melted.

Mayer, the Founder of the Theory.—Although the savans just mentioned were of the opinion that heat must be a kind of motion, and that a definite expenditure of work must generate an equivalent amount of heat, to Dr. Mayer of Heilbronn belongs the credit of not only stating clearly and definitely the principle of the equivalence of work and heat, but also of deducing a number of conclusions from it, so that he may be regarded as the founder of the mechanical theory of heat.

He claims in his treatises that any natural force, as light or heat, cannot be destroyed, either in whole or in part, any more than matter itself. That what appears to us to be destruction is nothing more than a transformation. He shows that the heat received from the sun by a plant enables it to extract its nourishment from the air and earth—to give out oxygen and absorb hydrogen and carbon—and that this absorbed hydrogen and carbon can furnish again the same quantity of heat received by the plant. That just as these elements furnished heat by ordinary combustion, so they generated heat in animal organisms, and enabled them to perform work, to carry loads, etc. He points out that the carbon, or, in other words, the food, which an animal consumes is proportional to the work it performs—that an increase of work raises the temperature or necessitates an increase of nourishment.

Thus he computes the consumption of carbon by a man in climbing a mountain 10,000 feet high, and puts it at 0.155 lbs. That is, the heat furnished by the combustion of 0.155 lbs. of carbon corresponds to the mechanical effect of raising the man 10,000 feet. Further, he determined by the agitation of water in a vessel the work necessary to raise one kilogram of water one degree, and obtained 365 meter-kilograms, which is thus considerably less than the result obtained by Rumford.

Exact Determination of the Mechanical Equivalent of Heat by Joule in Manchester.—Although the equivalence between work and heat was clearly expressed and proved by Mayer, still there was wanting an exact determination of the amount of work necessary in order to heat one pound or one kilogram of water one degree. Neither the result of Rumford nor that of Mayer can lay claim to great accuracy. Accordingly, the English physicist, Joule, undertook a large number of very careful experiments, in order to determine this number as exactly as possible. He adopted different methods. He caused an iron paddle-wheel, set in motion by appropriate cords and weights, to revolve in a vessel filled with water, and by means of a very accurate thermometer observed the increase of temperature of the water when the weights had fallen through a certain distance. Then he took other liquids, as mercury, oil, etc., and found for these also the increase of temperature for a certain expenditure of work. He also caused two cast-iron plates, immersed in water, to rub one against the other, and compared here also the rise of temperature with the work expended. In another series of experiments he forced water through capillary tubes, and determined the rise of temperature and the work expended. Finally, he determined the increase of temperature when air is compressed in a receiver.

From all these experiments he found that by the expenditure of a certain amount of work an equivalent amount of heat was generated, entirely independent of the nature of the substances experimented upon.

After seven years spent in study of the subject, he undertook, in 1849, another large series of experiments, in which he availed himself of all the precautions which his long experience had made familiar. He thus found for the work necessary to raise one pound of water one degree Fahrenheit, the following results:

772.692 foot lbs. for friction with water—mean of 40 experiments.

774.083 foot lbs. for friction with mercury—mean of 50 experiments.

774.987 foot lbs. for friction with cast-iron—mean of 20 experiments.

When we consider the difficulties attending such experiments, and the care and labor required for the exact determination of the work and temperature, the coincidence of these results is most remarkable. These experiments belong, in fact, to the most memorable in the domain of physics, and entitle Joule to a prominent place in the history of the mechanical theory of heat.

Of all these results, Joule considered those given by water as the most reliable. After making several necessary corrections, he gave the equivalent as 772 foot lbs. for 1° Fahr., or $772 \times \frac{9}{5} = 1389.6$ foot lbs. necessary to raise *one pound* of water 1° Centigrade.

Hence, the work necessary to raise *one kilogram* of water one degree C. is $1389.6 \times 0.3048 = 423.55$ meter-kilograms.

We shall hereafter arrive by entirely different considerations at almost exactly the same result, so that we may regard it as settled that for the generation of one heat unit[*] a work expenditure of 423.55, or, in round numbers, of 424 meter-kilograms is necessary. We call this work the "MECHANICAL EQUIVALENT OF HEAT."

Inversely, by the expenditure of one unit of work we can generate only $\frac{1}{424}$th of a heat unit, and this number we may call the "*thermal equivalent of work.*"

Since one horse-power (French) represents 75 meter-kilograms in one second, we have $\frac{424}{75} = 5.65$ horse-power necessary to raise one kilogram of water one degree in one second.

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[*] The term "heat unit" is usually used to denote that quantity of heat required to raise *one pound* of water one degree Fahrenheit, and the term "Calorie" is employed to denote that quantity of heat required to raise *one kilogram* of water one degree Centigrade. As we use throughout this work French units, the term "heat unit," unless otherwise stated in the text, must be understood as meaning the French heat unit, or *calorie*. In order to distinguish these two heat units from that amount of heat required to raise one pound of water one degree *Centigrade*, we may call this latter amount a "*therm*," or "thermal unit."

EXAMPLE.

Required to raise 2 kilograms of water in 20 minutes from 0° to 100°, what expenditure of work per second is necessary?

The 2 kilograms require $100 \times 2 = 200$ heat units, or a work of $424 \times 200 =$ 84,800 meter-kilograms. Since this work is to be performed in 20 minutes, or 1,200 seconds, the necessary work per second is $\frac{84800}{1200} = 70.66$ meter-kilograms per second, or nearly one horse-power (French). (One French horse-power, or 75 meter-kilograms per second, is about 542.5 foot lbs. per second, or somewhat less than an English horse-power, which is 550 foot lbs. per second.)

Hirn's Determination of the Mechanical Equivalent.—Hirn, a civil engineer at Colmar, has made a careful and difficult determination of the mechanical equivalent of heat. He adopted a different method from Joule, and attempted to determine the heat set free by the impact of inelastic bodies.

Two heavy blocks, one of wood the other of iron, were suspended like pendulums. The wooden block had an iron plate upon the side in contact with the other. The iron block was now raised a certain height, and a hollow lead cylinder placed upon the iron plate, which was struck by the descending iron block, and thus compressed and heated.

From the weight of the iron block, and the height through which it fell, Hirn calculated its living force, or mechanical effect. Moreover, the height to which the wood block was driven by the shock, as well as that to which the iron block rebounded, gave the work remaining in the masses after impact. Now, directly after the shock, the lead cylinder was filled with water, and the increase of temperature of the water determined. After skillfully determining the work expended in the compression of the lead cylinder, it was easy to calculate the work expended in the heating. The mean of various determinations gave 425 meter-kilograms, or the same, almost, as given by Joule.

Performance of Mechanical Work by Heat.—In the preceding we have seen that heat is generated by work, or, generally, that when heat appears work disappears, and that by a certain expenditure of work we can always generate a certain amount of heat. The question now arises, Can we generate mechanical work by heat, or does an equivalent amount of heat always disappear when work appears? We have, at once, in the expansion of solid bodies, a most striking proof of the performance of work by heat. Upon the surface of bodies we have the air pressure of about 10,334 kilograms per square meter (15 lbs. per sq. inch), and this pressure is overcome through a certain distance during the expansion of the body. Since, however, when we heat a body, its temperature also rises, it is difficult to determine how much of the heat imparted goes to overcome this pressure, and how much contributes to the rise of temperature. It is, therefore, difficult to determine from the expansion of solid bodies the relation which exists between the heat imparted and the mechanical work obtained.

The gaseous bodies afford the easiest proof of the generation of work by heat, or of the disappearance of heat when work is obtained. If, for example, we allow compressed air to issue from a receiver, there is a decrease of temperature, sometimes so great that drops of water near the orifice may be frozen. The work of the air is here the overcoming of the outside air pressure as it expands.

Joule found, by similar experiments, a work performed of 820 foot lbs. for every degree Fahr. through which the air was cooled. This corresponds to $\frac{9}{5} \times 820 \times 0.3048 = 448.88$ meter-kilograms for every degree C., or somewhat greater than already found. The method of determination, however, is less exact, and the discrepancy was to be expected.

During the expansion of ordinary atmospheric air, also, heat disappears. If we assume air of atmospheric pressure under the piston *EF* in the cylinder *ABCD*, Fig. 8, and if there is a vacuum above it, the piston, if not held fast, will rise, while the temperature of the air will sink. The heat which thus disappears is the equivalent of the work done in raising the weight of the piston through the distance traversed.

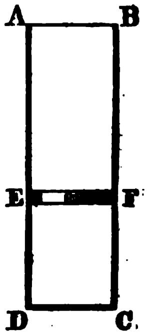

Fig. 8.

Although, now, heat thus disappears when work is done by the air, as when it rushes into the atmosphere or raises a weight, we should not expect to find any such disappearance when the air expands in a vacuum, because in such case no work is performed. This point, also, Joule has experimentally investigated.*

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* See, also, page 22 of Introduction.

He made use of two copper vessels connected by a pipe. Both were placed in a vessel of water, the temperature of which was determined directly before and after the experiment. Connection being closed, the air was compressed to 22 atmospheres in one vessel, and was exhausted from the other. The cock was then turned, and the compressed air rushed into the empty vessel until the pressure was the same in both. There was found to be neither a rise nor fall of temperature while the transfer took place, the temperature of the water in which the vessels were immersed remaining the same both before and after the experiment.

We may explain this as follows: As soon as a part of the air has passed from one vessel into the other, the air continuing to enter must, to be sure, perform work in compressing the air already there. There must be, therefore, in the vessel from which the air flows, a decrease of temperature, but in the other, where the air is being compressed, an increase of temperature precisely equal, so that, on the whole, heat is neither lost nor gained.

That this is actually the true explanation was proved by inclosing the vessels in two separate cisterns, and observing the temperature of each, both before and after the experiment.

It is therefore proved, at least for air, that when heat disappears work is gained; there is also no room for doubt that for every unit of heat disappearing, the same mechanical work is gained which we have already found to be necessary for the generation of one unit of heat, even although Joule found for each disappearing heat unit a somewhat greater work. One heat unit is therefore equivalent to a mechanical work of 424 meter-kilograms.

But Hirn has proved the correctness of this principle for steam also, by means of the steam engine. In fact, this machine shows us at once how heat can be transformed into work, and it was this especially which early led physicists to the conclusion that there must be a certain equivalence between heat and mechanical work.

Hirn determined first the temperature and tension of the steam in the boiler and the steam consumption per stroke. He was thus able to determine the number of heat units carried per stroke to the cylinder. Then he observed the temperature and quantity of the condensing water used for each stroke, as well as the temperature of this water after the condensation of the steam. He was thus able to determine the loss of heat experienced by the steam during its action in the cylinder, as well as that due to the resistances in the steam pipe, in the valve box, and in the exhaust pipe. Finally, he endeavored to estimate the loss of heat due to conduction and radiation between the boiler and the condenser. He then determined the mechanical work imparted to the fly-wheel, and also, as accurately as possible, the work absorbed by the prejudicial resistances, such as the friction of the piston, the slide valve, etc. He was thus able to calculate the work for each unit of heat disappearing in the cylinder. He found as a mean of many determinations 413 meter-kilograms, a number which agrees quite closely with Joule's results, especially when we consider the very great difficulties which Hirn had to contend with.

Still another apparatus affords us clear proof that heat can be transformed into mechanical work, viz., the Gifford injector — an apparatus which has for several years been used for furnishing feed water to locomotives and stationary engines. In this contrivance a pipe leads from the steam space in the boiler to the water space. This pipe joins another at a suitable place, which brings the feed water. The steam flows toward the water space and forces the air with it. A partial vacuum is thus caused in the feed-water pipe, and the water rises in it until it meets the steam current. By this it is carried along with great velocity and forced into the boiler.

Since, now, the water in the boiler is acted upon by the steam pressure, it is not at once evident how it can be possible that the steam flows round toward the water, instead of the water being forced into the steam space. When we remember, however, that the steam contains a large amount of latent heat, which the water does not possess, or, in other words, that the work inherent in the steam is greater than that in the water, we can easily understand the possibility of the action.

When the steam comes in contact with the cold feed water, a part of the latent heat goes to heat the water, but another part is transformed into mechanical work, and this it is which forces the water into the boiler.

QUESTIONS FOR EXAMINATION.

Give instances of heat generated by mechanical action. What conclusion do such instances point to? Describe experiments which test this conclusion. What did Count Rumford conclude? What chances for error were there in his experiment? Describe Davy's experiment. What precautions did he take against possible objections? Do you regard his experiment as conclusive? Why? What was the part played by Mayer as regards the theory? In what respect was it different from that of Rumford and Davy? What conclusions did he deduce from his views? Who first made an exact determination of the mechanical equivalent? Describe some of the methods he employed. How long did he devote himself to the subject? What did he consider on the whole the most exact result? Define precisely what you understand by the "mechanical equivalent of heat." Illustrate. What is a "foot-pound?" What is a "horse-power?" What are the equivalent French units of measurement? What is a meter-kilogram? What constitutes a French horse-power? How does it differ from the English? What system is used in this work? Define exactly what you understand by a "heat unit." What do you understand by "calorie?" Do we use the term calorie in this book? Why not? What is the mechanical equivalent in foot-pounds for Fahrenheit scale? What for Centigrade scale? What is it in French measure? Show how to reduce one to the other. Define exactly what you understand by "thermal equivalent of work." Illustrate.

What work is necessary to raise 2 kilograms of water from 0° to 100° C.? What horse-power (French) is required to perform this work in 20 minutes? What work is necessary to raise 2 pounds of water from 0° to 100° C.? From 0° to 100° F.? What horse-power is required in each case to perform these works in 20 minutes? How do you reduce Centigrade degrees to Fahrenheit, and *vice versa?* Define "work." What other term is sometimes employed? What methods did Hirn employ in order to determine the mechanical equivalent? Did it confirm the result obtained by Joule? Is the mechanical equivalent constant? Were it to vary, what impossible result could you logically deduce? (*See Introduction.*) Deduce this consequence. (*Intr.*) Is perpetual motion possible? Why not? (*Intr.*) Are you fully satisfied of this impossibility? (*Intr.*, *Notes.*) Can any scheme for attaining it rightfully claim attention? Why not?

Give instances of mechanical work performed by heat. Describe Joule's experiments in this direction. When air expands why does its temperature fall? When it expands into a vacuum does its temperature fall? Why not? Describe Joule's experiment here. Under what circumstances does the temperature remain constant in this experiment? Under what does it vary? Is this in full accord with our principle?

Who first proved this principle for steam? How was it done? What sources of error must be guarded against? Enumerate in order the principal steps of the experiment. Does this experiment afford additional proof of the correctness of our principle? What other apparatus illustrates it? Explain the secret of its action. Are you convinced that our principle is the expression of a natural law? What do you understand by a "natural law?" State concisely the law of equivalence of heat and work. What do you understand by "equivalence?" Are things equivalent necessarily identical? Is our numerical determination exact? If not, would that affect your belief in the truth of the law? Why? What did Davy and Rumford suspect "heat" to consist of? What do you understand by "work?" If their suspicions were correct, how would "heat" differ from "work?" Does the proof or truth of our principle rest upon any such hypothesis? Does it rest upon any hypothesis? Are you firmly convinced of its generality and truth?

CHAPTER II.

HEAT A KIND OF MOTION.

FROM what has been said, we may regard it as settled that with a certain amount of mechanical work we can always generate a certain amount of heat, and inversely, that with a certain amount of heat a definite amount of mechanical work can be produced; that the amount of heat which can raise the temperature of one kilogram of water one degree is equivalent to a mechanical work of 424 meter-kilograms, and that, inversely, by the expenditure of this work one kilogram of water can be heated one degree.

If, now, such an equivalence exists between heat and mechanical work, we are compelled to assume that heat is properly nothing else than mechanical work in another form. And since heat is possessed by bodies, since it can enter from without into bodies, we must expect to find here something similar or equivalent to mechanical work. We must therefore assume that either the atoms of a body, or the molecules or groups of atoms, or that some other substance between the atoms is set into some sort of motion. Such a substance is the *ether*, which is universally regarded as the medium for the transmission of light.

One theory, applicable to gaseous bodies, was first propounded by Daniel Bernoulli. This was further developed by Krönig and Clausius, and applied to solid and liquid bodies also. It numbers at present the most adherents.

Another theory, which was at an earlier period the most widely accepted, is due especially to Poisson and Cauchy, and was developed principally by Redtenbacher.

Redtenbacher's Theory.—According to Redtenbacher the body atoms attract each other, while between the ether atoms there is a mutual repulsion. The body atoms are incomparably larger than the ether atoms, and these last have inertia, but are not affected by gravity.

In consequence of these properties each body atom is surrounded with an envelope of ether atoms. In proximity to the atom this envelope has its greatest density, which diminishes with the distance from the center. Thus the ether surrounds each atom just as the atmosphere surrounds the earth, provided that the atoms are round. An atom with its envelope Redtenbacher calls a "*dynamide.*" Between every two dynamides there is a space incomparably greater than the dynamide.

The heat of a body consists now in a pulsating motion of the ether envelopes. It is assumed that each ether atom vibrates in the direction of the radius of the dynamide. The quicker and stronger the pulsations of the envelope, so much the hotter is the body.

If the attraction of the body atoms is equal to the repulsion of the ether envelopes, the body is either solid or liquid. If, on the other hand, the repulsive force of the latter is greater than the attractive force of the former, the body is gaseous.

In the solid state the body atoms are generally closer together than in the liquid; their mutual attractions are greater, the ether envelopes are denser, and their atoms vibrate through shorter distances. Displacement of the particles is thus difficult. A complete union of the atoms, however, cannot occur, for the nearer the atoms approach, the greater is the repulsion of the ether envelopes, until finally there is again equilibrium between attraction and repulsion.

As now the ether envelopes of a solid body pulsate faster and stronger, or as the body becomes warmer, the body atoms recede from each other, their attraction becomes less, until finally ever so small a force can displace the particles. The body is then liquid.

When the ether envelopes no longer pulsate, that is, when there is no motion within the body, the body possesses no heat—it is then *absolutely cold.* We shall soon see to what degree of the thermometer this "absolute zero" of temperature corresponds.

Other Views as to the Nature of Heat.—The other view as to the nature of heat starts also with the assumption that the atoms, or groups of atoms, the so-called molecules, are set into vibratory motion. This view, as already indicated, was indorsed by the famous English chemist Sir Humphry Davy. He says, in his treatise upon heat and light, that heat, or that force which prevents the direct contact of the body atoms, and gives rise to the sensations of heat and cold, may be defined as a special, and, in all probability, vibrating motion of the body atoms which tends to separate them. This may be called a repulsive action. Since there is also an attractive force, we may conceive the body atoms as acted upon by two opposite forces, that of attraction and that of repulsion. The first of these forces is the combined action of cohesion, which strives to hold the atoms in contact; of gravity, which tends to collect them into masses, and of the pressure exerted by exterior bodies. The second of these forces is due to a certain vibrating motion which tends to keep them apart, and which may be generated, or rather increased, by friction or impact. The action of cohesion, in causing the body atoms to approach, is precisely similar to the attraction of gravitation upon the large masses of the universe, and the repulsive force, to the centrifugal force of the planets. In his "Chemical Philosophy" also, Davy says that all the phenomena of heat can be explained by assuming that in solid bodies the atoms are in a permanent condition of vibratory motion, and that these vibrations become more rapid and larger as the temperature increases. In liquid and gaseous bodies we must assume that the atoms, besides their vibratory motion, have also a motion around their axes, and that both these motions are greatest in gaseous bodies.

According to Clausius, who has contributed much to the development of the mechanical theory of heat, it is less the atoms themselves than the groups of atoms, or the molecules, which are in motion. The manner in which the atoms combine to form a molecule, and the form of the same, determine the properties of the body. Hence it may happen that the same simple bodies or elements possess now these, now those properties, and thus we may have the so-called allotropic condition. It is evident that in chemically compound bodies also there may be such a various grouping of atoms; indeed, in such case the variety of grouping can be even greater.

While now, in solid bodies, there is in all probability a vibrating motion of the molecules, we are obliged to assume in liquids a rotating motion also, since in such bodies a molecule easily separates from its neighbors.

The vaporization of liquids, according to this view, may be explained as follows:

As heat is imparted to a liquid so that the velocity of vibration becomes greater and greater, and the force of cohesion ever less and less, the molecules finally break loose and move, like the particles of gaseous bodies, in straight lines. This may be illustrated by an experiment. Suppose at the end of a spiral spring a weight fastened, and one end is held in the hand while the weight is swung round in a circle. The spring will be extended and the weight will recede as the velocity increases. Finally the centrifugal force becomes greater than the tenacity of the spring, which then breaks, and the weight flies off in a tangential direction.

As the attractive force of the molecules is not the same for different liquids, some require more, some less heat to convert them into vapor.

It is also evident that vaporization takes place more rapidly at the surface of a body than in the interior, because the molecules are not restrained by the pressure of those above. Thus, if the centrifugal force of a molecule in the interior of a body is not sufficient to overcome the action of those above it, it is obliged to retain its earlier vibratory or rotating motion which it possessed as a liquid molecule. In such case we say that the liquid "simmers." Further, it is evident that the air molecules which impinge against the surface of the liquid must partially impede the process of vaporization. We say partially, because we assume that there are large spaces between the air molecules through which the liquid molecules can move. Yet it may often happen that an air molecule comes in contact with a liquid molecule just as it is about to leave the surface. If therefore the density of the air diminishes, or if it is entirely removed, the vaporization goes on quicker for the same temperature, more particles breaking loose. These results are, as we know, confirmed by experience.

When vaporization takes place in the interior of a liquid, the centrifugal force of the molecules must be so great as to overcome the pressure upon them. This pressure consists not only of the weight of superincumbent liquid, but also of the pressure of the air upon the surface of the liquid. The centrifugal force must therefore be greater the deeper the molecules lie beneath the surface. If they still break loose, their centrifugal force must be equal to the pressure of the air, increased by the weight of superincumbent liquid—and hence it may be that, for instance, in deep vessels the same mass of water is made to boil with more difficulty than in shallow.

Let us now notice briefly how combustion is regarded. We may call attention, first, to the following observations:

If we allow a weight to fall from a certain height upon a plate of lead or iron, its living force is entirely or partially destroyed by the impact, according as the bodies are entirely or partially inelastic. But if we now examine the plate, we shall find that it has been heated, and heated more according as more of the living force of the weight has disappeared. The mechanical work inherent in the weight has thus been transformed after impact into heat.

In combustion the process is similar, but instead of large masses we have here to do with body particles which elude observation even with the microscope. Thus when we heat, for example, pure carbon, the particles are not only set into much more rapid vibration, that is, their living force increased, but, at the same time, they recede from each other. It is thus more easy for the impinging oxygen atoms to penetrate between the atoms of carbon, and by reason of greater adhesion to unite with them. By this impact the rectilinear motion of the oxygen atoms is transformed into the vibratory or rotating motion of the carbon molecules, that is, generates heat. At the same time every carbon atom unites with two oxygen atoms to make a molecule of carbonic acid, which then takes a rectilinear motion.

Heat Conduction and Radiation.—In the foregoing we have briefly reviewed the different views as to the nature of heat, and have sought to explain some of the best known phenomena in accordance with these views. Let us now inspect somewhat more closely the phenomena of radiation and conduction, in order to deduce that the heating of a body *is not so much the motion of ether envelopes, but rather consists in a greater velocity of* *vibration of the body molecules.* Although the facts are familiar to us all, yet it may not be uninteresting to call attention to them again.

We say that a body is heated by conduction when it receives heat from another with which it is either in direct contact or by the intervention of another.

In such case the molecules of the warmer body impart their greater living force in part to those of the colder, until the molecules of both possess the same living force, or are equally warm.

A body is heated by radiation when it receives heat from another without the intervention of a third. If, for example, we stand near a hot stove, or expose ourselves to the sun's rays, we receive heat even when the surrounding air is cold. *Heat rays* must therefore proceed from the source of heat, just like light rays from a source of light, which rays excite in us the sensation of heat.

The researches of physicists have shown that these heat rays follow the same laws as those of light; that, for example, they are in similar manner reflected and refracted, and show the same phenomena of interference which have made us acquainted with the nature of light.

We may at present assume with certainty that light rays are propagated in a similar manner, and arise in a similar manner, to sound waves in the air or water waves in the water.

Luminous bodies possess the power of putting into vibration the ether which pervades all space and all bodies. These vibrations reach our eyes, excite the retina, and thus cause sight. Just as high tones are caused by quicker, and low tones by slower vibrations of the sounding body and of the air, so different colors are caused by quicker and slower vibrations of the ether.

Since, now, radiant heat follows the same laws as light, *we must attribute its origin and transmission to the vibrations of the ether atoms.* Of this the following experiment will convince us:

If we allow a ray of light to enter a dark room through an aperture, and to pass through a triangular prism held in front of the aperture, the ray will be not only deviated or refracted, but the ray originally white will be split up into various colors, of which we may distinguish especially seven, the so-called prismatic colors, or colors of the rainbow. All the colors bear the name of "spectrum," and when obtained by the decomposition of the sun's rays, of the solar spectrum.

Without the prism, we obtain a light strip upon the screen, of the form of the aperture, in the straight line drawn from the aperture to the sun. With the prism, the light strip is very much broader, is deviated toward the refracting angle of the prism, and is vividly colored. The colors, beginning with those least refracted, are red, orange, yellow, green, blue, indigo blue, and violet.

If we examine the spectrum formed upon the screen with a sensitive thermometer, or a thermo-electric pile, which shows least differences of temperature, we shall find that the violet rays have the least heating power, and the red the greatest. Indeed, if we examine beyond the red rays, where no color can be perceived, we find that the heat is still greater than in the spectrum itself, and these invisible rays extend beyond the spectrum a distance about equal to its own length. Thus the so-called heat spectrum has about double the length of the color spectrum.

Hence it appears that the heat rays are less refracted than the light rays, since they are less deviated.

For the sake of completeness we may also add the following:

If we extend our examination beyond the violet, we find here rays which have the greatest influence upon chemical combinations. In fact, these rays have the most powerful photographic effect, and hence they constitute the chemical spectrum.

Physicists have computed that for red light the ether must make not less than 481 billion vibrations in a second, while for violet light 764 billions are necessary. The heat rays, then, are caused by ether vibrations, on the whole less than 481 billion per second. On the other hand, the number of vibrations of the chemical rays is more than 764 billion per second.

When, now, the vibrating ether meets a body, it sets in vibration either the ether existing in the body, or the atoms of the body, or both. If the ether in a body is so constituted that it can transmit the vibrations which it receives from white light, it is transparent and colorless. If it can only transmit those vibrations which it receives from the violet rays, while the others are lost, it appears violet in color, and so on. In an opaque body the ether is so constituted that it cannot transmit the vibrations communicated to it by a source of light.

While, now, those ether vibrations generated by a source of light, and which, we must assume, possess a greater velocity but a less amplitude of vibration than heat rays, set into vibration the *ether* contained in bodies, it is those vibrations which arise from a source of heat, and which have a less velocity but a greater amplitude of vibration, which, chiefly by impact, set the *atoms* of a body into vibration, and thus heat it. Since, however, the heat spectrum is about double the length of the light spectrum, and since, therefore, there are heat rays of different refrangibility and velocity of vibration, we might expect that the molecules of one and the same body are differently excited by different heat rays, and the body differently heated—indeed, that certain heat rays may not excite the molecules of certain bodies at all, but only the contained ether, and thus that these bodies are not heated by such rays. Science furnishes a number of confirmations of this conclusion.

If, for example, we let fall upon a plate of fluor-spar of about 2.6 millimeters in thickness, rays from various sources, such as the electric lamp, glowing platinum, heated copper, the heat transmitted is found to be in each case as the numbers

$$78 : 69 : 42.$$

Thus the rays from the lamp excite the ether in the fluor-spar more than the atoms or molecules, while those from the copper act inversely.

Still more striking is the transmission of different heat rays by beryl, calkspar, rock-crystal, etc.

Different bodies also transmit different amounts of heat from one and the same source. Thus, for example, clear rock-salt transmits 92 per cent. of the rays from the electric lamp, fluor-spar, on the other hand, only 78, colorless alum 9, and very clear ice only 6 per cent.

Of all bodies rock-salt, and air especially, transmit equally well rays from different sources.

Following Melloni, an Italian physicist to whom we are indebted for valuable researches upon radiant heat, we call those bodies which transmit the heat rays, diathermanous, and those which do not, athermanous.

If a body cannot transmit through its mass the heat rays which strike its surface, these rays are either reflected, according to the same laws as light rays, or they are absorbed and heat the body. The greater the amount of heat transmitted, the less is that absorbed. Thus, for example, rock-salt becomes heated but little when exposed to the sun's rays or other sources of heat, because it is very diathermanous, while smoked glass, which transmits but few rays, is heated much more.

Hence it follows that in absorption of heat, also, there is considerable difference among bodies, and that the same body does not receive equal amounts of heat from different sources. Thus while, for example, soot absorbs almost completely the rays from the electric light, white lead absorbs only 0.53, shel-lac only 0.43, and a metal surface only 0.14.

On the other hand, the absorption power of white-lead for rays from the electric light, from glowing platinum, or from copper heated up to 400°, is as

$$53 : 56 : 89.$$

The same holds for most other bodies. Soot alone seems to absorb equal amounts of heat from all sources. Just as in respect to light it is perfectly opaque, so is it in respect to the heat rays.

If, now, we investigate the heat rays which a body heated by absorption again emits, we shall find that these rays are entirely independent of the nature or quality of those absorbed. Very extensive experiments have shown that *the heat rays emitted by a body are always the same, whatever the source from which the body is heated, whether the electric lamp, glowing platinum, or even contact with a warmer body.*

This seems to indicate plainly that the heat rays absorbed by a body are completely altered in character. It seems also indicated by the fact that the absorbed heat is propagated very slowly in the interior of a body, while the radiant heat has a velocity equal to that with which heat travels through air and space—a velocity probably not less than that of light itself.

We can therefore hardly assume that the heating of a body by absorption of heat rays consists in setting the body ether into more rapid vibration; for in such case it would be hard to see why the heat is not, as in diathermanous bodies, entirely or partially transmitted, and why the propagation in the body is so slow. It is much more probable that *the atoms or molecules of athermanous bodies are excited by the impinging heat rays, and put into quicker vibration, and that this is the cause of the heating. This* conclusion seems not without interest.

QUESTIONS FOR EXAMINATION.

What amount of work is required to raise the temperature of one kilogram of water one degree ? How many pounds make a kilogram ? What two theories are there ? Give Redtenbacher's theory. What do you understand by "atom ?" What by "ether ?" What by "molecule ?" What by "dynamide ?" In what does the heat of a body consist according to Redtenbacher ? Under what conditions is the body solid or liquid ? When gaseous ? What was Davy's view ? How did Clausius differ ? Explain, according to this view, the solid, liquid, and gaseous states. In what respects does experience confirm it ? How is combustion explained ? When is a body heated by conduction ? What is the theoretical process ? When is a body heated by radiation ? Explain the process. What are heat rays ? What facts prove that heat and light rays are identical ? What is the solar spectrum ? What different rays do we distinguish in it ? What physical difference is there between the heat, light, and chemical rays ? What physical similarity ? What facts go to prove that it is the motion of the molecules of a body rather than its contained ether atoms which constitute its heat ? When is a body said to be transparent ? Opaque ? Athermanous ? Diathermanous ? Are the heat rays emitted from a body always the same in kind ? What does this indicate ? What, then, seems most probably to constitute the heat of a body ? Are good radiators of heat good absorbers ? Why ? Explain now, in detail, your ideas of the heating of a body by a distant source, and the processes of radiation and conduction. Do these or any other theoretical views affect in any degree the validity of our general principle of the equivalence of heat and work ? Does our principle depend upon any hypothesis as to the nature of heat ? Upon what does it depend ? Have these views any value ?

CHAPTER III.

INNER AND OUTER WORK.—LATENT AND SPECIFIC HEAT.

WHATEVER may be the motion in the interior of a body, whether a motion of ether envelopes according to the views of Redtenbacher, or a motion of the molecules as assumed by Clausius and most other physicists; whatever may be the character of the motion, this much at least is established—that heat is some sort of motion of the particles, atoms, or molecules. If the mass of a particle is m, and its velocity v, then $\frac{1}{2}mv^2$ is its living force. If we assume the entire weight of the body to be G, then

$$\frac{v^2}{2g} G$$

is the entire inner living force of the body, or its entire "*inherent energy*," if v is the mean velocity of an atom.

The imparting of heat to the body has, in general, three effects.

1st. *The temperature of the body rises—its "sensible heat" is increased.*

2d. *The body expands—its volume is increased.*

3d. *In this expansion the exterior pressure—generally that of the atmosphere—is overcome.*

Different Works performed by the Heat.—We see, then, that the heat performs a threefold work.

1st. Since the rise of the temperature consists in an increase of the living force of the particles, it must perform a work equivalent to this increase.

2d. Since the molecules mutually attract each other, a certain work is necessary to alter their mutual positions; or, what is the same thing, to move their common center of gravity.

3d. Since the body is pressed upon on all sides by the air, a certain work is required to overcome this pressure through a certain distance.

That work which goes to increase the velocity of the molecules, and therefore to increase the sensible heat of the body, we call the "*vibration work,*" while that work which is necessary to displace the particles we call the "*disgregation work.*"

Outer and Inner Work.—Since, in the practical applications of the mechanical theory of heat, that work necessary to overcome the outer pressure is of especial importance, we shall follow Zeuner, and call the work which increases the velocity of vibration, together with that which changes the aggregation of the particles, that is, the "vibration work" and the "disgregation work," the "*inner work,*" while we shall call the other, which overcomes the outer pressure, the "*outer work.*"

Starting, then, from the experimentally proved law that "Heat and mechanical work are equivalent," we can lay down the following most important fundamental principle of the mechanical theory of heat:

The amount of heat (expressed in heat units) imparted to a body is directly proportional to the simultaneously produced inner and outer work.

Specific Volume—Specific Pressure.—In our discussions we shall, in general, take the *kilogram* (about 2.2 pounds) as the unit of weight. The volume of any body of this weight we call the *specific* volume, and shall denote it by v. V indicates the volume of a body which weighs more or less than just one kilogram. The pressure upon each square meter of surface of any body we call the *specific* pressure, and shall denote it by p. We therefore assume that the pressure upon each unit of surface is the same. We also assume in what follows *that the pressure upon the interior of the surface of a body, at any instant, is just equal to the outer pressure, or varies from it by an infinitely small amount.* That is, the "body tension" is at any moment just equal to the outer pressure. Whenever we deviate from this assumption we shall specially indicate it.

[We shall do well to distinguish between the "body tension" and the outer pressure. If we conceive the body inclosed by a tight-fitting envelope, or skin, the tension of this skin is the body tension. This may or may not be equal to the external pressure. Unless distinctly stated to the contrary, we suppose that the body tension at any moment differs from the outer pressure only by an indefinitely small amount, and hence that any change from one state to another is continuous, and therefore reversible. The outer pressure is thus a property of the body.

The case is quite different when we assume that equilibrium between the outer pressure and body tension does not exist during the changes of condition, and therefore that equilibrium is only attained at the beginning and end of the change, when the body has passed from one condition of equilibrium to another. The deportment of a body during such a transition is evidently of a very different character, and we are able to follow the transition only in a few special cases, in which, by certain assumptions upon the law of change of the outer pressure (which is considered quite independent of the body tension), we are able to determine the final condition of the body after the occurrence of a new state of equilibrium. The two cases may be represented by a rod, which in the first case is stretched by a force at any instant greater only by an infinitely small amount than the force with which at that instant the rod resists extension, and in the second case, acted upon suddenly by a constant force. In the first case the rod is gradually extended, and the work of extension can be easily shown to be $\frac{Gl}{2}$, where G is the final force and l is the extension. In the second case it can be shown that the rod is elongated twice as far, and the work up to the moment of greatest elongation is $2Gl$. Further, the end of the rod will vibrate up and down like a pendulum whose length is l, and finally come to the same state of equilibrium as the first.

In every case, therefore, unless otherwise stated, we assume that during change of state by accession or withdrawal of heat the outer pressure is equal at any instant to the body tension. Thus in the case of a gas expanding, it overcomes a pressure just equal to its own tension or pressure upon the piston at any instant. When this is the case the change of state is always *reversible*, i. e., by compression the gas passes through all intermediate states in reverse order back to original state.]

Fundamental Equations of the Mechanical Theory of Heat.—If, now, we impart to the unit of weight of any body one heat unit (that is, so much heat as will raise the temperature of one kilogram *of water* from 0° to 1° C.), we increase its total energy by 424 meter-kilograms; that is, the increase of its inner and outer work is equivalent to a work of 424 meter-kilograms. If, however, we impart Q heat units, then the work performed upon the body, or received by it, is $424 \times Q$. If we denote this work by E, we have for the energy imparted to each unit in weight by the reception of Q units of heat,

$$E = 424 \times Q,$$

and inversely,

$$Q = \tfrac{1}{424} E.$$

The fraction $\frac{1}{424}$, or that portion of one heat unit, or that amount of heat which is equivalent to one unit of work, we designate in general by A.

We thus have

$$Q = A \times E.$$

If, now, W is that portion of the whole energy which goes to the increase of the vibration work, J that portion which goes to disgregation work, and L that portion which corresponds to the outer work, we have $E = W + J + L$, and hence

$$Q = A(W + J + L) \quad . \quad . \quad . \quad . \quad . \quad (I.)$$

Since we call both W and J together the inner work, let us represent it by U,* or $U = W + J$, and we then have

$$Q = A(U + L) \quad . \quad . \quad . \quad . \quad . \quad . \quad (II.)$$

Change of Sign of the Terms in Equation I.—Now in Equation I., any one or more of the terms in the parenthesis may be zero, or may be negative. In such case, for the same amount of heat imparted, Q, the remaining terms must be greater. We may illustrate by a few examples.

It has been already remarked, that, in order to convert ice at 0° into water at 0°, not less than about 80 heat units are necessary. When, therefore, we add heat to the ice, we do not increase its vibration work, *i. e.*, its sensible temperature at all, but the heat imparted performs disgregation work and outer work. In Equation I., therefore, W is zero. This is the case with almost all other bodies. The heat which thus disappears, apparently, since it is not sensible to the thermometer or to our nerves while a body is melted, we call the "*latent* heat."

The experiments of Regnault and Person give the latent heat of several bodies as follows:

Water......	79.25	Tin	14.252	Zinc.....	28.13
Phosphorus.	5.034	Lead	5.369	Mercury..	2.83
Sulphur....	9.368	Bismuth..	12.640		

In passing from the liquid to the gaseous state heat also disappears, and the amount thus disappearing is in general greater than in liquefaction.

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* [Kirchoff calls the quantity U the "working function;" Thomson, "the mechanical energy of a body in a given state;" Clausius understands by "inner work" only that portion denoted by J, which we call "disgregation work."]

Thus, for example, according to Brix, the latent heat of steam is 540; that is, to convert one kilogram of water at 100° into steam at 100°, 540 heat units are necessary; that is, just as much heat as is required in order to heat 540 kilograms of water from 0° to 1°. Thus, referring to theory, a greater expenditure of work is required to give the particles a rectilinear motion, such as we assume for gas, than to impart those rotary motions which are supposed to constitute a liquid.

For alcohol, sulphuric ether, and oil of turpentine, Brix found the latent heat of evaporation, 210, 89.96, and 74.04, respectively.

If we heat water from 0° to 4°, it does not expand, as is generally the case when bodies are heated, but contracts, and thus the molecules approach each other. The outer work is thus negative. In fact the outer pressure here assists the heat, so to speak. Without this pressure we would have to impart more heat in order to raise the same water up to 4°. If, therefore, we increase the pressure, the heat necessary to be imparted is less.

We have still greater contraction when we convert ice at 0° into water at 0°. Ermann observed that the volume of water at 0° is only $\frac{9}{10}$ths of that of the ice at the same temperature. Here, then, the outer work has a still greater negative value, and therefore, under increased pressure, the latent heat of the ice ought to be diminished, *i. e.*, its melting point lowered. Mayer predicted this in his contributions to the mechanical theory of heat, and the experiments of Mousson have completely confirmed it. This physicist showed by a very ingenious experiment that under a pressure of about 13,000 atmospheres the melting point of ice was lowered about 18°, that therefore, under this pressure, ice became liquid at − 18°, instead of 0°.

Cast iron and bismuth also contract when they pass from the solid to the liquid condition. The most striking example of contraction when heated is, according to Sir William Thomson, shown by vulcanized rubber. When a tube of this material is fastened at one end, and a weight of ten or more pounds is hung from the other, this weight is raised by heating the tube. We see here, therefore, very plainly, that the outer work performed is negative.

Increased Pressure can also Raise the Melting Point.—If a body expands under the action of heat, this expansion will be less when the pressure is increased, if the heat imparted is the same in amount. And inversely, if the expansion is the same, more heat must be imparted. If a body in melting, then, follows the general rule and expands, its melting point must rise when it is subjected to a greater pressure. In this case the outer work, L performed during expansion, is greater, and accordingly the heat imparted, Q, must be also greater. These theoretical conclusions are also confirmed by numerous experiments. Thus Hopkins has found for the melting point of various substances, under varying pressures, the following results:

Pressure in Atmospheres.	MELTING POINT.			
	Spermaceti.	Wax.	Sulphur.	Stearine.
1	51° C.	64.5° C.	107° C.	72.5° C.
519	60°	74.5°	135.2°	73.6°
792	80.2°	80.2°	140.5°	79.2°

Heat which must be Imparted to Gases under different Conditions for equal Rise of Temperature.—In the case of gases, whose molecules are not bound together by mutual attractions, no work is expended in changing the state of aggregation. When, therefore, we impart heat to a gas, the vibration work is increased, and, under certain circumstances, outer work is performed. It is not difficult here to do away with the outer work also, and then a less quantity of heat is necessary to heat the gas a certain number of degrees than when outer work is performed during the expansion. This may be illustrated by the following experiment:

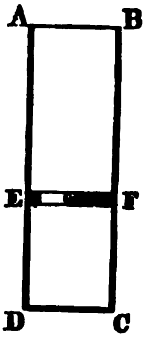

Fig. 9.

Suppose that air is inclosed in the cylinder $ABCD$, Fig. 9, below the air-tight piston EF. When the air is heated it expands and raises the piston. The heat imparted then goes to increase the vibration work and to perform outer work. Let us now make the piston fast, so that expansion is impossible; then the outer work is zero, and less heat is necessary in order to raise the air the same number of degrees as before.

We can easily make here the outer work negative. We have only to suppose that, while the air is being heated, weights are gradually applied to EF, so that, in spite of the rise of temperature, the piston sinks, and the air is compressed. The work performed thus by the sinking piston corresponds to a certain amount of heat, and the whole amount imparted, in order to raise the air to the same temperature, is less than before by just this amount.

Change of the Fundamental Equations when Heat is Abstracted.—If we take heat from a body, inverse phenomena occur; the vibration work is in general diminished, and there is therefore a decrease of temperature; also the molecules approach nearer, and the outer work is negative. Equation I. accordingly takes the form

$$-Q = A(-W - J - L).$$

Just as in Equation I. different terms in the parenthesis can be zero or negative, so here different terms can be zero or positive.

We have an example in the case of water. When this is cooled from 4° to 0° it does not contract, but expands, the outer work, L, is therefore positive. This positive outer work is still greater when we abstract from the water at 0° its latent heat of liquefaction, and thus convert it into ice at 0°. In this case the vibration work, or sensible heat, remains the same, and hence $W = 0$. Since, now, $W = 0$, and L is positive, J must have a so much greater negative value, that is, the molecules are the more strongly attracted. We can explain this only by assuming that the molecules are arranged in a definite and regular manner, that is, that the ice has a crystalline constitution.

As in the freezing of most other bodies there is a contraction, the outer work is negative when the latent heat of liquefaction is abstracted, and hence the decrease of the vibration work is zero. For these bodies, for the same withdrawal of heat, the attraction of the molecules increases relatively less rapidly than for water.

Also, when steam loses its latent heat of vaporization and becomes liquid, the decrease of its vibration work, or its sensible heat is zero. As we say, the steam is *condensed*, since steam occupies a very much greater space than the liquid from which it is generated, the outer work, *L*, must have a very great negative value.

This work Papin sought to utilize. The cylinder *ABCD*, Fig. 10, contained water, which was heated until all the air was driven out. Then the air-tight piston was inserted, and cold water applied to the cylinder, thus condensing the steam. The atmospheric pressure then forced the piston *HH* down, and thus raised the rod *I* by means of the lever *FG*. When the piston arrived at the bottom the water was again heated, the expansive force of the steam balanced the air pressure, and the weight of *I*, which was heavier than the piston, then raised the latter to the top of the cylinder. The steam was then again condensed, and so on.

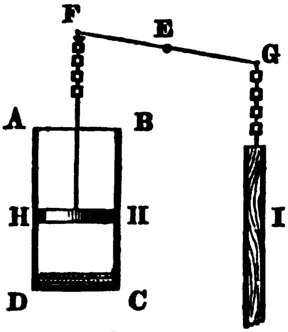

Fig. 10.

As is well known, Newcomen first practically utilized this idea in England, in his atmospheric engine, which Watt later converted into the steam engine.

This process, by which a body, as the water in this case, is changed from one condition to another, and from this back again to the first, is called a "*cycle process.*"

We have, also, in hot-air engines a similar cycle process, but instead of water it is air which is made to change its condition. First, a certain volume of air is compressed, then heated, and thus outer work is performed, then cooled back to its original condition, and so on.

Specific Heat.—If we impart equal amounts of heat to different bodies, the increase of vibration work, that is, of their sensible heat, is very different.

We may illustrate this by an experiment. If we mix one kilogram of water at 10° with one at 30°, the temperature of the mixture is $\frac{1 \times 10 + 1 \times 30}{2} = 20°$. For, since we require, in the one case, to raise one kilogram of water to 10°, $1 \times 10 = 10$

heat units, and, in the other, 1 × 30 = 30 heat units, there are required for both $10 + 30 = 40$ heat units, or $\frac{40}{2} = 20°$ for each kilogram.

If, now, we mix one kilogram of water at 10° with one kilogram of *iron* at 30°, we shall find that the temperature of the mixture is 12°. Thus the iron has lost $30 - 12 = 18°$, and this heat has been imparted to the water. But this heat does not raise the kilogram of water 18°, but only 2°. What the iron has lost is, apparently, not gained by the water.

If, on the other hand, we mix one kilogram of water at 30° with one kilogram of iron at 10°, we shall find the temperature of the mixture to be 28°.

The 2°, or the 2 heat units lost by the water, thus raise the temperature of the iron 18°. One heat unit, therefore, will raise one kilogram of iron 9°.

We see, therefore, that the same amount of heat which causes in water a certain rise of temperature, has an effect nine times as great for iron.

If, again, we have one kilogram of water at about 80° (more exactly 79.25°), it will, as we know, render completely liquid one kilogram of ice at 0°; and, on the other hand, with one kilogram of iron at 80° we can melt only $\frac{1}{9}$th of a kilogram of ice at 0°.

Also, if we allow equal quantities of water and iron, at the same temperature, to cool in the air, we shall find that the temperature of the iron sinks nine times as fast as that of the water. But since the air abstracts from both bodies, in the same time, equal quantities of heat, the water must possess nine times as much heat, at the same temperature, as the iron.

Experiments have shown that two bodies seldom occur for which equal weights are raised by equal amounts of heat the same number of degrees.

We call that *amount of heat*, expressed in heat units, which is necessary to raise one kilogram, or generally, one unit of weight, of a body one degree, the "*specific heat*" of the body. It is evidently the same whether French or English heat units are used.

In the following table we have the specific heat of various bodies, as determined *under constant pressure*, by the exact and careful experiments of Regnault:

Antimony	0.0508	Mercury	0.0333
Bismuth	0.0308	Nickel	0.1110
Carbon	0.2414	Phosphorus	0.1887
Cobalt	0.1070	Platinum	0.0324
Copper	0.0952	Silver	0.0570
Gold	0.0324	Sulphur	0.2026
Iron	0.1138	Tin	0.0562
Lead	0.0314	Zinc	0.0956
Manganese	0.1217		

Volume Capacity.—The specific heat defined above is the quantity of heat expressed in heat units necessary to raise one kilogram, or one *unit of weight* of a body, one degree. We may therefore call it the heat capacity of one unit of weight, or generally, the "*weight capacity.*"

But we may also easily determine the quantity of heat required to raise *equal volumes* of different bodies one degree.

One kilogram of water occupies a space of one cubic decimeter, since one cubic meter of water weighs 1,000 kilograms. Now, the density of chemically pure iron is 7.8439 times as great as that of water, hence one cubic decimeter of iron weighs 7.8439 kilograms.

Since, now, one kilogram of iron requires 0.1138 heat units to raise its temperature one degree, 7.8439 kilograms, or one cubic decimeter, requires 7.8439 × 0.1138 = 0.8926 heat units. This quantity of heat we may call the "*volume capacity*" of the iron. In general, we understand by volume capacity of a body *that amount of heat necessary to raise equal volumes one degree in temperature.*

We may obtain it, as shown by the example above, *by multiplying the specific gravity of the body by its weight capacity, or its "specific heat."*

[We owe the latest researches upon the specific heat of gases to Regnault; but these researches give only the specific heat for constant pressure, that for constant volume has not been as yet directly determined. Let c_p be the weight capacity for constant pressure, and ω_p the volume capacity for constant pressure, which, as we have seen above, can be found from the weight capacity by multiplying by the specific weight. Then we have

	c_p.	ω_p.
Air	0.23751	0.00030714
Nitrogen	0.24380	0.00030625
Oxygen	0.21751	0.00031099
Hydrogen	3.40900	0.00030533

The values of the weight capacity for constant pressure are, as we see, different for different gases. Hydrogen is most noticeable. Its specific heat is inded greater than for any other body, solid or liquid. After hydrogen comes water, whose specific heat (c_p) is 1. For others the specific heat for constant volume is less, and indeed for most much less than 1.

Regnault also found the specific heat of gases, especially of air, *constant* for different pressures and temperatures, a beautiful confirmation of a prediction made by Clausius, as early as 1850, upon theoretical grounds alone.

As regards the volume capacities for constant pressure, we see that they differ but little, so little that one is inclined to assert that the deviations are due merely to errors of observation. If we calculate, however, the volume capacity for the other gases given by Regnault, we find that approximate equality exists only for those gases which are furthest from their point of liquefaction, and in which, therefore, the disgregation work is of little or no account. We can, therefore, conclude that equality exists only for "perfect" gases, and this conclusion is supported by theory.

As to the specific heat of gases for *constant volume*, as already remarked, a direct determination is not yet attained. We can, however, determine this value indirectly, at least for air. If c_p is the weight capacity of air for constant pressure, and c_v for constant volume, we can in various ways determine the ratio $k = \frac{c_p}{c_v}$. Such a determination we shall hereafter give when we come to apply the principles of the mechanical theory of heat to the solution of different problems. One of these methods, used by Gay-Lussac, Clement, and Desormes, and later by Masson (*Wüllner, Experimental Physik*, Bd. 2, p. 279), gave respectively $k = 1.372$, 1.357, and 1.419. Another method, by Hirn (*Theorie mécanique de la chaleur*, p. 69), and Weisbach (*Civilingenieur*, Bd. 5, p. 46), gave 1.3845 and 1.4025. Further, a comparison of the results of the formulæ for the velocity of sound, by Dulong, with observations upon the progression of sound in air, gave $k = 1.421$, and Dulong found similar values also for nitrogen, oxygen, and hydrogen. A similar comparison, with the results of observation by Moll and Van Bech upon the velocity of sound, gave for air $k = 1.410$.

This last value is regarded at present as the most reliable, especially as it is justified by the results of other researches.

Taking for air, then, $k = \frac{c_p}{c_v} = 1.41$, and c_p according to Regnault 0.23751, we have for the specific heat of air for constant volume $c_v = 0.16844$.

Since the outer pressure may be supposed indefinitely varied, there may be, strictly speaking, an indefinite number of values for the specific heat of a body, one for each different law of variation of pressure. We have considered above only two special cases, viz., for constant pressure and for constant volume. We shall see later how to find the specific heat for any given law of variation of pressure with volume. In Regnault's experiments the bodies were subjected to the constant pressure of the atmosphere, and in accepting his results we must not therefore neglect the fact that they strictly hold good only for constant outer pressure.

For solid and most liquid bodies the expansion is very slight, and therefore the heat converted into work insignificant in such case. For gases, however, as we have seen, the difference is great.

It is evident that the specific heat for constant pressure c_p must always be greater than that for constant volume c_v, because in the first case heat is absorbed by the outer work, and for the same rise of temperature more heat must be imparted.

In general, we have for the heat imparted for a rise of temperature dt, for the *unit of volume* heated under constant volume, in which case dL the outer work is zero,

$$dQ_v = \omega dt + AdJ.$$

For a perfect gas dJ, or the increase of disgregation work, is zero, and then ω is the specific heat of the unit of volume for constant volume (volume capacity). If, however, J is not zero, then the value which we have until now called the volume capacity has a complex significance. Without doubt dJ is, under the above conditions, positive, since we must admit that the smallest particles act attractively upon each other, and dJ represents the work which even by constant volume is applied to disgregation of the molecular groups. Rankine calls the value of ω in the above equation the "real specific heat," and hence we should conclude that the "apparent specific heat" deviates the more from the real, and is so much greater than it, the more the gas departs from the perfect condition.

The *total heat*, then, imparted in order to raise a body one degree in temperature is the *apparent* specific heat; if from this be subtracted all the heat expended in performing interior and exterior work, the remainder is the *real* specific heat, because it alone measures the actual heat of the body.]

The Disgregation Work, in Solid and Liquid Bodies, is very small in comparison with the Vibration Work.—We have already repeatedly defined the "unit of heat" as that quantity of heat which must be imparted to one kilogram of water in order to raise its temperature one degree. But now, if we impart to, say one kilogram of water, so much heat that its temperature is raised one degree, we have not only increased the vibration work or sensible heat, but also the disgregation work as well. The heat imparted must therefore be greater than it would have been had we only increased the sensible heat of the water. Of this fact no account was taken when the idea of the heat unit was first formed and the specific heats of bodies determined, because at that time the principles of the mechanical theory of heat were unknown. If, then, the disgregation work were considerable and varied much for different bodies, these determinations of specific heats would have little or no value. Fortunately this is not the case. Thus Regnault, whose experiments are the most reliable, made his observations upon bodies subjected only to the pressure of the atmosphere. The outer work and disgregation work thus performed were but slight in comparison with the vibration work.

For example, let us suppose one kilogram of water at 4°, that is at its greatest density, contained in a cylindrical vessel of one square decimeter in cross-section. The depth of the water is then exactly one decimeter. If this water is raised one degree in temperature it expands, according to the elaborate experiments of Kopp, about 0.000006 of its volume at 4°. The water thus rises in the vessel about 0.000006 of a decimeter, and through this distance the atmospheric pressure of about 103.34 kilograms per square decimeter is overcome. We have therefore the outer work equal to $103.34 \times 0.000006 = 0.00062004$ decimeter kilograms. But since the mechanical equivalent of the heat imparted is 424 meter kilograms, the outer work is, as we see, very small in comparison. The disgregation work, or the work expended in separating the particles, is also very small. Thus, according to the experiments of Grassi, the coefficient of compression of water, that is, the amount it is compressed by an increase of pressure of one atmosphere, is only 0.00005 of its volume when the compression takes place between 4° and 5°. If, then, we increase the pressure upon the water in our vessel by one atmosphere, the water is compressed about 0.00005 of a decimeter. This corresponds to a work therefore of $0.00005 \times 103.34 = 0.005167$ decimeter kilograms. We see thus that the disgregation work is also very small. We may, therefore, for slight rise of temperature, disregard the entire disgregation work for *solid and liquid* bodies.

Since for high temperatures the coefficient of expansion increases, that is, the increase of volume for a rise of 1° is greater, the disgregation work must increase, and, for the same amount of heat imparted, the vibration work or sensible heat be proportionally less. If the increase of this last is the same as before; that is, if the body at the higher temperature is raised also 1°, then the total heat imparted is greater than before. We see, therefore, the reason why *the specific heat increases with the temperature.*

It is also a fact that liquid bodies expand more for the same rise of temperature than solid. It is therefore very probable that the same body has a greater coefficient of expansion when in the liquid condition than when solid; and this renders it also probable *that the specific heat of a body when liquid is greater than when solid.* For many bodies this has already been confirmed by careful experiments.

Specific Heat for Constant Volume and for Constant Pressure.—If we conceive a body so confined that it cannot expand when it is heated, then all the heat imparted goes to increase the vibration work, and there is no disgregation work or outer work at all. For the same amount of imparted heat, therefore, the sensible heat will be greater than when the body is free to expand. Inversely, a less amount of heat would be necessary to cause a certain rise than when free to expand.

The amount of heat measured in heat units which must be imparted in order to raise one kilogram weight of any body 1° in temperature when expansion cannot take place, is called the "*specific heat for constant volume.*"

We call, on the other hand, that amount of heat which is necessary to raise the same unit weight, one kilogram of any body, 1° in temperature, when the body is allowed to expand under the constant pressure of the atmosphere, the "*specific heat for constant pressure.*" It must evidently be greater than the first, because heat is required to perform the disgregation work and outer work which take place in the second case.

[Both these specific heats refer to the *unit of weight*, and are therefore "weight capacities," the one under constant volume and the other under constant pressure. We might also have two "volume capacities" in the same circumstances. No use is made of such quantities. "Specific heat" refers always to the *unit of weight*, and indeed, unless distinctly stated, we always understand a *constant volume* to be presumed.]

[The following tables give the mean specific heats for constant pressure of the substances named, according to Regnault. These specific heats are average values, taken at temperatures which usually come under observation in technical applications. The actual specific heats of all substances, in the solid or liquid states, increase slowly as the body expands or as the temperature rises, and when great accuracy is required tables of specific heats must be consulted, which will give these quantities with greater definiteness at special temperatures.

SOLIDS.

Antimony	0.0508	Steel (soft)	0.1165
Copper	0.0951	Steel (hard)	0.1175
Gold	0.0324	Zinc	0.0956
Wrought Iron	0.1138	Brass	0.0939

SOLIDS—*Continued.*

Glass	0.1937	Ice	0.5040
Cast Iron	0.1298	Sulphur	0.2026
Lead	0.0314	Charcoal	0.2410
Platinum	0.0324	Alumina	0.1970
Silver	0.0570	Phosphorus	0.1887
Tin	0.0562		

LIQUIDS.

Water	1.0000	Mercury	0.0333
Lead (melted)	0.0402	Alcohol (absolute)	0.7000
Sulphur "	0.2340	Fusel Oil	0.5640
Bismuth "	0.0308	Benzine	0.4500
Tin "	0.0637	Ether	0.5034
Sulphuric Acid	0.3350		

GASES.

	Constant Pressure.	Constant Volume.
Air	0.23751	0.16847
Oxygen	0.21751	0.15507
Hydrogen	3.40900	2.41226
Nitrogen	0.24380	0.17273
Superheated Steam	0.4805	0.346—
Carbonic Acid	0.217	0.1535
Olefiant Gas	0.404	0.173
Carbonic Oxide	0.2479	0.1758
Ammonia	0.508	0.299
Ether	0.4797	0.3411
Alcohol	0.4534	0.3200
Acetic Acid	0.4125	——
Chloroform	0.1567	——]

[There are three methods employed for determining the specific heat of a body,

(1) Method of mixture.
(2) Method by fusion of ice.
(3) Method of cooling.

(1) *Method of mixture.*—The body whose specific heat is to be determined, is raised to a known temperature, and is then immersed in a mass of liquid of which the weight, specific heat, and temperature are known. When both the body and the liquid have attained the same temperature, this is carefully ascertained.

Now the quantity of heat lost by the body is the same as the quantity of heat absorbed by the liquid.

Let c, w, and t be the specific heat, weight, and temperature of the hot body, and c', w', and t' of the liquid. Let θ be the temperature the mixture assumes.

Then, by the definition of specific heat, $c \times w \times (t - \theta) =$ heat units lost by the hot body, and $c' \times w' \times (\theta - t')$ = heat units gained by the cold liquid. If there is no heat lost by radiation or conduction, these must be equal, and

$$cw(t - \theta) = c'w'(\theta - t'),$$

or

$$c = \frac{c'w'(\theta - t')}{w(t - \theta)}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (1)$$

The same method of mixture may be made use of to determine the latent heat of a body, or the heat required to reduce a unit weight of a solid body at the melting point to a unit weight of liquid at the same temperature.

Let a solid body of weight w_1 and at temperature t_1 have the known specific heat c_1, and let the specific heat of the *same substance* in the liquid condition be c_2, and the temperature of the melting point be t_2. Let the body be immersed in a liquid whose weight is w', temperature t', and specific heat c', and when it is all melted let the common temperature be θ. The solid body is then first raised to the melting point, which requires $c_1w_1(t_2 - t_1)$ heat units. It is then melted, and since, when melted, the resulting liquid has the same temperature of the melting point, all the heat required to melt it is latent. If x is the latent heat of fusion, in heat units per unit of weight, the total latent heat is xw_1. To now raise the resulting liquid to θ°, requires $c_2w_1(\theta - t_2)$ heat units.

Hence the total heat imparted is $c_1w_1(t_2 - t_1) + xw_1 + c_2w_1(\theta - t_2)$. The heat lost by the liquid in which the body is immersed is $c'w'(t' - \theta)$. We have then

$$c_1w_1(t_2 - t_1) + xw_1 + c_2w_1(\theta - t_2) = c'w'(t' - \theta),$$

or

$$x = \frac{c'w'}{w_1}(t' - \theta) - c_2(\theta - t_2) - c_1(t_2 - t_1). \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

If the temperature of the body before immersion is just at the melting point, $t_1 = t_2$ and

$$x = \frac{c'w'}{w_1}(t' - \theta) - c_2(\theta - t_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (3)$$

If, in addition, the liquid in which the body is immersed is the same substance as the solid body, $c_2 = c'$ and

$$x = \frac{c'w'}{w_1}(t' - \theta) - c'(\theta - t_2). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

This last is the case when ice at 0° C. is immersed in water at t' degrees C.

If a pound of ice at zero C. is immersed in a pound of water $79^\circ.25$ C. it is found that the temperature of the mixture when all the ice is melted is just 0°. We have, then, $c' = 1$, $w' = w_1$, $t' = 79.25$, $t_2 = 0$, $\theta = 0$, and hence $x = 79.25$ = latent heat of water.

From (1) we can, by experiment, determine the specific heat of any solid body. From (4) we can find the latent heat in the melting of any body, and from (2) or (3) we can find the specific heat c_2 of the solid when in a fluid condition.

We can also find from (1) the temperature of a mixture when we know the specific heat of the substances used.

(2) ***Method by fusion*** (**Black's Calorimeter**).—The substance whose weight is w, is heated to a known temperature t, placed in a block of ice and then covered with ice. When it has cooled to 0° C., the cavity in the ice is wiped out with a dry cloth, the increase in weight of the cloth giving the weight of ice melted. Let i = weight of ice melted. Then since it takes 79.25 heat units to melt one unit weight of ice,

$$cw(t-\theta) = i \times 79.25,$$

or, since in this case $\theta = 0$, $cwt = 79.25\,i$, or $c = \frac{79.25\,i}{wt}$.

This method is open to the objection that some ice is melted by heat from surrounding objects, etc.

Calorimeter of Laplace and Lavoisier.—This consists of three vessels. The two outer ones contain powdered ice, and in the third and interior one, the body is placed. If any heat passes into the apparatus from exterior objects it will liquefy some of the ice in the outer vessel, but will not affect the inner one. All the melting which takes place in the middle vessel is therefore due to heat from the body to be tested. The water thus produced is drawn off and weighed. We have then, as before, $c = \frac{79.25\,i}{wt}$.

The method is open to the objection that some of the water adheres to the ice and is not drawn off.

Bunsen's Calorimeter.—The weight of ice melted is determined by measuring the contraction which takes place in its volume. A small vessel (A) is fitted into a larger one. The space between is filled with water and the inner vessel is cooled, by the rapid evaporation of ether, until the surrounding water is converted into ice. The body is placed in A. As it cools it melts some of the surrounding ice. A narrow tube, the area of whose section is known, is fitted into the larger vessel, and partially filled with mercury. As the ice melts, the mercury recedes in the tube, and thus the amount of contraction is determined.

As ice loses about 9 per cent. of its volume when liquefying, the weight of ice melted can be deduced from the change in volume.

(3) ***The method by cooling.***—This consists in determining the time in cooling of equal weights of different bodies of equal surfaces—these times have the same ratio as the specific heats.

Example 1.—*A piece of iron weighing* 750 *grs., at a temperature of* 200° *C. is placed in ice at* 0° *C. Taking latent heat at* 79, *and specific heat of iron at* 0.114, *how much ice will be melted?* Ans. 216$\frac{36}{79}$ grs.

Example 2.—*A pound of iron at* 100° *C. is placed in a Laplace calorimeter, and the water which flows out is found to weigh* 0.144 *lb. Taking latent heat at* 79, *what is the specific heat of iron?* Ans. 0.11376.

Example 3.—*In Bunsen's calorimeter the area of cross-section of the mercury tube is* 0.01 *inch. It is found that* 78 *grains of iron introduced at the temperature* 100° *C. cause the mercury to move* 0.5 *inch. If the weight of a cubic inch of ice is* 227.25 *grains, and it contracts* $\frac{1}{10}$ *of its volume in melting, find the specific heat of iron.* Ans. 0.115085.]

QUESTIONS FOR EXAMINATION.

Whatever may be the views held as to the nature of heat, what have we thus far established? Is this firmly established? What are the proofs? What do you understand by "living force?" What by "inherent energy?" If a body of mass m has a velocity v, what work can it perform in coming to rest while overcoming resistance? Is the setting of a body in motion "work?" If heat is a motion of bodies, is it not then identical with work? If it is not such motion, should we expect to find any equivalence? Do we find any equivalence? What is it?

When a body is heated, what three effects are in general produced? In what does the rise of temperature consist? What three works are performed when a body is heated? What do you understand by vibration work? Disgregation work? Outer work? Inner work? How do we measure heat? What is a "heat unit?" To what is the amount of heat imparted to a body proportional? How many heat units are equivalent to 2120 meter-kilograms? How many meter-kilograms are equivalent to 12 heat units? What does A denote in our notation? If E is work expressed in meter-kilograms, and Q is amount of heat given in heat units, what is the general equation between them? What is the general relation between Q and the vibration work (W), disgregation work (J), and outer work (L)? What does U denote in our notation?

Define "specific volume"—"specific pressure." What does v denote? What does V denote? What do you understand by "body tension?" What is the relation between body tension and outer pressure which is assumed unless otherwise stated?

In the equation $Q = A(W + J + L)$, what do the letters denote? Give a familiar example where W is zero. What other examples can you think of? Define "latent heat?" What is the latent heat of steam? Give a familiar example in which L is negative. Can increased pressure raise the melting point? Under what circumstances is this true, and why? Give a familiar example where J is zero.

When a gas is heated, what works are performed? When a gas is not free to expand, does it require more or less heat to raise a given weight of it a given number of degrees? Why? When a gas can expand when heated, what works does the heat imparted perform? When it can not expand? What is the outer work in the latter case? Can the outer work ever be negative? Give a familiar example.

When we abstract heat from a body what occurs in general? Can L ever be positive in this case? Give an example. Is W ever equal to zero? Give examples.

Define exactly what you understand by "specific heat." Why is it called "specific?" What is the specific heat of water? If the specific heat of iron is $\frac{1}{10}$th, what does that mean? What do you understand by "volume capacity" for heat? What by "weight capacity?" How can you find the volume capacity from the weight capacity? What is real specific heat? Apparent specific heat?

What is the disgregation work in solids and liquids? How does it compare with the vibration work? Can you illustrate this? What do you understand by coefficient of expansion? Why does the specific heat increase with the temperature? Why should the specific heat of a body when liquid be greater than when solid? What do you understand by specific heat for constant volume? What for constant pressure? Which is the greatest? Why? When we simply say "specific heat," without further limitation, what do we mean?

What do you mean by latent heat of water? Would it be correct to say latent heat of ice? Why not? If c_v is the specific heat for constant volume, how many units of heat would be necessary to raise k kilograms of a body t degrees?

How many pounds of mercury at the temperature of 300° are required to raise 15 pounds of water from 60° to 70°?

If two liquids have the weights w and w', the temperatures t and t', and the specific heats c and c' respectively, what is the temperature of the mixture?

Reduce $-40°$ Fahr. to Centigrade degrees. Reduce $-273°$ C. to Fahrenheit degrees. How do you reduce generally Fahrenheit to Centigrade degrees, and *vice versa?*

What outer work is performed when 2 pounds of air are heated from 60° to 70° Fahr. under the pressure of the atmosphere? What, when 3 kilograms are heated from 0° to 1° C.?

What is the specific heat of air under constant volume? Under constant pressure? Show how to find from these the mechanical equivalent in French measures? In English measures?

See Examples for practice at end of volume.

CHAPTER IV.

EXPANSION OF GASES.—SPECIFIC HEAT OF GASES.—DETERMINATION OF MECHANICAL EQUIVALENT OF HEAT.

Expansion of Gases when Heated.—As we have just seen, the disgregation work and the outer work are very small indeed for solid and liquid bodies, and in comparison with the vibration work may be neglected. But for gaseous bodies it is different. Here there is no attraction between the molecules, or if any, it is exceedingly small, so that there is no disgregation work. All the heat imparted to a gas goes, therefore, to increase the vibration work, that is, to raise the temperature and to perform outer work. This last is, for gases, much greater than for solid and liquid bodies, because they expand much more for the same rise of temperature.

Let us now seek to ascertain the amount of this expansion, as well as the other properties of gases.

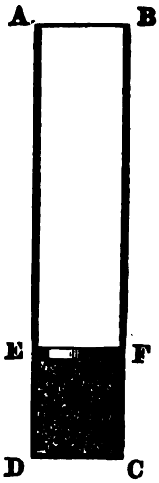

Fig. 11.

Suppose that below the piston *EF*, Fig. 11, in the cylinder *ABCD*, we have one cubic meter of air at 0° and ordinary tension, corresponding to $760^{\mathrm{mm.}}$ of the barometer. This air weighs, then, according to the experiments of Regnault, 1.29318 kilograms (one cubic foot of air at 32° weighs 0.080744 lbs.). If, now, we heat the air, it expands for every degree Centigrade $\frac{1}{273} = 0.00367$ of its volume at zero. This coefficient has been determined by Regnault and Magnus from a series of very careful experiments. It is therefore the *coefficient of expansion of air.*

If, now, this air volume of one cubic meter is heated to 2, 3, 4, etc., degrees, it expands $2 \times \frac{1}{273}$, $3 \times \frac{1}{273}$, $4 \times \frac{1}{273}$, etc., or 2×0.00367, 3×0.00367, 4×0.00367, and the original cubic meter becomes, at these several temperatures,

$1 + 2 \times 0.00367$ cubic meters,
$1 + 3 \times 0.00367$ " "
$1 + 4 \times 0.00367$ " "

etc. If heated to $t°$, we have, therefore,

$$1 + 0.00367t \text{ cubic meters.}$$

If instead of one cubic meter we had 2, 3, 4, or in general V cubic meters to start with, we should have when heated to $t°$,

$$V(1 + 0.00367t) \text{ cubic meters.} \quad . \quad . \quad . \quad \text{(III.)}$$

If heated to 100°, the one cubic meter becomes

$$1 + 0.00367t = 1 + 0.00367 \times 100 = 1.367 \text{ cubic meters,}$$

and if heated 273°, we have

$$1 + \tfrac{1}{273} \times 273 = 1 + 1 = 2 \text{ cubic meters.}$$

If, therefore, air is heated from 0° to 273° (or from 32° to 523° Fahrenheit), it expands to double its original volume.

This law, according to which air expands, is called the law of Gay-Lussac.

It is evident that the density of the air under the piston EF, or the weight of a unit of volume, diminishes as the volume increases. Since, for example, when heated 273°, our one cubic meter becomes two, and yet still weighs 1.29318 kilograms, the density at this temperature is only one-half of that at 0°.

Since the densities of two bodies are inversely as their volumes, provided that the weights are the same, we can find the density D, of the air for any temperature t, from the proportion,

$$1 : D = V(1 + 0.00367t) : V,$$

where we assume the density at $0° = 1$, and the volume at this temperature $= V$.

We obtain, therefore, the density for any temperature t,

$$D = \frac{V}{V(1 + 0.00367t)} = \frac{1}{1 + 0.00367t} . \quad . \quad . \quad \text{(IV.)}$$

[or for Fahrenheit degrees $D = \dfrac{1}{1 + 0.002038\,(t - 32)}$, where t is the temperature Fahrenheit.

One cubic foot of air at 32° F. weighs 0.080744 lbs.]

For example, for $t = 10°$ and $t = 100°$, we have $D = \frac{1}{1 + 0.0367} = 0.9646$, and $D = \frac{1}{1 + 0.367} = 0.7316$. One cubic meter of air at 10° weighs, therefore,

$$0.9646 \times 1.29318,$$

and at 100°, 0.7316×1.29318 kilograms.

[For $t = 10°$ Fah. and 100° Fah. we have the weight of one cubic foot in lbs. at 10° Fah.,

$$1.0469 \times 0.080744,$$

and at 100° Fah., 0.878×0.080744.]

The weight of one cubic meter of air at ordinary tension (760^{mm}, or 30 inches, of barometer) and $t°$ temperature, is, therefore,

$$G = \frac{1.29318}{1 + 0.00367t} \text{ kilograms.}$$

[For $t°$ Fahrenheit the weight of one cubic foot is

$$G = \frac{0.080744}{1 + 0.002038\,(t - 32)} \text{ lbs.]}$$

and that of V cubic meters is

$$G = \frac{1.29318}{1 + 0.00367t} \cdot V \text{ kilograms.} \quad . \quad . \quad . \quad \text{(V.)}$$

[For V cubic feet we have $G = \frac{0.080744\,V}{1 + 0.002038\,(t - 32)}$ lbs., where t is the temperature Fahrenheit.]

Thus, for example, 3 cubic meters of air, at ordinary pressure and 20° temperature, weigh

$$G = \frac{3 \times 1.29318}{1 + 0.00367 \times 20} = 3.615 \text{ kilograms,}$$

[and 3 cubic feet of air at ordinary pressure and 20° Fah. weigh

$$\frac{3 \times 0.080744}{1 - 0.002038 \times 12} = 0.2483 \text{ lbs.]}$$

We see from the preceding that the expansion of air for the same rise of temperature is much greater than for solid and liquid bodies. Experiments have also shown that this expansion is very nearly the same for all gases. Thus Regnault found for the coefficient of expansion of hydrogen 0.003661, and for carbonic acid 0.003710.

It seems also proved by experiment that the coefficients of expansion of such gases as are most easily liquefied by cold and pressure, as, for example, carbonic acid, are greater than for those which are most difficult of liquefaction. We accordingly assume that between the molecules of such bodies there is a certain, though small, amount of attraction, which even for the so-called "permanent" gases is not entirely zero.

Experiments have also shown that the coefficient of expansion increases as the pressure increases. In the case of air, as represented in the figure above, the piston is pressed by the atmosphere. If, however, the pressure is greater, the coefficient of expansion increases as shown in the following table:

	Pressure in Millimeters of Barometer.	Coefficient of Expansion.
Air	760	0.0036706
	2525	0.0036944
Hydrogen	760	0.0036613
	2520	0.0036616
Carbonic acid	760	0.0037099
	2545	0.0038455

[We give below the coefficients of linear, surface, and cubic expansion, that is, the amount by which a piece of unit length area or volume is increased for a rise of temperature of one degree C., counting from zero, under pressure of the atmosphere. For one degree Fahrenheit take $\frac{5}{9}$ths of tabular values.

	Cubic Expansion 3δ.	Surface Expansion 2δ.	Linear Expansion δ.	
Cast Iron	0.00003330	0.00002220	0.00001110	$\frac{1}{90100}$
Copper	0.00005155	0.00003436	0.00001718	$\frac{1}{58200}$
Brass	0.00005603	0.00003735	0.00001868	$\frac{1}{53500}$
Silver	0.00005726	0.00003817	0.00001908	$\frac{1}{52400}$
Bar Iron	0.00003705	0.00002470	0.00001235	$\frac{1}{81000}$
Steel (untempered)	0.00003236	0.00002158	0.00001079	$\frac{1}{92700}$
Steel (tempered)	0.00003791	0.00002479	0.00001240	$\frac{1}{80600}$
Zinc	0.00008825	0.00005883	0.00002942	$\frac{1}{34000}$
Tin	0.00005813	0.00003875	0.00001938	$\frac{1}{51600}$
Mercury	0.00018018	0.00012012	0.00006006	$\frac{1}{16600}$
Water*	0.00046600	0.00031066	0.00015533	$\frac{1}{6440}$
Gases	0.003665	....	....	.....

The *force of expansion or contraction* of a prismatic piece for a change of temperature of $t°$ is

$$P = \delta t F E$$

where δ is the coefficient of expansion, E the coefficient of elasticity, and F the area of cross-section.]

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* The expansion of water is very different for different temperatures.

Mechanical Work Performed by the Air during Expansion.—Let us now compute the mechanical work which one cubic meter of air performs during expansion.

We assume that the piston *EF* has an area of one square meter, and is therefore at a distance of one meter above the bottom of the vessel *ABCD*. If, then, the air is heated 273°, the piston will be raised one meter, and the air pressure will be overcome through this distance. This pressure, at 0° temperature and $760^{\text{mm.}}$ height of barometer, is 10334 kilograms per square meter (or 14.7 lbs. per square inch); so that the work performed by the expanding air is

$$10334 \times 1 = 10334 \text{ meter-kilograms.}$$

But one cubic meter of air at 0° weighs 1.29318 kilograms, and hence the work performed by one *kilogram* of air under the same conditions would be

$$\frac{10334}{1.29318} = 7991.15 \text{ meter-kilograms.}$$

When, therefore, we heat one cubic meter of air, free to expand under atmospheric pressure, from 0° to 273°, we not only increase the vibration work, but we also obtain an outer work of not less than 10334 meter-kilograms. We see that the outer work in the case of solids and liquids is not to be compared to this.

Heating under Constant Volume.—If we conceive the piston *EF* to be fixed, so that it cannot be raised when the air is heated, then evidently no outer work can be performed. All the heat imparted, therefore, goes to increase the vibration work or to raise the temperature. With the temperature the expansive force of the air or the pressure upon the piston also increases, and becomes, as shown by experiment, for each degree $\frac{1}{273}$d = 0.00367th greater; that is, the pressure increases in the same degree as the volume increased in the first case.

If we denote the pressure per square meter at 0° by p, then the pressure for a rise of $t°$ will be

$$p + t \cdot \tfrac{1}{273} p = p\left(1 + \tfrac{t}{273}\right) = p(1 + 0.00367t) \quad . \quad . \quad \text{(VI.)}$$

For $t = 273°$, the pressure is evidently $2p$, or twice as great as for 0°.

When, therefore, air is heated under constant volume, the expansive force increases with the temperature.

This law is also known as that of Gay-Lussac.

It may be explained according to our theoretical views as follows:

When heat is imparted to the air, the velocity of the atoms is increased. But the greater this velocity, the greater the number of impacts in the same time against the piston. The greater the number of impacts in the same time, the greater the pressure of the expansive force of the gas.

Absolute Zero of Temperature.—Since, now, for each rise of one degree, the expansive force, or what is the same thing, the living force of the atoms is increased $\frac{1}{273}$d of that at 0°, and for every fall of one degree is diminished $\frac{1}{273}$d; for a fall of 273°, the living force of the atoms must be zero. We call therefore the temperature

$$-273^\circ \text{ C.}$$

the "*absolute zero.*" *

It is from this point that we should properly reckon the velocity of the atoms of a body, or its living force, or finally, its temperature.

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* [It has been objected to this reasoning that the coefficient of expansion $\frac{1}{273}$d, is not the same for all gases, that it varies, especially near the point of liquefaction, and that it also depends upon the temperature in some relation not yet fully known. Thus for each gas there is a different absolute zero, and nothing justifies the assumption of this special one. It has even been termed "one of those false hypotheses which tend to retard the development of science."

It is true that the reasoning above seems open to these objections, but this is not really the reasoning by which the absolute zero is properly determined. The true reasoning cannot be presented in an elementary manner without the aid of the higher mathematics. It is proved generally by the principles of the mechanical theory of heat, that there *is* a point at which the living force of the atoms would be zero, and that this point *must be the same for all bodies*, whether there is disgregation work or not. In some cases part of the expansive force has to perform more disgregation work, in others less. In the text we have simply endeavored to simplify the determination, and to illustrate its physical significance by taking a body in which the disgregation work is nearly zero, and thus making our experiments upon a gas at a point for which this assumption is known to be approximately correct. Whether this seems perfectly correct or not, the fact remains that the absolute zero is a point which has a definite physical significance, and which is capable of more or less accurate determination, for all bodies, whether perfect gases or not. It has thus been found to be very closely -273° C. for bodies in which the disgregation work is not zero. But even if it had no physical significance, which it has, and if the above conclusion were founded upon the consideration of a body possessing purely hypothetical properties, which is not the case, still it would not follow that the determination of such a point would be without value. The *coefficient of elasticity* is also a purely supposititious force, which will stretch a purely hypothetical body by its own length, and yet it is of considerable use in determinations of strength and flexure of bodies, and can scarcely be considered as "retarding the development of science."

Let it be remembered, then, that there is an absolute zero, and that it is the same for *all* bodies, and is very closely -273° C., as determined by experiment.]

Since in the last experiment no outer work is performed, but all the heat imparted goes to increase the vibration work, we ought to expect the amount of heat necessary to be imparted for a certain rise of temperature to be less than in the first case, where outer work is performed. This conclusion is confirmed by many and various observations.

[We give here tables of the most remarkable temperatures :

Greatest artificial cold	– 140° C.
Mercury freezes	– 39.4°
Ice melts	0°
Greatest density of water	4°
Blood heat	36.6°
Water boils	100°
Red heat	526°

MELTING POINT OF DIFFERENT SUBSTANCES.

	C°.		C°.
Platinum	2500	Tin	230
Wrought Iron	1600	5 Tin, 1 Lead	194
Steel	1400	8 Bismuth, 3 Tin, 5 Lead	100
Cast Iron (gray)	1200	4 B.,1T.,1 Lead (Rose's metal)	94
" " (white)	1050	Sulphur	109
Gold 1100 to.	1250	Yellow Wax	61
Copper 1050 to.	1200	Soda	90
Silver	1000	Potash	58
Bronze	900	Stearine	49–43
Antimony	432	Paraffine	46
Zinc	360	Phosphorus	43
Lead	330	Oil Turpentine	– 10
Bismuth	260	Mercury	– 39
		Water	0

BOILING POINT OF DIFFERENT SUBSTANCES.

	C.		C.
Mercury	350	Linseed Oil	316
Sulphuric Acid	325	Nitric Acid	86
Sulphur	440	Alcohol (spec. gr. 0.79 at 20°).	78.4
Oil Turpentine	156	Sulphuric Ether	35
		Sulphurous Acid	–10]

Calculation of the Mechanical Equivalent of Heat.—When the heated air is free to expand and overcome the outer air pressure, the amount of heat necessary to be imparted to one kilogram, in order to cause a rise of temperature of one degree is 0.23751 heat units. This is the "*specific heat*" *of air for constant pressure.* The exact determination of this number is due to Regnault.

If the heated air cannot expand, if, therefore, the volume remains constant, only 0.1684 heat units are necessary in order to raise one kilogram one degree. This is the "*specific heat*" *for constant volume.*

The *specific heat* of the air, for constant pressure, is, therefore, $\frac{0.23751}{0.16847} = 1.410$ times greater than that for constant volume. This number has been determined by a score of observations made in different ways.

The excess of heat,

$$0.23751 - 0.16847 = 0.06904$$

heat units, is that which goes to the performance of outer work.

We have already seen that when one kilogram of air is heated from 0° to 273°, and is free to expand under the air pressure, the outer work is 7991.15 meter-kilograms. If heated from 0° to 1°, then the outer work is

$$\frac{7991.15}{273} = 29.272 \text{ meter-kilograms.}$$

This number we denote generally in the mechanical heat-theory by the letter R.* A work of 29.272 meter-kilograms corresponds to an expenditure of heat of 0.06904 heat units. One unit of heat, then, corresponds to

$$\frac{29.272}{0.06904} = 423.98 \text{ meter-kilograms.}$$

This result agrees perfectly with that found by Joule as the mean of a large number of experiments. We can now easily deduce a general formula for the mechanical equivalent.

If we denote the specific heat of air for constant volume, that is, the number 0.16847, by c_v, and that for constant pressure, or 0.23751, by c_p, then the difference $c_p - c_v$ denotes the amount of heat corresponding to the outer work of 29.272 meter-kilograms, or to the work R. For one unit of heat, therefore, we obtain the mechanical work (mechanical equivalent) $\frac{1}{A}$, from the proportion

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* [For Fahrenheit degrees and foot lbs. $R = 53.354$. The student will do well to make the calculation.]

$$c_p - c_v : 1 :: R : \frac{1}{A}.$$

Hence we have
$$\frac{1}{A} = \frac{R}{c_p - c_v}.$$

Since $c_p = 1.41 c_v$, we have, when we substitute for 1.41 the letter k,

$$\frac{1}{A} = \frac{R}{c_v k - c_v},$$

or, since it is customary to write c_v without the index,

$$\frac{1}{A} = \frac{R}{c(k-1)}. \quad \ldots\ldots \quad \text{(VII.)}$$

If we assume the older determination of the specific heat for constant pressure of Delaroche and Bérard, of 0.267, we obtain a smaller value for the mechanical equivalent. It is thus that Mayer found (1842) the number 365, Holtzmann (1845) 374, and Clausius (1850) 370.

[The value of R in French measures and Centigrade degrees is, as we have seen, for air 29.272 meter-kilograms. We give here the values of R for other gases both in French measures and in English measures for both Centigrade and Fahrenheit degrees.

	French Measures.		English Measures.		
	Centigrade.		Cent.	Fahr.	
	R.		R.		
Air	29.272	meter-kil.	96.0876	53.354	foot lbs.
Nitrogen	30.134	"	98.867	54.926	"
Oxygen	26.475	"	86.862	48.257	"
Hydrogen	422.612	"	1386.579	770.322	"

These can all be calculated from the formula

$$R = \frac{c_p - c_v}{A} = \frac{pv}{T} = \frac{p}{\gamma T},$$

where γ is the density, or weight of one cubic unit of volume.

The value of $\frac{1}{A}$ is 424 meter-kilograms for French measures and Centigrade degrees, 1,390 foot lbs. for English measures and Centigrade degrees, and 772 foot-pounds for English measures and Fahrenheit degrees.

We give below the values of c_p and c_v.

	c_v.	$k = \frac{c_p}{c_v}$.	c_p.	Weight of 1 Cubic Meter in Kilograms.	Weight of 1 Cubic Foot in Pounds.
Air	0.16847	1.4098	0.23751	1.29318	0.08073
Nitrogen	0.17273	1.4114	0.24380	1.25616	0.07860
Oxygen	0.15507	1.4026	0.21751	1.42980	0.08926
Hydrogen	2.41226	1.4132	3.40900	0.08957	0.00559]

Increase of the Expansive Force by Compression.—Mariotte's Law.—When we heat the air without allowing it to expand, its expansive force increases at the cost of the heat imparted, but its density remains the same. Let us now increase the expansive force in another way, viz., by compressing the air. According to our principles the action is evident; for by compressing the air we perform mechanical work, and this can always replace a certain amount of heat.

If, then, we assume below the piston EF, Fig. 12, one cubic meter of air at 0° temperature and atmospheric pressure, the expansive force will increase as the piston is forced down. If, for example, the air is compressed into half its former volume, its tension is twice as great; if compressed to one-fourth of its former volume, its tension is four times as great; and so on. In such case we assume, indeed, *that the temperature is kept constant, viz., at* 0°. In the same degree in which the tension increases, the density evidently increases also.

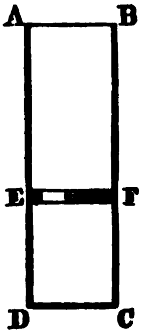

Fig. 12.

This law, according to which, *for constant temperature*, the tension of a gas increases as its volume decreases, is called *Mariotte's law.*

This law also is a necessary consequence of our assumptions as to the constitution of gases.

Every atom in the cubic meter of air makes at 0° a certain number of impacts upon the piston, and thus causes a certain pressure. If, at the same temperature, the gas only occupies half its original volume, the atom makes in the same time double as many impacts, because its velocity (temperature) is the same, and it has only half as far to go. Its pressure upon the piston is therefore twice as great, and since this is the case with all the atoms, the tension of the entire mass must be twice as great.

If we denote the volume of air by V_1 and its tension by p_1 (upon the square meter), then change the volume V_1 into V_2, and the pressure p_1 into p_2, we have, according to Mariotte's law,

$$V_1 : V_2 :: p_2 : p_1, \text{ or } V_1 p_1 = V_2 p_2.$$

If the volume V_2 with the tension p_2 is changed to V_3 with the new tension p_3, we have,

$$V_2 : V_3 :: p_3 : p_2, \text{ or } V_3 p_3 = V_2 p_2.$$

Comparing with the above, we have

$$V_1 p_1 = V_2 p_2 = V_3 p_3, \text{ etc.} \quad . \quad . \quad . \quad . \quad \text{(VIII.)}$$

We see, therefore, that the product of the volume and pressure is constant, *provided that the temperature is kept the same.*

As now the expansive force increases, the density also increases, and inversely.

If the density for the volume V_1 is D_1, and for $V_2 = D_2$, we have

$$V_1 : V_2 :: D_2 : D_1, \text{ or } V_1 D_1 = V_2 D_2.$$

On the other hand, $D_1 : D_2 :: p_1 : p_2$.

The density increases, therefore, inversely as the volume, and directly as the tension.

Example 1.—*What is the weight of $\frac{1}{8}$th cubic meter of air at 0° and 4 atmospheres?*

One cubic meter at 0° and atmospheric pressure weighs 1.29318 kilograms, hence $\frac{1}{8}$th of a cubic meter weighs 0.16165 kilograms. We have then

$$1 : 4 :: 0.16165 : x.$$

Hence

$$x = 4 \times 0.16165 = 0.6465 \text{ kilograms.}$$

[In the same way, the weight of $\frac{1}{8}$th cubic foot of air at 32° F. and 4 atmospheres is 0.040372 lbs.]

Example 2.—*What is the volume of one kilo. of air at 0° and $\frac{1}{5}$th atmosphere?*

One kilogram of air at 0° and at atmospheric pressure has a volume of $\frac{1}{1.29318} = 0.7733$ cubic meters. Its volume, therefore, at a tension of $\frac{1}{5}$th of an atmosphere, is given by $0.7733 : x :: \frac{1}{5} : 1$, or

$$x = 5 \times 0.7733 = 3.8665 \text{ cubic meters.}$$

[The volume of one lb. of air at 32° F. and $\frac{1}{5}$th atmosphere, is found in same way to be 61.9241 cub. ft.]

Mariotte's and Gay-Lussac's Laws Combined.—If we denote the volume at 0° and ordinary tension by V_0, then for pressures of 2, 3, 4 p_1 atmospheres, the volume will be, according to Mariotte's law,

$$\frac{V_0}{2}, \frac{V_0}{3}, \frac{V_0}{4}, \ldots \ldots \frac{V_0}{p_1}.$$

If the temperature of the air is $t_1°$, it is evident that for the same tension p_1 its volume will be greater. Since for each degree the air expands 0.00367 of its volume, the volume V_0 will become at $t_1°$ $V_0(1 + 0.00367t_1)$, and we have

$$V_1 = \frac{V_0(1 + 0.00367t_1)}{p_1}.$$

Again, if the tension at 0° were p_2, the volume would be

$$\frac{V_0}{p_2}$$

at 0°, and if the temperature were $t_2°$ instead of 0°, the volume would be

$$V_2 = \frac{V_0(1 + 0.00367t_2)}{p_2}.$$

We have, then,

$$V_1 : V_2 :: \frac{V_0}{p_1}(1 + 0.00367t_1) : \frac{V_0}{p_2}(1 + 0.00367t_2),$$

or,

$$\frac{V_1}{V_2} = \frac{p_2}{p_1} \cdot \frac{1 + 0.00367t_1}{1 + 0.00367t_2}. \quad . \quad . \quad . \quad \text{(IX.)}$$

The law expressed by this formula is known in physics as the combined law of Mariotte and Gay-Lussac.

Since, further, the volumes are inversely as the densities,

$$\frac{D_2}{D_1} = \frac{p_2}{p_1} \cdot \frac{1 + 0.00367t_1}{1 + 0.00367t_2}. \quad . \quad . \quad . \quad . \quad \text{(X.)}$$

[For temperature t_1 in Fahrenheit degrees we put 0.002038 $(t_1 - 32)$ in place of $0.00367t_1$, etc.]

EXAMPLE 1.—*A quantity of air of $V_1 = 1$ cubic meter, $t_1 = 10°$, and $p_1 = 1$ atmosphere, is compressed to $V_2 = 0.8$ cubic meter, and $t_2 = 100°$. What is the tension?*

We have from IX.,

$$\frac{1}{0.8} = \frac{p_2}{1} \cdot \frac{1 + 0.00367 \times 10}{1 + 0.00367 \times 100}, \quad \text{or} \quad 1.25 = p_2 \frac{1.0367}{1.367},$$

or

$$p_2 = \frac{1.25 \times 1.367}{1.0367} = \frac{1.70875}{1.0367} = 1.648 \text{ atmospheres.}$$

EXAMPLE 2.—*If a mass of air of $V_1 = 30$ cubic meters, $p_1 = 1$ atmosphere, and $t_1 = 10°$, in passing through the blowing apparatus of a blast furnace is heated to $t_2 = 200°$ and compressed to 1.26 atmospheres, what will be its new volume?*

We have from IX.,

$$\frac{30}{V_2} = \frac{1.26}{1} \cdot \frac{1 + 0.00367 \times 10}{1 + 0.00367 \times 200}, \text{ or}$$

$$V_2 = \frac{30 \times 1.7340}{1.26 \times 1.0367} = 39.8 \text{ cubic meters.}$$

[For $V_1 = 30$ cubic ft., $p_1 = 1$ at., $t_1 = 10°$ F., $t_2 = 200$ F., and $p_2 = 1.26$ at., we have $V_2 = 33.46$ cub. ft.]

EXAMPLE 3.—*If the density of air for $p_1 = 760^{mm}$ height of barometer, and $t_1 = 0°$ is 1, what would be the density for $p_2 = 750^{mm.}$ and $t_2 = 20°$?*

We have from X.,

$$\frac{D_2}{1} = \frac{750}{760} \cdot \frac{1 + 0.00367 \times 0}{1 + 0.00367 \times 20},$$

or

$$D_2 = \frac{75}{76} \cdot \frac{1}{1.0734} = \frac{75}{81.5784} = 0.9195,$$

and hence the weight of one cubic meter in the new condition would be only

$$1.29318 \times 0.9195 = 1.189 \text{ kilograms.}$$

[For $t_1 = 32°$ F. and $t_2 = 20°$ F., we have $D_2 = 1.01158$, and the weight of one cubic foot is 0.08168 lbs.]

Transformation of the last two Formulæ.—In the mechanical theory of heat these last formulæ are put into a simpler form. First we denote the coefficient of expansion by α, and can thus write

$$\frac{V_1}{V_2} = \frac{p_2}{p_1} \cdot \frac{1 + \alpha t_1}{1 + \alpha t_2}, \quad \text{or} \quad \frac{D_2}{D_1} = \frac{p_2}{p_1} \cdot \frac{1 + \alpha t_1}{1 + \alpha t_2}.$$

If now we divide numerator and denominator of the right side of these equations by α, we have

$$\frac{V_1}{V_2} = \frac{p_2}{p_1} \cdot \frac{\frac{1}{\alpha} + t_1}{\frac{1}{\alpha} + t_2}, \quad \text{and} \quad \frac{D_2}{D_1} = \frac{p_2}{p_1} \cdot \frac{\frac{1}{\alpha} + t_1}{\frac{1}{\alpha} + t_2}.$$

Since now $\alpha = \frac{1}{273}$, and hence $\frac{1}{\alpha} = 273$, we have

$$\frac{V_1}{V_2} = \frac{p_2}{p_1} \cdot \frac{273 + t_1}{273 + t_2} \qquad \frac{D_2}{D_1} = \frac{p_2}{p_1} \cdot \frac{273 + t_1}{273 + t_2}.$$

From the first of these equations we have

$$\frac{V_1 p_1}{273 + t_1} = \frac{V_2 p_2}{273 + t_2}.$$

In like manner we have for the volume V_3, the tension p_3, and the temperature t_3,

$$\frac{V_1 p_1}{273 + t_1} = \frac{V_3 p_3}{273 + t_3}.$$

We have, therefore, generally,

$$\frac{V_1 p_1}{273 + t_1} = \frac{V_2 p_2}{273 + t_2} = \frac{V_3 p_3}{273 + t_3}, \text{ etc.} \quad . \quad . \quad \text{(XI.)}$$

It thus appears that the above quotients have a constant value for any "perfect" gas.*

The formulæ hold good, evidently, when V_1 and V_2 are the "specific volumes," that is, the volumes of the unit of weight, as the kilogram.

If therefore v_0 is the specific volume for $t_0 = 0°$ and atmospheric pressure p_0, and v_1 that for the temperature t_1 and tension p_1, we have

$$\frac{v_0 p_0}{273 + t_0} = \frac{v_1 p_1}{273 + t_1}, \text{ etc.}$$

But now the volume v_0 for the pressure p_0 and temperature t_0 is known. For since 1.29318 kilograms, under these circumstances, is known to occupy the space of one cubic meter, one kilogram will occupy

$$\frac{1}{1.29318} \text{ cubic meters.}$$

[One pound at 32° Fah. occupies the space of

$$\frac{1}{0.080744} \text{ cubic feet.}]$$

This is therefore the specific volume v_0 at 0°.

If, now, we express p_0 in kilograms per square meter, or put $p_0 = 10334$ kilograms, we have

$$\frac{v_0 p_0}{273 + t_0} = \frac{\frac{1}{1.29318} \times 10334}{273 + 0} = \frac{10334}{1.29318 \times 273} = \frac{10334}{353.03814} = 29.272.$$

We have then

$$\frac{v_1 p_1}{273 + t_1} = \frac{v_2 p_2}{273 + t_2} = 29.272.$$

[Taking the temperature Fahrenheit and the volume of one pound and the pressure per square foot, we have

$$\frac{v_1 p_1}{491.4 + (t_1 - 32)} = 53.354.]$$

---

* [Meaning by "perfect gas" one between whose molecules there are no forces of attraction, or one so far removed from its point of liquefaction that the disgregation work may be disregarded.]

This number we have already found to be the outer work performed under atmospheric pressure by one kilogram of air when its temperature is raised from 0° to 1°. This, as already remarked, we denote by R, and have thus

$$R = \frac{v_0 p_0}{273 + t_0} = \frac{v_1 p_1}{273 + t_1} = \frac{v_2 p_2}{273 + t_2}, \text{ etc.,*}$$

where v_0, v_1, v_2, are the specific volumes, or volumes of one kilogram of the gas, and p_0, p_1, p_2 are the specific pressures, or pressures upon the square meter.

If p is expressed in atmospheres, we have

$$R = \frac{1}{353.03814} = 0.002833 \text{ for air.}$$

[For English measures the corresponding value of $R =$ 0.025208.]

For other gases, which have different densities, R has different values.

We have already remarked that the absolute zero of temperature lies at 273° below the zero of the centigrade scale. At this temperature the living force of the atoms is 0, and the body possesses no heat.

From this point the heat of a body, or its inner work, should be measured. If, then, a body has a temperature of 0° according to the thermometer, its actual temperature is 273°, and so for the temperatures t_1, t_2, t_3, etc., the absolute temperatures are $273 + t_1$, $273 + t_2$, $273 + t_3$, etc. [For Fahrenheit scale the corresponding absolute temperatures are $459.4 + t_1$, $459.4 + t_2$, etc.]

Thus we see that the denominators in the equations above give the absolute temperatures. If, then, we denote these by T_1, T_2, etc., we have

$$R = \frac{v_0 p_0}{T_0} = \frac{v_1 p_1}{T_1} = \frac{v_2 p_2}{T_2}.$$

That is, *if we divide the specific volumes, multiplied by their corresponding pressures, by the corresponding absolute temperatures, the quotients are constant and equal to R, or for air to* 29.272, [or 53.354 for English measures, and Fahrenheit degrees.]

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* Taking the temperature Fahrenheit and the volume of one pound and pressure per square foot, we have

$$R = \frac{v_0 p_0}{491 + (t_0 - 32)} = \frac{v_1 p_1}{491 + (t_1 - 32)} = \text{etc.,}$$

where $R = 53.354$.

From these equations we obtain

$$\left.\begin{aligned} RT_0 &= v_0 p_0 \\ RT_1 &= v_1 p_1 \\ RT_2 &= v_2 p \end{aligned}\right\}, \text{ etc.} \quad . \quad . \quad . \qquad \text{(XII.)}$$

In this form the combined laws of Mariotte and Gay-Lussac can be easily remembered. We shall have occasion to make frequent use of these formulæ.*

EXAMPLE 1.—*What is the volume v_1 of one kilogram of air at the temperature $t_1 = 100°$ and pressure $p_1 = 5000$ kilograms per square meter?*

$$\text{We have } 29.272\,(273 + 100) = v_1 \times 5000,$$

hence $$v_1 = \frac{29.272 \times 373}{5000} = \frac{10918.456}{5000} = 2.1837 \text{ cubic meters.}$$

[For 1 lb. at 100° Fah. and 1024 lbs. per square foot, we have $v_1 = 29.1467$ cubic feet.]

Therefore, 2, 3, 4 . . . kilograms of air would occupy the space of 2×2.1837, 3×2.1837, 4×2.1837 cubic meters, etc.

EXAMPLE 2.—*If one kilogram of air has a volume of 3 cubic meters and temperature of 200°, what is its pressure per square meter?*

$$\text{We have } 29.272\,(273 + 200) = 3\,p_1, \text{ or } p_1 = 4615.218 \text{ kilograms.}$$

EXAMPLE 3.—*What is the temperature t_1 of one kilogram of air whose tension is $p_1 = 3$ atmospheres and volume $v_1 = 0.5$ cubic meters?*

$$\begin{aligned} \text{We have } 0.002833\,(273 + t_1) &= 0.5 \times 3, \\ \text{or } 0.7734 + 0.002833 t_1 &= 1.5 \\ 0.002833 t_1 &= 0.7266 \\ t_1 &= 256.4 \text{ degrees.} \end{aligned}$$

[For 1 lb. of air, $p_1 = 3$ atmospheres and $v_1 = 14$ cubic feet, $t_1 = 1206.7°$ Fah.]

QUESTIONS FOR EXAMINATION.

What is the disgregation work in a perfect gas? What effects are produced by heat imparted? How does the outer work compare with that for solid and liquid bodies? What is the coefficient of expansion for air? If V cubic meters of air are heated $t°$ under atmospheric pressure, what is the new volume? What is the law of expansion of air called? State it concisely in words. How does the density vary with the temperature for constant pressure? How can you find the weight of V cubic meters of air at $t°$? Does the coefficient of expansion vary for different gases? How? For what gases is it least? Does it vary with the pressure? How? What is the cause of variation? What is a "perfect gas?" Would it vary for such a gas? If we heat one cubic meter of air under atmospheric pressure from 0° to 273° C., what is the outer work performed? What would it be for one kilogram of air? What is the disgregation work? The vibration work? The inner work?

When air is heated under constant volume, what effects does the heat produce? How does the pressure increase with the temperature ? If the pressure at 0° is p, what is it at 1° ? At 2° ? At t° ? What is this law called ? State it in words. How can you explain it according to theoretical views ?

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* [If V is the volume of G kilograms of gas, then $\frac{V}{G}$ is the volume of one kilogram, or $v = \frac{V}{G}$, and hence $GRT = Vp$.]

What do you understand by "absolute zero ?" Has this point a definite physical significance ? Is it the same for all bodies ? If it had no physical significance, would it necessarily follow that it is valueless in the theory ? Which is the greater, the specific heat for constant pressure or for constant volume ? Why ? What is the ratio of the two for air ? What does R denote in our notation ? Deduce a relation between the two specific heats R and A.

State concisely Mariotte's law. Under what assumption as to temperature does this law hold good ? Is it a consequence of our theoretical views as to the constitution of gases ? How ? If p_1 and v_1 are initial pressure and volume, what relation subsists between them and any other p and v ? Is this law exact for all gases ? For any gas ? What do we call a gas for which it is exact ? What constitutes a "perfect" gas ? Are there any such ? What relation subsists between volume and density ? Between pressure and density ? State again Gay-Lussac's laws. State algebraically the combined laws of Mariotte and Gay-Lussac. Deduce a relation which must exist, by virtue of these laws, between the volumes, pressures, and temperatures at two different states, and the coefficient of expansion. Between the densities, pressures, and temperatures. Deduce from these last the simplest expression of the combined laws. Does this relation between volume, pressure, and temperature hold good for all perfect gases ? What does R signify ? What is it for air ?

What is the volume of 8 kilograms of air when its pressure is 5000 kilograms per square meter and temperature 100° C. ?

If one kilogram of air has a volume of 3 cubic meters and a temperature of 100°, what pressure must it have ?

If 2 kilograms of air have a tension of 3 atmospheres and a volume of 1 cubic meter, what must be the temperature ?

How many kilograms per square meter correspond to one atmosphere ?

If 8 cubic feet of air are heated from 0° to 8° C., what is the new volume ? From 0° to 8° Fahr. ? What is the density in each case ?

What is the weight of 3 cubic feet of air at atmospheric pressure and 25° C. temperature ? At 25° Fahrenheit ?

If 2 cubic feet of water are heated under atmospheric pressure from 0° to 100° C., what expenditure of work is equivalent to the heat imparted ? From 40° to 212° Fahrenheit ?

What is the coefficient of expansion for air ? If one cubic foot of air is heated from 0° C. to 273° C., what is the new volume ? If it is further heated from 273° to 274°, what is the increase of volume ? Is this increase $\frac{1}{273}$d of the volume at 273°, or of the volume at 0° ? If 10 cubic feet of air are heated from 11° C. to 12° C., what is the increase of volume ? Is this increase $\frac{1}{273}$d of the 10 cubic feet, or $\frac{1}{273}$d of what the volume of the 10 cubic feet *would be* at 0° ? Has pressure any influence upon the coefficient of expansion ? What influence ? Does the coefficient of expansion vary for different gases ? Define, then, exactly what is meant by coefficient of expansion ?

We insert here the following values of R, for air, which may be found useful in calculations:

p	given in	kilograms per sq. meter,	v	vol. of 1 kil. in cub. meters, Temper. Cent.,	$R = 29.272$.
p	"	atmospheres,	v	" " " " " "	$R = 0.002833$.
p	"	lbs. per sq. ft.,	v	" 1 lb. in cub. feet, Temperature Fah.,	$R = 53.354$.
p	"	" " inch,	v	" " " " " "	$R = 0.3703$
p	"	inches of mercury,	v	" " " " " "	$R = 0.75427$.
p	"	atmospheres,	v	" " " " " "	$R = 0.025208$.
p	"	millimeters mercury,	v	" " " " " "	$R = 19.1562$.
p	"	" "	v	" 1 kil. in cub. meters, Temper. Cent.,	$R = 2.15274$.
p	"	lbs. per sq. ft.,	v	" 1 lb. in cubic feet, " "	$R = 96.0376$.
p	"	atmospheres,	v	" " " " " "	$R = 0.045374$.
p	"	lbs. per sq. inch,	v	" " " " " "	$R = 0.666368$.
p	"	inches of mercury,	v	" " " " " "	$R = 1.35768$.
p	"	millimeters "	v	" " " " " "	$R = 34.4847$.

CHAPTER V.

HEAT CURVES AND THE MECHANICAL WORK WHICH A GAS PERFORMS DURING EXPANSION AND RECEIVES DURING COMPRESSION.

Isothermal Curve.—Let OX and OY be two lines perpendicular to each other, the so-called "co-ordinate axes," OX being the axis of abscissas, and OY the axis of ordinates.

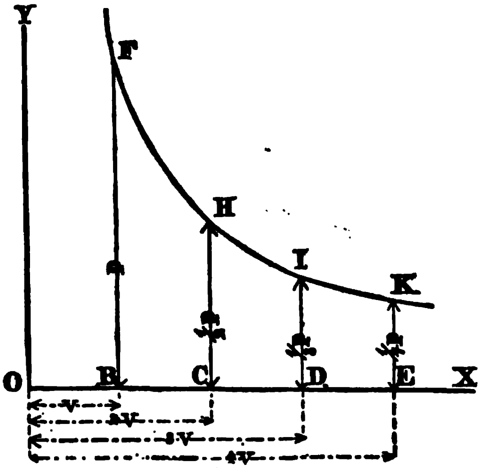

Fig. 13.

Suppose that we have in a cylinder the unit of weight of air (one kilogram) of the volume $OB = v$, and tension $BF = p$, and the temperature t. If this air expands to double its volume, or to $OC = 2\,OB = 2v$, and if we suppose that *during the expansion the temperature t is constant*, that is, that heat is imparted to the air from without as its temperature falls during expansion, then the tension becomes $CH = \frac{1}{2}BF = \frac{1}{2}p$. If the volume becomes $OD = 3OB = 3v$, the tension becomes $DI = \frac{1}{3}BF = \frac{1}{3}p$, etc. If we join the points $FHIK$, the curve gives the law of variation of the tension with the volume. We call the curve thus obtained the "isothermal curve for permanent* gas." It represents graphically the law of Mariotte.

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* [Late investigations have shown that all the so-called "permanent" gases can be liquefied. The term is therefore to be taken merely as applying to those gases which, under ordinary circumstances, are so far from their point of liquefaction that the disgregation work in expansion can be neglected.]

Mechanical Work which the Air performs during Expansion and receives during Compression.—In order to find the mechanical work which the air performs during its expansion, we must suppose *EE* divided into an indefinitely large number of parts, that is, we must suppose the volume v to increase little by little, and find the corresponding tensions. The pressure during the small increase of volume may be regarded as constant, and we thus obtain the work during this increase by multiplying the pressure by the change of volume. The sum of the products thus obtained gives the mechanical work during expansion.

[Such a summation can easily be made by means of the calculus.

Thus if the initial tension is p_1 and volume v_1, and if after expansion we have p and v, we have

$$p_1 : p :: v : v_1, \qquad \text{whence} \qquad p = \frac{p_1 v_1}{v}.$$

During the small expansion dv, the pressure p may be regarded as constant, and the work performed is therefore

$$pdv = \frac{p_1 v_1 \, dv}{v}.$$

Integrating this between the limits v and v_1, we have the work

$$L = p_1 v_1 \log \text{nat} \frac{v}{v_1},$$ [*]

or in common logarithms

$$L = 2.3026\, p\, v \log \frac{v}{v_1} \quad . \quad . \quad . \quad . \quad (\text{XIII.})$$

where v is the greater volume and v_1 the less.

Since $p_1 : p :: v : v_1$, we have also

$$L = 2.3026\, pv \log \frac{p_1}{p}, \quad . \quad . \quad . \quad . \quad (\text{XIV.})$$

where p_1 is the greater tension and p the less.]

The same formulæ hold good when the specific volume v and tension p are by compression under constant temperature changed into the less volume v_1 and greater tension p_1. They evidently give the work per unit of area of the piston or surface pressed upon, since p is the pressure per unit of area.

---

[*] When using the formulæ either for expansion or compression, remember that $\frac{v}{v_1}$ must always have *the larger of the two volumes in the numerator*, so also for p_1. For compression, then, we should have $\frac{v_1}{v}$ or $\frac{p}{p_1}$. The formulæ all apply to the unit of weight.

EXAMPLE 1.—*What work must be expended in order to convert* 0.33 *cubic meters of air at ordinary tension* (760$^{\text{mm.}}$) *into air of* 814$^{\text{mm.}}$ *tension, the temperature being kept constant?*

If we denote the original volume by V, and tension by p, and the new greater tension by p_1, we have $L = 2.3026\ Vp \log \frac{p_1}{p}$, or substituting the values given,

$$L = 2.3026 \times 0.33 \times 10334 \log \tfrac{814}{760} = 233.99 \text{ meter-kilograms.}$$

[For 10 cubic feet of air, we have

$$L = 2.3026 \times 10 \times (14.7 \times 144) \log \tfrac{814}{760} = 1452.98 \text{ ft. lbs.}]$$

EXAMPLE 2.—*What is the mechanical work which the unit of weight of air performs when it expands to double its volume, the temperature being constant, its volume* v_1 *being* $\frac{1}{1.29318} = 0.7733$ *cubic meters,* $p_1 = 10334$ *kilograms per square meter, and* $v = 2v_1$?

We have $$L = 2.3026\ p_1 v_1 \log \frac{v}{v_1}, \text{ or}$$

$$L = 2.3026 \times 0.7733 + 10334 \log 2 = 5538.6 \text{ meter-kilograms,}$$

[For 1 lb. of air the volume $v_1 = \frac{1}{0.080744} = 12.3848$ cubic feet, $p_1 = 14.7$ lbs. per square inch, and

$$L = 2.3026 \times 12.3848 \times (14.7 \times 144) \times \log 2 = 18169.8 \text{ ft. lbs.}]$$

EXAMPLE 3.—*The piston of a steam engine has an area of 0.14 square meters. We have beneath it a volume of steam* 0.395 *meter high and* 3 *atmospheres tension. What mechanical work is performed when the piston moves* 0.658 *meter, the temperature remaining constant?*

The original volume V_1 is 0.14 × 0.395 cubic meters. The volume V after expansion is 0.14 (0.395 + 0.658). Hence

$$\frac{V}{V_1} = \frac{0.395 + 0.658}{0.395} = \frac{1.053}{0.395}. \text{ We have then}$$

$$L = 2.3026 \times 0.14 \times 0.395 \times 3 \times 10334 \log \frac{1.053}{0.395} = 1679 \text{ meter-kilograms.}$$

The work of the steam during the full pressure is 0.14 × 10334 × 3 × 0.395 = 1714 meter-kilograms.

Hence the total work performed is 1679 + 1714 = 3393 meter-kilograms. If this is performed in one second, we have a work of $\frac{3393}{75} = 45.2$ horse power (French). [If the area is 13 sq. ft., the volume 1.3 ft. high, the tension 3 atmos., and the piston moves 2.16 ft., we have

$$L = 2.3026 \times 13 \times 3 \times 14.7 \times 144 \log \frac{58.474}{1.3} = 61010.7 \text{ ft. lbs.,}$$

or about 111 horse power.]

Amount of Heat imparted or abstracted during Expansion or Compression, according to Mariotte's Law.—We know from what has preceded that when a gas expands while performing work its temperature must sink, because the outer work is performed at the expense of the inner.

If, therefore, during the expansion the temperature remains constant, heat must be imparted from without.

Since, now, the temperature or vibration work remains the same, the outer work performed is the exact equivalent of the heat imparted. If therefore we denote the heat imparted during expansion, measured in heat units, by Q, we must have

$$424\ Q = 2.3026\ vp \log \frac{v}{v_1}.$$

Since we denote the mechanical equivalent of heat, 424 (or 772 ft. lbs.), by $\frac{1}{A}$, we have generally

$$\frac{Q}{A} = 2.3026\ vp \log \frac{v}{v_1},$$

or

$$Q = 2.3026\ Avp \log \frac{v}{v_1}. \quad . \quad . \quad . \quad . \quad (\text{XV.})$$

where, in general, the greater volume is put in the numerator.

Just as during the expansion of a gas we must impart heat in order to preserve the temperature constant, we must abstract heat during compression. The work performed upon the gas goes to increase its vibration work, or its sensible temperature. The heat abstracted must therefore be equal to the work performed upon the gas. We have thus, in this case also,

$$Q = 2.3026\ Avp \log \frac{v}{v_1},$$

or

$$Q = 2.3026\ Avp \log \frac{p_1}{p},$$

where v is always the greatest of the two volumes and p_1 of the pressures.

*Isodynamic Curve.**—For solid and liquid bodies the case is not so simple. When such a body is compressed, that is, when outer work is performed upon it, we cannot directly determine how much of this outer work goes to increase the vibration work, and how much to disgregation work. It may be that one or the other of these parts is zero or negative, as we have already seen.

We have therefore for solid and liquid bodies the *isodynamic curve*, which gives the relation between pressure and volume when the *inner work* (that is, *both* the vibration work and the disgregation work) is constant.

Since in gases there is very little, if any, mutual action between the molecules, and therefore no work is required to bring them nearer, the outer work performed upon the gas only increases the vibration work. Here then the isodynamic curve becomes identical with the isothermal. All that has been said with reference to the isothermal curve *for gases* holds good therefore for the isodynamic curve also.

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* [Sometimes called also "isenergic curve," or curve of equal energy. When there is no disgregation work the isothermal curve corresponds to the isenergic or isodynamic.]

If in the formula

$$Q = 2.3026\, Avp \log \frac{v}{v_1},$$

or

$$Q = 2.3026\, Av_1p_1 \log \frac{v}{v_1},$$

we put in place of pv or p_1v_1 the values from Equation XII., viz., RT and RT_1, we have

$$\left.\begin{aligned} Q &= 2.3026\, ART \log \frac{v}{v_1} \\ Q &= 2.3026\, ART_1 \log \frac{p_1}{p} \end{aligned}\right\} \quad . \quad . \quad . \quad (XVI.)$$

where, as before, v is the greater volume and p_1 the greater pressure.

From these formulæ we can determine Q when the initial and final volumes or pressures and the temperature are known. Since $pv = p_1v_1$, so also $RT = RT_1$, or the temperature is constant, as should be.

EXAMPLE.

We have one kilogram of air inclosed in a cylinder. The temperature is $t = 30°$. What work will it perform when it expands from the less volume v_1 to the greater $v = \frac{4}{3} v_1$, and how many units of heat must be imparted to keep the temperature constant?

We have for the work

$$L = 2.3026\, RT_1 \log \frac{\frac{4}{3}v_1}{v_1} = 2.3026\, RT_1 \log \tfrac{4}{3}.$$

Or by substituting the numerical values

$$L = 2.3026 \times 29.272\,(273 + 30) \times 0.125 = 2549 \text{ meter-kilograms.}$$

Since now $\frac{Q}{A} = L$ or $Q = AL$, we have for Q

$$\tfrac{1}{424} \times 2549 = 6.012 \text{ heat units.}$$

[For one lb. at 86° Fah., we have $T_1 = 459.4 + 86 = 545.4°$, and $R = 53.354$, hence, $L = 2.3026 \times 53.354 \times 545.4 \log \frac{4}{3} = 83753$ ft. lbs. $= 108.5$ heat units.]

*Adiabatic Curve.**—If a gas expands, all the time performing work, *without any heat being imparted to it from without*, the outer work which it performs can only be at the expense of the vibration work or temperature. The temperature then diminishes as the expansive force diminishes. This last then diminishes for two reasons; by reason of the increase of the volume, and by reason of the decrease of temperature.

---

* [Sometimes called also "*isentropic* curve," or curve of equal "entropy;" entropy being defined as that property of a body that remains constant when the body undergoes any change, but without receiving or losing any heat.]

We see at once that the work of the gas for the same expansion must be less than when the temperature is kept constant by imparting heat from without.

If v_1 is the specific volume and p_1 the tension of air, and v and p that after expansion, we have now no longer, according to Mariotte's law,

$$v : v_1 :: p_1 : p, \qquad \text{or} \qquad pv = p_1 v_1,$$

but we have the relation, first proved by Poisson,*

$$v^{1.41} : v_1^{1.41} :: p_1 : p,$$

---

* The above relation was proved by Laplace and by Poisson upon the hypothesis of the caloric theory of heat. It is easily deduced from the mechanical theory of heat by the aid of the calculus. For those who wish to understand the method of deduction we give it here. Others must accept it simply as an accurate expression of the law of relation of pressure and volume during adiabatic change.

We have for every perfect gas

$$pv = RT,$$

or

$$pdv + vdp = RdT,$$

hence

$$dT = \frac{pdv + vdp}{R} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1.)$$

If we denote inner work by U and outer by L, then

$$Q = A(U + L),$$

or

$$dQ = A(dU + dL).$$

For adiabatic change, $dQ = 0$, and hence

$$dU + dL = 0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2.)$$

Now for a perfect gas there is no disgregation work, and dU represents vibration work or change of temperature only. Hence

$$dU = \frac{c_v}{A} dT.$$

Also, since the pressure for a very small change of volume may be considered as constant,

$$dL = pdv.$$

Substituting in (2) and referring to (1) we have

$$\frac{c_v}{AR}(pdv + vdp) + pdv = 0$$

for the differential equation of the adiabatic curve.

But from Equation VII. we have $\frac{c_v}{AR} = \frac{1}{k-1}$, hence

$$\frac{1}{k-1}(pdv + vdp) + pdv = 0,$$

or
$$pv^{1.41} = p_1 v_1^{1.41},$$

or
$$\left(\frac{v}{v_1}\right)^{1.41} = \frac{p_1}{p}, \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(XVII.)}$$

where v is the greater volume and p_1 the greater pressure. The law is very similar, as we see, to Mariotte's, only the volumes are raised to the power denoted by 1.41, or in general by $\frac{c_p}{c_v}$, *i. e.*, the ratio of the specific heat by constant pressure to the specific heat by constant volume. This law has therefore been very appropriately called by Redtenbacher the *exponential law of Mariotte*, and as such it may easily be remembered.

If we make use of logarithms, we have

$$1.41 \log \frac{v}{v_1} = \log \frac{p_1}{p}.$$

If we denote the ratio 1.41 by k, we have

$$k \log \frac{v}{v_1} = \log \frac{p_1}{p}.$$

If we assume the initial volume $v_1 = 1$, and the initial tension $p_1 = 1$ atmosphere, we have for the tension p when the air has expanded to double its volume, or $v = 2v_1$, at the expense of its inherent heat, that is, without any heat being imparted from without,

$$1.41 \log 2 = \log \frac{1}{p},$$

---

or
$$pdv + vdp + kpdv - pdv = 0,$$

or
$$kpdv + vdp = 0.$$

Dividing by pv, we obtain

$$\frac{dp}{p} + \frac{kdv}{v} = 0.$$

Integrating this betewen the limits of the initial pressure and volume (p_1 and v_1), and the final pressure and volume (p and v), we have

$$\log p - \log p_1 = k \log v_1 - k \log v,$$

or
$$\log \frac{p}{p_1} = k \log \frac{v_1}{v}, \quad \text{or} \quad \frac{p}{p_1} = \left(\frac{v_1}{v}\right)^k.$$

Hence
$$pv^k = p_1 v_1^k = \text{etc.}$$

For air the ratio of the specific heats $k = 1.41$, hence we have

$$pv^{1.41} = p_1 v_1^{1.41}.$$

or $$1.41 \times 0.3010 = \log \frac{1}{p}, \text{ or } 0.42441 = \log \frac{1}{p},$$

or $$\frac{1}{p} = 2.6571, \text{ or } p = \frac{1}{2.6571} = 0.376 \text{ atmosp.}$$

If the air had expanded according to Mariotte's law, that is, if heat *had* been imparted from without in such a manner that while expanding the temperature remained constant, we should have had

$$p = 0.5 \text{ atmospheres.}$$

It is now easy to calculate in similar manner the tension p, which the air has, when, without receiving heat from without, it expands to 3, 4, etc. times its original volume. If we should thus actually compute these tensions, and lay them off as ordinates, with the corresponding volumes as abscissas, the curve *BCDE* thus obtained would give the relation of volume to pressure. This curve is called the *adiabatic curve* for permanent gases. We see that it approaches the axis of abscissas much more rapidly than the isothermal or isodynamic curve, which is represented by the broken line. Let us now determine the decrease of temperature of the air during the expansion.

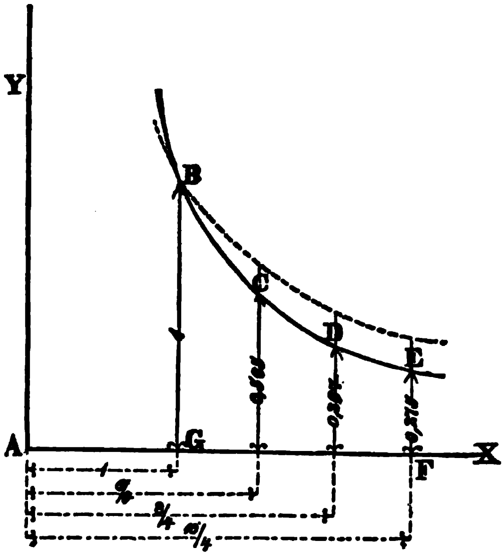

Fig. 14.

We have found by Equation XII., for the combined laws of Mariotte and Gay-Lussac,

$$pv = RT,$$
$$p_1v_1 = RT_1, \text{ etc.,}$$

where T and T_1 are the absolute temperatures at the volumes

v and v_1. From Equation XVII. we have for adiabatic expansion

$$\left(\frac{v}{v_1}\right)^{1.41} = \frac{p_1}{p} \quad \text{or} \quad \frac{v^{1.41}}{v_1^{\,1.41}} = \frac{p_1}{p}.$$

If we multiply both sides by v_1 and divide by v, we have

$$\frac{v_1 v^{1.41}}{v v_1^{\,1.41}} = \frac{p_1 v_1}{pv} \quad \text{or} \quad \left(\frac{v}{v_1}\right)^{0.41} = \frac{p_1 v_1}{pv}, \quad . \quad \text{(XVIII}a.)$$

or generally

$$\left(\frac{v}{v_1}\right)^{k-1} = \frac{p_1 v_1}{pv}. \quad . \quad . \quad . \quad . \quad \text{(XVIII}b.)$$

The right side of this equation becomes $\frac{RT_1}{RT} = \frac{T_1}{T}$, and hence

$$\left(\frac{v}{v_1}\right)^{0.41} = \frac{T_1}{T} = \frac{273 + t_1}{273 + t}, \quad . \quad . \quad \text{(XIX}a.)$$

or generally, putting k for the ratio $\frac{c_p}{c_v}$,

$$\left(\frac{v}{v_1}\right)^{k-1} = \frac{T_1}{T} = \frac{273 + t_1}{273 + t}, \quad . \quad . \quad . \quad \text{(XIX}b.)$$

We can therefore find the final temperature t, from the initial temperature t_1 and the expansion ratio $\frac{v}{v_1}$.

We see, then, from the formula, *that the* 0.41 *powers of the specific volumes are inversely proportional to the corresponding absolute temperatures.*

EXAMPLE.

If the specific volume v_1 *of the air has the temperature* $t_1 = 30°$, *what will be its temperature* t *when it has expanded to* $v = 2v_1$, *performing work and without receiving heat from without?*

We have $\left(\frac{2v_1}{v_1}\right)^{0.41} = \frac{273 + 30}{273 + t}$ or $2^{0.41} = \frac{303}{273 + t}$,

or

$1.3286(273 + t) = 303$ or $362.7078 + 1.3286t = 303$. $1.3286t = -59.7078$.

Hence $t = -44.9°$.

[For $t_1 = 86°$ Fah., we have

$1.3286(459.4 + t) = 545.4$ or $t = -48.9°$ Fah.]

Since in the formula $\left(\frac{v}{v_1}\right)^{0.41} = \frac{T_1}{T}$, the expansive force p or p_1 does not occur, the end temperature depends solely upon the initial temperature and the expansion ratio. Whether therefore the specific volume is small or great, and hence the expansive force great or small, makes no difference in the final temperature, if only the initial temperature remains the same.

Just as we have determined the final temperature from the initial and end volumes, so we can also determine it from the initial and final tensions.

Thus we have for the law of adiabatic expansion,

$$\left(\frac{v}{v_1}\right)^{1.41} = \frac{p_1}{p}.$$

This can be written

$$\text{XX}a \ldots \frac{v}{v_1} = \frac{p_1^{\frac{1}{1.41}}}{p^{\frac{1}{1.41}}} = \left(\frac{p_1}{p}\right)^{0.7093} \quad \text{or} \quad \frac{v}{v_1} = \left(\frac{p_1}{p}\right)^{\frac{1}{k}}. \; . \; (\text{XX}b.)$$

If we multiply the numerators by p, and the denominators by p_1, we have

$$\frac{vp}{v_1 p_1} = \frac{p p_1^{0.7093}}{p_1 p^{0.7093}}.$$

This gives us

$$\frac{vp}{p_1 v_1} = \frac{p^{0.2907}}{p_1^{0.2907}} \quad \text{or} \quad \frac{vp}{p_1 v_1} = \left(\frac{p}{p_1}\right)^{\frac{k-1}{k}}.$$

Hence

$$\frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{0.2907} = \frac{273 + t}{273 + t_1}. \; . \; . \; (\text{XXI}a.)$$

or generally

$$\left(\frac{p}{p_1}\right)^{\frac{k-1}{k}} = \frac{T}{T_1}. \; . \; . \; . \; . \; (\text{XXI}b.)$$

Hence, *the 0.2907 powers of the pressures are directly proportional to the absolute temperatures.*

EXAMPLE.

We have in a cylinder one unit in weight of air, at a tension $p_1 = 1\frac{1}{2}$ atmospheres, and a temperature of $t_1 = 30°$. What will be its temperature t when the air has expanded adiabatically until its tension p is only one atmosphere?

We have

$$\left(\frac{1}{1\frac{1}{2}}\right)^{0.2907} = \frac{273 + t}{273 + 30} \quad \text{or} \quad \left(\frac{2}{3}\right)^{0.2907} = \frac{273 + t}{303}.$$

Hence

$$0.8888 = \frac{273 + t}{303} \quad \text{or} \quad 273 + t = 269.306 \quad \text{or} \quad t = -3.694°,$$

or in round numbers $t = -3.7°$.

The temperature therefore falls 33.7°.

[If we have 1 lb. of air at 86° Fah., we have

$$0.8888 = \frac{459.4 + t}{545.4} \quad \text{or} \quad t = 25.35° \text{ Fah.}]$$

From Equation XXa $$\frac{v}{v_1} = \left(\frac{p_1}{p}\right)^{0.7093}$$

we can now find at once the expansion ratio, or the ratio of the final volume to the original.

We thus have $$\frac{v}{v_1} = \left(\frac{3}{2}\right)^{0.7093};$$

hence $$\frac{v}{v_1} = 1.333 \quad \text{or} \quad \frac{v}{v} = \frac{3}{4}.$$

While, then, the tension falls in the ratio of 3 to 2, the volume increases in the ratio of 3 to 4.

Outer Work Performed by Air when expanding Adiabatically.—The question now arises, What *work* does the air perform when it expands, performing work, without receiving heat from without?

Since the expansion occurs at the expense of the vibration work, or of the temperature, the work performed must depend upon the initial and final temperatures.

Now we know that under constant volume we must impart 0.16847 heat units, in order to raise the temperature of one weight unit of air one degree. This heat we have called the specific heat of air for constant volume. In like manner we must abstract 0.16847 heat units from each kilogram for every degree that we cool it, under constant volume. But 0.16847 heat units correspond to a mechanical work of

$$0.16847 \times 424 \text{ meter-kilograms,}$$

or 0.16847 × 772 foot-pounds for standard lb. and degree Fah., and this work must be performed when the unit weight of air is cooled one degree by expansion, while performing work, because the work performed is the equivalent of the heat which disappears, since no heat is imparted or abstracted during expansion.

We denote the specific heat for constant volume, or, in the case of air, the number 0.16847, by c, and the mechanical equivalent of heat (424 meter-kilograms) we denote by $\frac{1}{A}$, and hence the work is

$$c \times \frac{1}{A} = \frac{c}{A} \text{ meter-kilograms, or foot-pounds,}$$

according to the value of $\frac{1}{A}$ which is 772 for standard pound and 1° Fah.

If, therefore, the specific air volume has the temperature

t_1, and hence the absolute temperature $273 + t_1$, or $(459.4 + t_1)$ Fah., the inherent vibration work in it, or the "intrinsic energy," as it is called, is

$$u_1 = \frac{c}{A}(273 + t_1) \text{ meter-kilograms, where } \frac{1}{A} = 424,$$

or $$u_1 = \frac{c}{A}(459.4 + t_1) \text{ foot-pounds, where } \frac{1}{A} = 772.$$

If now this volume gradually expands, overcoming an outer pressure which at any moment is less than the air pressure by an infinitely small amount, until its temperature is t, so that t is less than t_1, then the inner work inherent in it will be

$$u = \frac{c}{A}(273 + t) \text{ kilograms.}$$

The inner work which disappears is thus

$$u_1 - u = \frac{c}{A}(273 + t_1) - \frac{c}{A}(273 + t)$$

$$= \frac{c}{A}\Big((273 + t_1) - (273 + t)\Big) = \frac{c}{A}(t_1 - t) \text{ meter-kilograms,}$$

and this is evidently exactly equal to the outer work performed, since no heat has been imparted from without. If we denote this outer work by L, we have

$$L = \frac{c}{A}(t_1 - t), \quad . \quad . \quad . \quad . \quad \text{(XXIIa.)}$$

or also,

$$L = \frac{c}{A}(T_1 - T) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XXIIb.)}$$

$$= 71.431\,(t_1 - t) \text{ meter-kilograms.}^{*}$$

$$= 130.0588\,(t_1 - t) \text{ foot-lbs, where temperature is Fah.}$$

Thus we see *that the outer work is proportional to the difference between the initial and final temperatures*, as might have been at once concluded.

The area $BCDEFG$ (see last Figure) inclosed by the adiabatic curve $BCDE$ and the ordinates BG and EF, represents the work performed during expansion, or the decrease in the inherent vibration work.

The final temperature t in the last formula can be found from Equation XIXa,

$$\left(\frac{v}{v}\right)^{0.41} = \frac{273 + t_1}{273 + t},$$

---

[*] [This is the work performed by one unit of weight. For G units of weight we have $71.431\,G\,(t_1 - t)$. If we use Fahrenheit degrees and English measures we have $130.06\,G\,(t_1 - t)$. If we use Centigrade degrees and English measures we have $234.173\,G\,(t_1 - t)$. The student will do well to make the reductions.]

or from Equation XXI*a*,

$$\left(\frac{p}{p_1}\right)^{0.2907} = \frac{273 + t}{273 + t_1}.$$

If Q is the number of disappearing heat units, we have

$$L = \frac{1}{A} Q, \text{ and hence}$$

$$L = \frac{1}{A} Q = \frac{c}{A} (t_1 - t),$$

or $$Q = c (t_1 - t) \text{ heat units.}$$

If L or the outer work is equal to zero, that is, if the air performs no outer work during expansion—if there is no outer pressure overcome—we have

$$0 = \frac{c}{A} (t_1 - t) \quad \text{or} \quad 0 = t_1 - t,$$

$$\text{or } t = t_1.$$

That is, *the final temperature is equal to the initial temperature, and the temperature of the air remains unchanged,* as the experiment of Joule, already noticed, clearly proves.

EXAMPLE.

What is the work performed by the air in the last example, when the temperature sinks from + 30° to − 3.7° ?

Here $t_1 - t = 33.7°$, hence

$$L = 71.431 \times 33.7 = 2407.225 \text{ meter-kilograms.}$$

The number of heat units disappearing is

$$Q = 0.16847 \times 33.7 = 5.6774.$$

We may also determine the work L directly from the initial and final volumes, or from the initial and final tensions, as well as from the initial and final temperatures.

Thus we have

$$L = \frac{c}{A} (273 + t_1) - \frac{c}{A} (273 + t).$$

If we divide by $273 + t_1$, we have

$$\frac{L}{273 + t_1} = \frac{c}{A} - \frac{c}{A} \cdot \frac{273 + t}{273 + t_1}.$$

The factor on the right is, according to Equation XXI$a = \left(\frac{p}{p_1}\right)^{0.2907}$, and since $R\,(273 + t_1) = v_1 p_1$, or $273 + t_1 = \frac{v_1 p_1}{R}$, we have

$$\frac{LR}{v_1 p_1} = \frac{c}{A} - \frac{c}{A}\left(\frac{p}{p_1}\right)^{0.2907},$$

or

$$L = \frac{c}{AR} v_1 p_1 \left[1 - \left(\frac{p}{p_1}\right)^{0.2907}\right] \quad . \quad (\text{XXIV}a.)$$

Again, since according to Equation XIXa, $\frac{273 + t}{273 + t_1} = \left(\frac{v_1}{v}\right)^{0.41}$, we have also

$$L = \frac{c}{AR} v_1 p_1 \left[1 - \left(\frac{v_1}{v}\right)^{0.41}\right] \quad . \quad (\text{XXIV}b.)$$

Compression of Air when Heat is neither Imparted nor Abstracted. —When air is compressed, the opposite phenomena take place. The work expended in the compression is transformed into heat. The vibration work is therefore increased. The tension of the air is then increased, for two reasons. First, the density is increased by the compression, and the atoms strike oftener against the piston which causes the compression, and this alone causes an increase of tension. Second, the living force, or the velocity of the particles, is increased by the heat due to the transformation of the work, and this also causes an increase in the expansive force. This last must increase according to the same law as before, during expansion, it diminished.

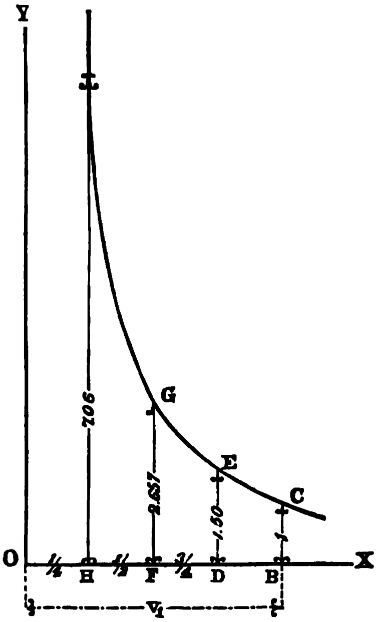

FIG. 15.

That air is heated by compression has long been known. We are all familiar with the "pneumatic syringe." This consists of a glass or metal cylinder, in which moves an air-tight piston. Upon the under side of the piston is a piece of tinder. If now the piston is pressed quickly down, the air is compressed, heat is developed, and the tinder ignited.

If we denote the specific volume of air inclosed in a cylinder by v_1 and the pressure by p_1, and if v and p are the volume and pressure after compression, we have from the law already found, when heat is not imparted nor abstracted,

$$\left(\frac{v}{v_1}\right)^{1.41} = \frac{p_1}{p}.$$

Let now, Fig. 15, $OB = v_1 = 1$ be the initial volume, and $p_1 = 1$ atmosphere be the corresponding pressure. If we compress the air to $\frac{3}{4}$ of its original volume, we have

$$\left(\tfrac{3}{4}\right)^{1.41} = \frac{1}{p},$$

or $$3^{1.41}p = 4^{1.41};$$

hence $$p = \left(\tfrac{4}{3}\right)^{1.41} = 1.5 \text{ atmos.} = DE.$$

For $v = \frac{1}{2} = OF$, we have

$$\left(\tfrac{1}{2}\right)^{1.41} = \frac{1}{p}, \text{ hence } p = 2^{1.41},$$

or $$p = 2.657 \text{ atmospheres} = FG.$$

If $v = \frac{1}{4} = OH$, we have

$$\left(\tfrac{1}{4}\right)^{1.41} = \frac{1}{p}, \text{ hence}$$

$$p = 4^{1.41} = 7.06 \text{ atmospheres} = HI.$$

The curve joining the points $CEGI$ thus found gives the law of the increase of pressure as the volume diminishes. We see how rapidly this curve rises for great diminution of volume.

The area $BCEGHI$, inclosed by the curve and the ordinates BC and HI, gives the mechanical work expended in compressing the body, and which is therefore completely transformed into heat.

EXAMPLE 1.—In a cylinder we have one kilogram of air of 0° and atmospheric pressure. What amount must it be compressed adiabatically in order to raise the pressure to 2 atmospheres; how much is the air heated; what work is necessary for compressing it; and how many heat units appear?

We have from Equation XX*a*,

$$\frac{v}{v_1} = \left(\frac{p_1}{p}\right)^{0.7093} = \left(\frac{1}{2}\right)^{0.7093},$$

hence $$v = 0.61188v_1.$$

Since v_1 is the specific volume, or volume of unit of weight, at 0°, which volume is $\frac{1}{1.29318} = 0.773$ cubic meters, we have for the volume of v

$$v = 0.773 \times 0.61188 = 0.4731 \text{ cubic meters.}$$

The final temperature t is given by Equation XXI*a*,

$$\left(\frac{p}{p_1}\right)^{0.2901} = \frac{273 + t}{273 + t_1} \quad \text{or} \quad \left(\frac{2}{1}\right)^{0.2907} = \frac{273 + t}{273 + 0}.$$

Hence $$2^{0.2907} \times 273 = 273 + t = 334.01,$$

or $$t = 334.01 - 273 = 61.01°.$$

The air therefore becomes heated to 61.01°. The mechanical work necessary for the compression is, from Equation XXII.,

$$L = \frac{C}{A}(t_1 - t) = 71.431\ 5\ (0 - 61.01) = -\ 4358 \text{ meter-kilograms.}$$

The negative sign denotes that this work is received by or performed upon the gas, instead of performed by it.

The quantity of heat generated is

$$Q = AL = \frac{-\ 4358}{424} = -\ 10.28 \text{ heat units.}$$

If we compress only ½ or ¼ of a kilogram to 2 atmospheres, only ½ or ¼ as much work would be necessary, and we should generate only ½ or ¼ as many heat units. But the rise of temperature would be just the same, as, evidently, this depends only upon the expansion ratio.

If we have one pound of air at 32° Fah. and atmospheric pressure, we have as before, $v = 0.61188v_1$. Since $v_1 = \dfrac{1}{0.080744}$, we have $v = 7.578$ cubic feet. The final temperature is given by $459.4 + t = 601.218$, hence $t = 141.818$. The work is $L = 180.0588 \times (32 - 141.818) = -\ 14282.797$ ft. lbs. The heat is $\dfrac{14282.797}{772} = 18.5$ heat units.

EXAMPLE 2.—If a caloric engine compresses adiabatically at each stroke ¼ of a kilogram of air at 10° and 1 atmosphere to 4 atmospheres, what mechanical work is necessary, and how much is the air heated ?

We have from Equation XXI*a*,

$$\left(\frac{p}{p_1}\right)^{0.2907} = \frac{273 + t}{273 + t_1} \quad \text{or} \quad \left(\frac{4}{1}\right)^{0.2907} = \frac{273 + t}{273 + 10},$$

hence $$t = -\ 273 + 283 \times 4^{0.2907} = -\ 273 + 423.62 = 150.62°.$$

The mechanical work is

$$L = \tfrac{1}{4}\frac{c}{A}(t_1 - t) = \frac{71.431}{4}(10 - 150.62)$$

$$= 17.86 \times -\ 140.62 = -\ 2511.47 \text{ meter-kilograms.}$$

If this work is performed in one second, we have

$$\frac{2511.47}{75} = 33.48 \text{ horse-power.}$$

If the air thus compressed is confined and heated 273°, its pressure would be doubled, or 8 atmospheres. The temperature after heating is then 273 = 150.62 = 423.62°.

What mechanical work can the air now perform, when it expands adiabatically, until its tension becomes again one atmosphere and temperature at 10° ?

We have $$L = \tfrac{1}{4}\frac{c}{A}(t_1 - t) = \tfrac{1}{4} \times 71.431\ (423.62 - 10)$$

$$= 17.86 \times 413.62 = 7387.25 \text{ meter-kilograms,}$$

or if the work is performed in one second,

$$\frac{7387.25}{75} = 98.5 \text{ horse-power.}$$

The compression of the air consumes thus about ⅓ of the work which the heated air can perform. If we assume that the resistances consume about 40 per cent., we have the total loss of work, 40 + 33⅓ = 73⅓ per cent. Only about 26⅔ per cent. remain as useful work. In most cases the efficiency would probably be lower.

[As we often have occasion to make use of the formulæ for adiabatic expansion or compression, the following table will be found useful in abridging calculations. The table gives for different values of the ratio $\frac{p_2}{p_1}$ the corresponding ratios of the temperatures, volumes, etc. It is made out for compression, or p_2 greater than p_1. It applies equally well to expansion if we simply suppose all the subscripts at head of columns *interchanged*. Before giving the table, we group below, for convenience of reference, the adiabatic formulæ already deduced.

$$\frac{p_2}{p_1}=\left(\frac{v_1}{v_2}\right)^k=\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}; \quad \frac{v_2}{v_1}=\left(\frac{p_1}{p_2}\right)^{\frac{1}{k}}=\left(\frac{T_1}{T_2}\right)^{\frac{1}{k-1}};$$

$$\frac{T_2}{T_1}=\left(\frac{v_1}{v_2}\right)^{k-1}=\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}.$$

For air,

$$k=1.41, \quad k-1=0.41, \quad \frac{1}{k-1}=2.44, \quad \frac{1}{k}=0.7093,$$

$$\frac{k-1}{k}=0.2907, \quad \frac{k}{k-1}=3.44.$$

Work done,

$$L=\frac{c}{A}(T_1-T_2)=\frac{c}{A}T_1\left(1-\frac{T_2}{T_1}\right); \quad \frac{c}{A}(t_1-t_2),$$

or

$$L=\frac{c}{AR}v_1p_1\left[1-\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}\right]=\frac{c}{AR}v_1p_1\left[1-\left(\frac{v_1}{v_2}\right)^{k-1}\right]$$

$$=\frac{c}{AR}v_1p_1\left[1-\frac{T_2}{T_1}\right]=\frac{c}{AR}v_2p_2\left[1-\frac{T_1}{T_2}\right].$$

In these formulæ, for air

$$c=0.16847, \qquad \frac{c}{AR}=2.44,$$

and

$\frac{1}{A}=424$, $\frac{c}{A}=71.431$, $R=29.272$ in French measures and Centigrade degrees.

$\frac{1}{A}=1390$, $\frac{c}{A}=234.1783$, $R=96.0376$ in English measures and Centigrade degrees.

$\frac{1}{A}=772$, $\frac{c}{A}=130.0588$, $R=53.354$ in English measures and Fahrenheit degrees.

$T=t+273$ Centigrade $=t+459.4$ Fahrenheit.

1 atmosphere = 10334 kil. per sq. meter = 14.7 lbs. per sq. inch.

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[*] All formulæ for work are for one unit of weight. The formulæ as given apply to expansion. For compression exchange subscripts.

TABLE FOR ADIABATIC CHANGE OF AIR.

$\frac{p_2}{p_1}$	$\frac{T_2}{T_1}$		$\frac{T_1}{T_2}$		$1-\frac{T_1}{T_2}$	$\frac{V_1}{V_2}$		$\frac{V_2}{V_1}$	
	Numbers.	Differences.	Numbers.	Differences.	Numbers.	Numbers.	Differences.	Numbers.	Differences.
1.2	1.0543	481	0.9485	415	0.0515	1.1384	1311	0.8784	907
1.4	1.1024	436	0.9070	344	0.0930	1.2695	1255	0.7877	709
1.6	1.1460	399	0.8726	294	0.1274	1.3950	1222	0.7168	577
1.8	1.1859	367	0.8432	253	0.1568	1.5172	1177	0.6591	474
2.	1.2226	343	0.8179	223	0.1821	1.6349	1143	0.6117	400
2.2	1.2569	321	0.7956	198	0.2044	1.7492	1114	0.5717	340
2.4	1.2840	303	0.7758	178	0.2242	1.8606	1087	0.5377	299
2.6	1.3193	287	0.7580	162	0.2420	1.9693	1062	0.5078	260
2.8	1.3480	272	0.7418	146	0.2582	2.0755	1041	0.4818	230
3.	1.3752	260	0.7272	135	0.2728	2.1796	1021	0.4588	205
3.2	1.4012	248	0.7137	124	0.2863	2.2817	1003	0.4383	185
3.4	1.4260	238	0.7013	116	0.2987	2.3820	985	0.4198	167
3.6	1.4498	230	0.6897	107	0.3103	2.4805	970	0.4031	151
3.8	1.4728	220	0.6790	100	0.3210	2.5775	955	0.3880	139
4.	1.4948	213	0.6690	94	0.3310	2.6370	941	0.3741	127
4.2	1.5161	206	0.6596	89	0.3404	2.7671	928	0.3614	117
4.4	1.5367	200	0.6507	83	0.3493	2.8599	916	0.3497	109
4.6	1.5567	193	0.6424	79	0.3576	2.9515	904	0.3388	101
4.8	1.5760	188	0.6345	75	0.3655	3.0419	894	0.3287	93
5.	1.5948	865	0.6270	322	0.3730	3.1313	4322	0.3194	388
6.	1.6813	769	0.5948	260	0.4052	3.5635	4117	0.2806	290
7.	1.7582	694	0.5688	217	0.4312	3.9752	3949	0.2516	228
8.	1.8276	636	0.5471	183	0.4529	4.3701	3806	0.2288	183
9.	1.8912	579	0.5288	159	0.4712	4.7507	3685	0.2105	152
10.	1.9491	554	0.5129	140	0.4871	5.1192	3582	0.1953	127
11.	2.0045	512	0.4989	125	0.5011	5.4774	3486	0.1826	110
12.	2.0557	483	0.4864	111	0.5136	5.8260	3408	0.1716	94
13.	2.1040	457	0.4753	101	0.5247	6.1668	3327	0.1622	83
14.	2.1497	434	0.4652	92	0.5348	6.4990	3360	0.1539	74
15.	2.1931		0.4560		0.5440	6.8350		0.1465	

The subscript 1 denotes initial and 2 final state. Thus p_1 is initial and p_2 final pressure, etc.

The Table holds good as it is for compression. For expansion exchange subscripts.

We can illustrate the use of this table by the examples already given. Thus,

Example 1.—In a cylinder we have one kilogram of air of 0° and atmospheric pressure. What amount must it be compressed adiabatically in order to raise the pressure to two atmospheres? how much is the air heated?

Here $\frac{p_2}{p_1} = 2$. Opposite 2 in the table we find at once $\frac{v_2}{v_1} = 0.6117$, hence $v_2 = 0.6117\, v_1$, the same as already found by calculation.

We have also at once from the table, corresponding to $\frac{p_2}{p_1} = 2$, $\frac{T_2}{T_1} = 1.2226$; hence $T_2 = 1.2226\, T_1$, or $t_2 + 273 = 1.2226 \times 273$; hence $t_2 = 333.77 - 273 = 60.77°$.

For 32° Fah. we have $t_2 + 459.4 = 1.2226 \times 491.4$ or $t_2 = 141.3°$ Fah.

Example 2.—If a caloric engine compresses adiabatically at each stroke ¼ of a kilogram of air at 10° and 1 atmosphere, to 4 atmospheres, how much is the air heated?

Here $\frac{p_2}{p_1} = 4$, and we have at once from the table

$$\frac{T_2}{T_1} = 1.4948, \quad \text{or} \quad t_2 + 273 = 1.4948 \times 283, \quad \text{or} \quad t_2 = 150°.$$

If t_1 is 50° Fah., we have $t_2 + 459.4 = 1.4948 \times 509.4$, or $t_2 = 302°$ Fah.

The new volume is $\frac{v_2}{v_1} = 0.3741$, or $v_2 = 0.3741\, v_1$.

Example 3.—If ¼ kilogram of air at 4 atmospheres tension and 150° *expands* adiabatically, performing work, till its tension is 1 atmosphere, what is its new temperature?

For expansion we simply have *to invert the subscripts at the head of the columns.* The student will do well to make a special note of this. Considering, then, all the subscripts reversed, we have from table for $\frac{p_1}{p_2} = 4$, $\frac{T_2}{T_1} = 0.6690$; hence $T_2 = 0.6690 T_1 = 0.6690 \times 423$, or $t_2 + 273 = 282.98$, or $t_2 = 9.98°$, or about 10°, as should be.

In similar manner we can easily find pressure and temperatures when volume ratio is given, or pressure and volume when temperature ratio is given. The student will do well to propose other examples and solve them both by table and calculation. (See examples at end of Part I.) The use of the last two columns for $\frac{t_1}{t}$ will be explained hereafter. *They have no reference to temperature at all.*]

Transference of Air from one Adiabatic Curve to Another.—We have now to deduce a very important thermodynamic principle, of which we shall have occasion to make frequent use.

Suppose the specific volume v_1, with the tension p_1, and the absolute temperature T_1, to be compressed, without any heat being added to it from without or abstracted from it. The compression thus takes place according to the adiabatic curve, or, as we say, this curve represents the law of the change of condition of the air. If the volume $Ov_1 = v_1$ is compressed to $Ov_2 = v_2$, let p_2 and T_2 be the corresponding tension and absolute temperature. If the volume is $Ov_3 = v_3$, let p_3 and T_3 be the corresponding tension and absolute temperature.

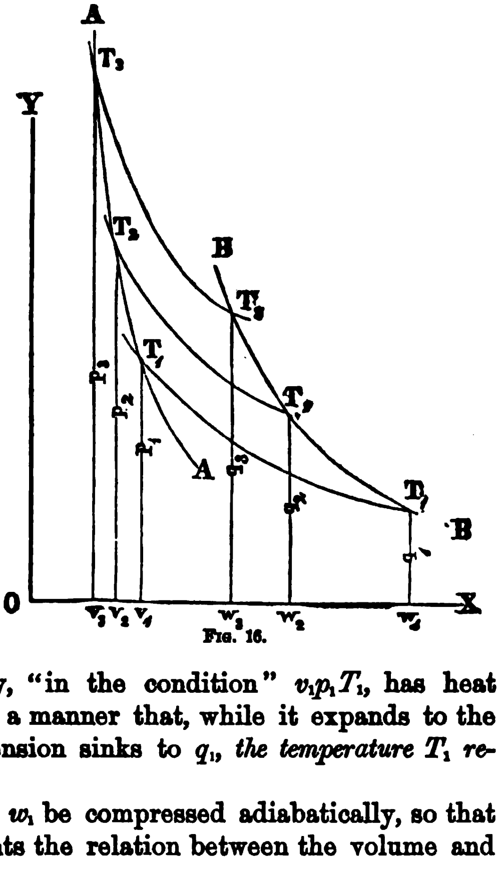

FIG. 16.

Now suppose that the air with the volume v_1, the pressure p_1, and the absolute temperature T_1, or, as we may say briefly, "in the condition" $v_1p_1T_1$, has heat imparted to it in such a manner that, while it expands to the volume w_1, and the tension sinks to q_1, *the temperature* T_1 *remains constant.*

Then let the volume w_1 be compressed adiabatically, so that the curve *BB* represents the relation between the volume and tension at any instant.

During the passage of the air from the condition $v_1p_1T_1$ to the condition $w_1q_1T_1$ (while therefore the tension changes according to the isothermal or isodynamic curve T_1T_1), let the quantity of heat imparted be Q_1.

Suppose, again, the air in the condition $v_2p_2T_2$ to expand, heat is imparted to it in such a manner that the temperature T_2 is preserved constant, until it arrives at the condition $w_2q_2T_2$ in the adiabatic curve *BB*, and let the heat imparted be Q_2.

Then we shall prove this relation:

$$Q_1 : Q_2 :: T_1 : T_2,$$

or,

the heat imparted in the first case is to that imparted in the second, as the corresponding temperatures T_1 and T_2, at which the addition of heat commenced.

If, again, the air passes from the condition $v_3p_3T_3$ to the condition $w_3q_3T_3$, and if Q_3 is the heat imparted, then we shall have

$$Q_2 : Q_3 :: T_2 : T_3.$$

We have then generally,

$$Q_1 : Q_2 : Q_3 = T_1 : T_2 : T_3.$$

Let us now seek to prove this relation.

The law of the adiabatic curve AA is from Equation XXI*a*

$$\left(\frac{p_2}{p_1}\right)^{0.2907} = \frac{T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (a.)$$

or it is also

$$\left(\frac{p_3}{p_1}\right)^{0.2907} = \frac{T_3}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (b.)$$

In like manner the law of the adiabatic curve BB is given by

$$\left(\frac{q_2}{q_1}\right)^{0.2907} = \frac{T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (c.)$$

and

$$\left(\frac{q_3}{q_1}\right)^{0.2907} = \frac{T_2}{T_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (d.)$$

The amount of heat Q_1 which is necessary to change the gas from the condition $v_1p_1T_1$ to the condition $w_1q_1T_1$, along the isothermal, is from Equation XVI.

$$Q_1 = 2.3026\, ART_1 \log \frac{p_1}{q_1}.$$

In like manner

$$Q_2 = 2.3026\, ART_2 \log \frac{p_2}{q_2}.$$

and

$$Q_3 = 2.3026\, ART_3 \log \frac{p_3}{q_3}.$$

We have therefore

$$Q_1 : Q_2 : Q_3 = T_1 \log \frac{p_1}{q_1} : T_2 \log \frac{p_2}{q_2} : T_3 \log \frac{p_3}{q_3}.$$

From Equations (a) and (c) we have

$$\frac{p_2}{p_1} = \frac{q_2}{q_1} \quad \text{or} \quad \frac{p_1}{q_1} = \frac{p_2}{q_2}.$$

In like manner from (b) and (d),

$$\frac{p_1}{q_1} = \frac{p_3}{q_3}.$$

We have then generally

$$\frac{p_1}{q_1} = \frac{p_2}{q_2} = \frac{p_3}{q_3}.$$

Hence in the above proportion the logarithms of the quotients $\frac{p_1}{q_1}$, $\frac{p_2}{q_2}$, $\frac{p_3}{q_3}$ are equal, and we obtain

$$Q_1 : Q_2 : Q_3 = T_1 : T_2 : T_3,$$

which was to be proved.

If therefore we wish to change the condition of air, as determined by a certain point on an adiabatic curve, into another condition which lies on another adiabatic curve, by the addition of heat, under constant temperature, the quantity of heat which must be imparted is proportional to the temperature.

We can also prove, as we shall hereafter, that this principle holds good when the curves T_1T_1, T_2T_2, T_3T_3 are not isothermals, but simply curves of the same kind, that is, which follow the same law of change of pressure and volume, *whatever that law may be.*

From the proportion

$$Q_1 : Q_2 : Q_3 = T_1 : T_2 : T_3,$$

we obtain

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2} = \frac{Q_3}{T_3}, \text{ etc.} \quad . \quad . \quad . \quad . \quad \text{(XXV.)}$$

Equations for the Expansion of Air under constant Pressure.—Let us now consider the case in which the specific air-volume expands under *constant pressure*, while it is heated. The heat imparted serves here, 1, to increase the *vibration work*, and 2, to perform *outer work.*

In the beginning, let us have the volume v_1 at the temperature t_1 and pressure p_1. After expansion, let the volume be v, and the temperature t. Then the increase of vibration work is

$$U=\frac{c}{A}(t-t_1).$$

Let now $Ov_1 = v_1 =$ the initial volume, Fig. 17, and $Ov = v =$ the final volume, then the outer work performed (L_1) will be represented by the rectangle v_1ABv, or by the difference of the products [*]

$$pv - p_1v_1 = p_1v - p_1v_1,$$

since the pressure is constant, and hence $p = p_1$.

We have, therefore,

$$L_1 = p(v - v_1) = p_1(v - v_1) \quad . \quad . \quad \text{(XXVI.)}$$

or since

$$pv - p_1v_1 = R(T - T_1) = R(t - t_1),$$

$$L_1 = R(t - t_1).\dagger \quad . \quad . \quad . \quad . \quad \text{(XXVII.)}$$

By means of these two equations, we can calculate the increase of the inner work and the outer work performed.

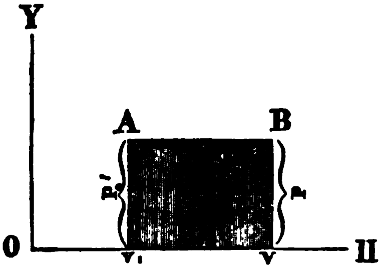

Fig. 17.

As in general the *ratio* between the initial and final temperatures is given, we can easily determine the temperature t at the end of expansion. Thus, as we have already shown,

$$\frac{vp}{273+t} = \frac{v_1p_1}{273+t_1},$$

or

$$\frac{v_1p_1}{vp} = \frac{273+t_1}{273+t},$$

and since, in the present case, $p = p_1$, we have

$$\frac{v}{v_1} = \frac{273+t}{273+t_1}.$$

For Fahrenheit degrees put 459.4 in place of 273.

EXAMPLE.

Suppose in a cylinder one unit of weight of air at a temperature $t = 80°$. What temperature will it possess when, being heated, it expands under constant pressure until it is $\frac{4}{3}$ds of its original volume? What outer work will it perform? and what is the increase of its inner work?

---

[*] Any curve or line, like *AB*, which represents the states of a body when the pressure remains constant, is sometimes called an "*isopiestic* line" or curve, that is, a line or curve of equal pressure. Such a line is also sometimes called an "*isobar*." In like manner, a line which represents the states of a body, when the *volume* is constant, is sometimes called an "*isometric* line," or line of equal volume.

[†] If there are G kilograms, we have $L = GR(t - t_1)$.

Since $v = \frac{4}{3}v_1$, we have

$$\frac{\frac{4}{3}v_1}{v_1} = \frac{273 + t}{273 + 30}, \qquad \tfrac{4}{3} \times 303 = 273 + t,$$

$$t = 404 - 273, \quad \text{or} \quad t = 131^\circ.$$

For $t_1 = 86^\circ$ Fah. we have

$$\tfrac{4}{3} \times 545.4 = 459.4 + t, \quad \text{or} \quad t = 267.8^\circ \text{ Fah.}$$

The increase of inner work is

$U = \frac{c}{A}(131 - 30) = 71.431 \times 101 = 7214.53$ meter-kil. for each kilogram,

or $130.0583 \times 131.8 = 23644$ ft. lbs. for each pound,

and the outer work performed is

$L = R(131 - 30) = 29.272 \times 101 = 2956.47$ meter-kilograms for each kilogram,

or $53.354 \times 181.8 = 9699$ ft. lbs. for each pound.

The amount of heat consumed in each work may be easily found. We know that the specific volume, when heated under constant pressure, requires 0.23751 heat units, or 1.41 times as much as when the air is heated under constant volume. Since we denote the specific heat for constant volume by c, that for constant pressure is $1.41c$ for air, or, in general, kc.

If, therefore, we heat one kilogram of air $t - t_1$ degrees under constant pressure, we have

$$Q = kc(t - t_1)$$

heat units necessary to be imparted.

In the present case, then,

$Q = 0.23751\,(131 - 30) = 23.9885$ heat units for each kilogram,

or $0.23751\,(267.8 - 86) = 43.18$ heat units for each pound.

The preceding enables us to find an expression for the mechanical equivalent of heat in a more general method than before.

The quantity of heat Q, in the last expression, is equivalent to the work AL, hence

$$AL = kc\,(t - t_1) \quad \text{or} \quad L = \frac{kc}{A}(t - t_1).$$

But we must have this work L equal to the vibration work plus the outer work, or

$$L = U + L_1, \quad \text{or}$$

$$\frac{kc}{A}(t - t_1) = \frac{c}{A}(t - t_1) + R\,(t - t_1).$$

If we divide through by $t - t_1$, we have

$$\frac{kc}{A} = \frac{c}{A} + R, \quad R = (k - 1)\frac{c}{A}$$

$$\frac{1}{A} = \frac{R}{c(k - 1)}$$

Inserting numerical values as determined for air,* we have

$$\frac{1}{A} = \frac{29.272}{0.16847 \times 0.41} = \frac{29.272}{0.0607} = 423.8 \text{ meter-kilograms.}$$

QUESTIONS FOR EXAMINATION.

What do you understand by an isothermal curve or line? What is the law of this curve? What kind of a curve then is it? What do you mean by "permanent" gas? Are there any such? What is the graphical representation of Mariotte's law? Find the expression for the mechanical work of gas expanding according to this law. Find the expression for the amount of heat imparted.

What do you understand by an isodynamic curve or line? When is this curve the same as the isothermal, and for what kind of bodies? What is an isenergic curve? What does "isenergic" mean? What do you understand by energy?

What is an adiabatic curve? Isentropic curve? What does "isentropic" mean? Define "entropy." What does "adiabatic" mean? What is the law of the adiabatic curve? Deduce it. Why has it been called the exponential law of Mariotte? In what respect is the term appropriate? Which curve, the adiabatic or the isothermal, approaches the axis of X most rapidly? Why? Deduce from the general law a relation between volume and temperature. Between pressure and temperature.

Show how to determine the outer work performed by air expanding adiabatically. To what is the outer work solely proportional in this case? Why might this have been at once concluded? Deduce the expression for the heat units converted into work. Discuss in similar manner the case of air when compressed adiabatically.

When a gas, as air, is made to pass according to any given law, from one adiabatic curve to another, what is the relation between the heat imparted and the temperature? Prove this relation.

What is an isopiestic line? What does "isopiestic" mean? When air expands under constant pressure what effects does the heat imparted produce? What is the expression for the increase of vibration work? What is the expression for the outer work performed? What is the relation between volume and temperature?

If one kilogram of air has a temperature of 30° C., what will be its temperature when it is heated and made to expand under constant pressure until it is twice its original volume? What outer work will it perform? What is its increase of inner work? What amount of heat is imparted? Prove that $\frac{1}{A} = \frac{R}{c(k-1)}$.

What work is performed by 10 pounds of air at 2 cubic feet volume and 5 atmospheres pressure, when it expands to 7 cubic feet, overcoming an outer pressure equal at any moment to the tension, the temperature being kept constant?

What is this constant temperature? What is the final pressure? How much heat must have been imparted in order to keep the temperature constant?

If one kilogram of air is heated, under the pressure of the atmosphere, from 0° to 1° C., how much work does it perform during expansion?

If one pound of air is heated, in same manner, from 32° to 33° Fahr., what work is performed? If heated from 0° to 1° C., what work is performed?

What is the weight of one cubic foot of air at atmospheric pressure and 32° F.? What is weight of one cubic meter of air at same pressure and 0° C.?

If two pounds of air at a temperature of 40° Fahr. expand adiabatically, performing work, till the volume is doubled, what is the final temperature?

What is the original volume? What is final pressure, if the initial pressure is one atmosphere? What is the work performed?

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* Page 148.

CHAPTER VI.

THE SIMPLE REVERSIBLE CYCLE PROCESS.*—ILLUSTRATION OF THE PROCESS BY ANALOGOUS PRINCIPLES OF MECHANICS.

SUPPOSE in the cylinder *CC*, Fig. 18, one unit in weight of air of the volume v_1, tension p_1 and absolute temperature T_1.

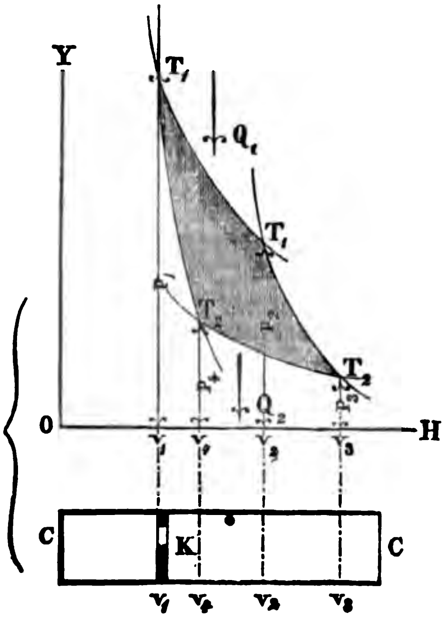

FIG. 18.

Let this volume expand, performing work (the outer pressure being always equal to the tension, or differing only by an infinitely small amount), until it has the volume $Ov_2 = v_2$, and the tension p_2. Let us also assume that the temperature T_1 remains the same, so that heat must be imparted from without. The expansion therefore takes place along the isothermal line T_1T_1, and the heat imparted is Q_1. The area $v_1T_1T_1v_2$ represents the work performed by the air during expansion.

Now let the air still expand from the volume $Ov_2 = v_2$ to $Ov_3 = v_3$, still performing work, but without receiving or part-ing with heat, that is, let it expand adiabatically. The expansion then follows the adiabatic curve T_1T_2, and the area $T_1T_2v_3v_2$ represents the outer work performed. Since this work must be at the expense of the vibration work or temperature, this latter must decrease, and T_1 falls to T_2.

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* [A cycle is termed "*closed*" if the body after any series of changes returns to its original condition, otherwise it is called "*open*." A closed cycle is termed "*simple*" if its bounding curves are of only two types. If of more than two types, it is called compound. A closed cycle process is "*reversible*" when the changes of state are continuous. In such a cycle all the changes may take place in reverse order.]

The work thus far performed is therefore given by the area $T_1T_1T_2v_3v_1T_1$, and the amount of heat imparted is Q_1 heat units.

Let now the air, whose volume is $Ov_3 = v_3$ and temperature T_2, be compressed, the temperature T_2 being kept the same, until its volume is $Ov_4 = v_4$. During this process a certain amount of heat Q_2 must be abstracted in order that the temperature may remain the same. The compression follows then the isothermal line T_2T_2, and the mechanical work necessary for compression is represented by the area $T_2v_3v_4T_2$.

The volume $Ov_4 = v_4$ may be so chosen that when the air is finally compressed adiabatically to the volume v_1, the air shall be again in its initial condition. In this case, the work performed upon the air is given by the area $v_4T_2T_1v_1$, the temperature rises from T_2 to T_1, and the expansive force rises from p_4 to p_1 again.

A process of this kind is called by Clausius a "*simple reversible cycle process.*" *

As remarked, the work done by the air is given by the area $T_1T_1T_2v_3v_1T_1$, and that performed upon the air, or used in the compression, is given by the area $T_2v_3v_1T_1T_2T_2$. The difference of these two areas, or the area $T_1T_1T_2T_2$, which is shaded in the Fig., and is inclosed by the two isothermal and adiabatic curves, is the *excess of the work performed by the air over that performed upon it.*

Since, however, heat and mechanical work are equivalent; since the generation of work requires heat, and the generation of heat requires work; the heat Q_1 *imparted* during the change from T_1 to T_1, as shown by the arrow, must be greater than the heat abstracted, Q_2, which is also indicated by an arrow. In other words, the difference of the heat, units $Q_1 - Q_2$, transformed into work, is represented by the area $T_1T_1T_2T_2$. Since the heat $Q_1 - Q_2$ is equivalent to the work

$$\frac{1}{A}(Q_1 - Q_2)$$

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* See preceding note.

we have, if we denote the area $T_1T_1T_2T_2$ by F,

$$\frac{1}{A}(Q_1 - Q_2) = F \quad . \quad . \quad . \quad . \quad (\text{XXVIII.})$$

or

$$Q_1 - Q_2 = AF.$$

We have thus far assumed that the initial condition of the air is given by the quantities $v_1p_1T_1$, and that this condition passes gradually into the condition T_1, p_2, v_2, or T_2, p_3, v_3, or, what is the same thing, that the end of the ordinate p_1 traverses the outline of the area $T_1T_1T_2T_2$ in the direction $T_1T_1T_2T_2T_1$. We may have its motion, however, in the opposite direction, $T_1T_2T_2T_1T_1$.

In such case we have evidently the work of compression greater than that performed by the air during the expansion, and the difference of the two is still given by the area $T_1T_1T_2T_2$.

Since, then, work is performed upon the air, an equivalent amount of heat is generated in it, or, in other words, the heat Q_1 abstracted during the change T_1T_1 must be greater than the heat Q_2 imparted during the change T_2T_2. We have therefore obtained heat instead of work.

Since, then, mechanical work has disappeared, and heat been obtained, we must have the same equation as before, but with opposite sign. Thus

$$-\frac{1}{A}(Q_1 - Q_2) = -F \quad . \quad . \quad . \quad (\text{XXIX.})$$

or

$$-(Q_1 - Q_2) = -AF.$$

In fact, the arrows in the Fig. must now have an opposite direction; the lower should point toward the curve T_2T_2, the other away from T_1T_1.

The reason for the change of sign is also seen in that, in the present case, where heat is generated, the end of the ordinate p_1 goes around the area F in a direction opposite to that in which it went when heat disappeared and work was given out. Since the above process can be performed in either way, we call it a "*simple* REVERSIBLE *cycle process.*"

Intermediate Body in the Cycle Process.—It is evident that for the performance of a cycle, we must have, in addition to the body which goes through the cycle of changes, and which we may call the "working body" or the "*intermediate body*," also two other bodies, one of which gives out and the other of which absorbs heat. In the hot-air engine the fire is the *source* of heat, the air in the engine is the *intermediate body*, and the outside air is the body which absorbs heat, or the *refrigerator*. In the steam engine the water is the intermediate body, the hot gases of combustion the source, and the cold condensing water or the outside air the refrigerator.

If the absorption and rejection of heat by the intermediate body takes place as represented by the cycle diagram, it is evident that the source of heat, which we denote by K, must possess so much heat as to replace each time the loss of heat of the intermediate body, and that this latter body must be able to at once receive each new accession.

In like manner the refrigerator, which we denote by K_1, must be able at every moment to absorb the heat excess from the intermediate body.

In practice these conditions are seldom perfectly fulfilled, and hence we generally find the calculated work varies more or less from that obtained by direct observation.

Now that we have introduced the bodies K and K_1, let us consider once more our cycle process.

While the end of the ordinate p_1 describes the circumference of the shaded area in the direction T_1 T_1 T_2, etc. The intermediate body, or in this case the air, receives the heat Q_1 while passing from T_1 to T_1. The source K then, which imparts this heat Q_1, sinks in temperature. While the intermediate body passes from T_2 to T_2, it gives up the quantity of heat Q_2 to the refrigerator K_1, and hence the temperature of K_1 rises. But now the heat Q_1 is greater than Q_2 and the excess corresponds to the work performed by the intermediate body. The body K, therefore, imparts more heat than the body K_1 receives. The disappearing heat is transformed into work.

In a steam engine then, where the steam is the intermediate body, *the heat which the steam possesses before performing the work must be greater than that which it possesses after, and which it imparts to the condensing water or to the air, and the difference, transformed into work, acts upon the piston.* The same holds good for the hot-air engine. The heat which the hot air gives up to the outer air is less than the heat imparted to it, by an amount equivalent to the work performed.

Carnot, to whom the cycle process is due, laid down the following principle : "When heat passes from one body K, through an intermediate body, to a third K_1, work is performed, but the heat received by K_1 is *equal* to that lost by K." This last clause contradicts, as we see, the fundamental principle of thermodynamics, and is incorrect.

If the end of the ordinate p_1 passes round the shaded area in the direction T_1 T_2 T_2, etc., then along the path T_2' T_2, the heat Q_2 is absorbed, and the temperature of K_1 sinks. Along the path T_1 T_1 the heat Q_1 is rejected, and this heat Q_1 is greater than Q, by the amount of heat *equivalent* to the work performed upon the intermediate body. This heat $Q_1 - Q_2$ is now given up to the body K, which thus receives more heat than the first gave out.

Transformation of Equation XXVIII.—We can now give to the Equation,

$$\frac{1}{A}\left(Q_1 - Q_2\right) = F,$$

or

$$Q_1 - Q_2 = AF \, . \; . \; . \; . \; . \; . \; . \; . \qquad (1).$$

another form.

Since the curves T_1 T_2 and T_1 T_2 are adiabatic curves, and T_1 T_1 and T_2 T_2 are isothermal curves, we may, according to Eq. XXV., express the ratio of the quantities of heat Q_1 and Q_2, in terms of the absolute temperatures.

We have, according to that Equation,

$$\frac{Q_1}{T_1} = \frac{Q_2}{T_2},$$

therefore

$$Q_2 = \frac{Q_1 T_2}{T_1}; \; Q_1 = \frac{Q_2 T_1}{T_2}.$$

If we substitute the value of Q_2 in (1) we have

$$Q_1 - \frac{Q_1 T_2}{T_1} = AF, \quad \text{or} \quad \frac{Q_1 T_1 - Q_1 T_2}{T_1} = AF,$$

or $$F = \frac{Q_1}{AT_1}(T_1 - T_2). \quad . \quad . \quad . \quad . \quad (\text{XXX.})$$

If we substitute the value of Q_1 in (1) we have

$$\frac{Q_2 T_1}{T_2} - Q_2 = AF, \text{ or } \frac{Q_2 T_1 - Q_2 T_2}{T_2} = AF,$$

or $$F = \frac{Q_2}{AT_2}(T_1 - T_2). \quad . \quad . \quad . \quad (\text{XXXI.})$$

Since $\frac{1}{A} = 424$ meter-kilograms, we have also

$$F = \frac{424 Q_1}{T_1}(T_1 - T_2)$$

$$F = \frac{424 Q_2}{T_2}(T_1 - T_2).^*$$

For Fahrenheit degrees put 772 in place of 424.

We have, therefore, the following important principle: *The work performed by the intermediate body in a simple reversible cycle process, or, when the process is reversed, the work performed upon this body is directly proportional to the amount of heat absorbed or rejected, and also to the difference between the highest and lowest temperatures.*

The correctness of this principle is also seen at once from our Figure. The greater T_1 or the less T_2, or the higher the end of the ordinate p_1 and the lower that of p_2, so much the greater is the vertical depth of the shaded area. Also, the more the gas expands under constant temperature along the isothermal $T_1 T_1$—the more heat, therefore, is imparted to it in order to maintain its temperature constant—the further apart are the ordinates p_1 and p_2, and so much the greater is the length of the shaded area.

The heat Q_1 absorbed, or the heat Q_2 rejected by the gas in passing over the isothermals $T_1\,T_1$ or $T_2\,T_2$, may be calculated from Equation XV., when the initial and end volumes, v_1 and v, and the initial and end pressures, p_1 and p, are known. But from Equation XVI. we can determine Q_1 and Q_2, when we know the initial and end volumes, as well as the temperatures T_1 and T_2.

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* For Centigrade degrees, $T = 273 + t$. For Fahrenheit, $T = 459.4 + t$.

EXAMPLE.

A hot-air engine is so constructed, that each unit of air in it makes, for each double stroke of the engine, a complete, simple, reversible cycle process. The initial temperature T_1 is, for the greatest compression, 273 + 300 = 573°, the lowest temperature T_2 is 273° + 0 = 273°. What work will the air perform when it expands from the initial volume v_1 to $\frac{4}{3} v_1$ under constant temperature, and then expands adiabatically?

The heat imparted is, from Equation XVI.,

$$Q_1 = 2.3026\ ART_1 \log \frac{\frac{4}{3}v_1}{v_1}$$

$$= \frac{2.3026 \times 29.273 \times 573 \times 0.125}{424} = 11.39 \text{ heat units for each kilogram.}$$

The mechanical work is therefore, from Equation XXX.,

$$F = \frac{11.39 \times 424}{573}(573 - 273)$$

$$= 8.43 \times 300 = 2529 \text{ meter-kilograms for each kilogram.}$$

If we have $T_1 = 459.4 + 572 = 1031.4°$ Fah., and $T_2 = 459.4 + 32 = 491.4$, we have $Q = \frac{2.3026 \times 53.354 \times 1031.4 \times 0.125}{772} = 20.51$ heat units for each pound,

and $F = \frac{20.51 \times 772}{1031.4}(1031.4 - 491.4) = 8290$ ft. lbs. for each pound.

If we substitute in the formula

$$F = \frac{Q_1}{AT_1}(T_1 - T_2)$$

the value of Q_1 from XVI., we have

$$F = 2.3026R(T_1 - T_2) \log \frac{v}{v_1}. \quad . \quad (\text{XXXII.})$$

The mechanical work, therefore, increases with the expansion ratio $\frac{v}{v_1}$ and the difference of the absolute temperatures T_1 and T_2.

Since $T_1 - T_2 = (273 + t_1 - [273 + t_2])$, we have also

$$F = 2.3026R(t_1 - t_2) \log \frac{v}{v_1}. \quad . \quad (\text{XXXIII.})$$

where t_1 and t_2 are the temperatures in centigrade or Fahrenheit degrees. We shall have occasion to refer to these equations in our discussion of the engines of Ericsson and Lehmann.

Illustration of these Principles by Analogous Mechanical Principles.—In illustration of the formula

$$F = \frac{Q_1}{AT_1}(T_1 - T_2)$$

or
$$F = \frac{Q_2}{AT_2}(T_1 - T_2)$$

we may, as has been done by Zeuner, make use of the following mechanical considerations.

Let AA_1, BB_1, and CC_1 be three planes one above another. The distance between AA_1 and CC_1 is h_1, and between BB_1 and AA_1, h_2.

Suppose a weight G placed upon BB_1, then the uniform sinking of this weight through the height h_2 will perform the work

$$Gh_2.$$

If, for example, G is the weight of a quantity of water which arrives every second, the fall of this water may put in motion a vertical water-wheel, and, for uniform motion of the wheel, give a mechanical effect every second expressed by

$$Gh_2.$$

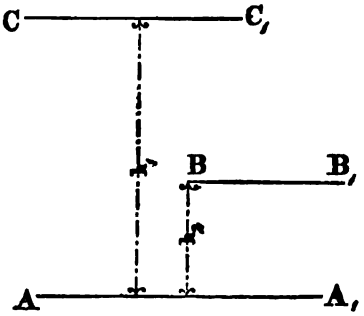

FIG. 19.

If we denote this product by F_2, we have

$$F_2 = Gh_2, \text{ or}$$

$$G = \frac{F_2}{h_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

If the same mass were to sink, performing work, from the plane CC_1, we should obtain the work Gh_1. If we denote this by F_1, we have

$$F_1 = Gh_1, \quad \text{or} \quad G = \frac{F_1}{h_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

If we raise the weight G from the plane BB_1 to the plane CC_1, the work performed is

$$F = G(h_1 - h_2).$$

This would also be the work obtained if we should allow G to sink uniformly from the upper plane to the lower, and then raise it with the same uniform velocity from the lowest plane to the plane BB_1.

If now, we substitute in the last equation the value of G from (2) and (3), we have

$$F = \frac{F_1}{h_1}(h_1 - h_2),$$

or

$$F = \frac{F_2}{h_2}(h_1 - h_2).$$

These equations have precisely the same form as those already given, viz.:

$$F = \frac{424 Q_1}{T_1}(T_1 - T_2),$$

and

$$F = \frac{424 Q_2}{T_2}(T_1 - T_2).$$

There, the products $424 Q_1$ and $424 Q_2$ represent the mechanical works corresponding to the quantities of heat Q_1 and Q_2, just as now, F_1 and F_2 represent the mechanical works obtained by the sinking of the weight G from the heights h_1 and h_2. If we allow the weight G to fall freely through the distances h_1 and h_2, the products $Gh_1 = F_1$ and $Gh_2 = F_2$ will represent the work potential in the weight in the form of living force. But if the products $424Q_1$ and $424Q_2$, or what is the same thing, $\frac{Q_1}{A}$ and $\frac{Q_2}{A}$ represent mechanical work, we may consider the quotients

$$\frac{424Q_1}{T_1}, \quad \frac{424Q_2}{T_2}, \quad \text{or} \quad \frac{Q_1}{AT_1} \quad \text{and} \quad \frac{Q_2}{AT_2}$$

as weights, the so-called "*heat weights*," of Prof. Zeuner.[*] Since further, the difference $T_1 - T_2$ is equivalent in significance with $h_1 - h_2$, we may call the difference $T_1 - T_2$ the "*temperature fall*."

---

[*] [The "*heat weight*" of Prof. Zeuner is identical with the "*thermodynamic function*" of Rankine, or "*entropy*," as defined by Clausius. The term "entropy" has been used by Tait, Thomson, Maxwell, and others, with an entirely different signification. The term "thermodynamic function" is perhaps good enough as a name for a certain function of the heat and temperature, which occurs so often as to render a special name for it desirable. The term "heat weight," not only answers this end, but also gives an analogical significance to the term, which is of real service in using it. We therefore use it exclusively. Those who prefer to call it "thermic weight" can do so.]

We may therefore express the principle of the simple reversible cycle process, as follows: *The work performed by the intermediate body in the simple reversible cycle process, is directly proportional to the heat weights* $\left(\frac{Q_1}{AT_1}\right.$ and $\left.\frac{Q_2}{AT_2}\right)$ *imparted or given out, as well as to the temperature fall* $(T_1 - T_2)$.

QUESTIONS FOR EXAMINATION.

What is a cycle process? When is such a process said to be "closed?" What is a simple cycle process? What is a compound? When is a cycle process reversible? What is a simple reversible cycle process? By how much does the heat imparted exceed that abstracted? Draw the Figure, and show what represents the work performed. What is the intermediate body? Give examples. In a steam engine is the heat of the steam before performing work greater than that after? What becomes of the difference? State Carnot's principle. What is incorrect in this statement, and why? Deduce an expression for the work in a simple reversible cycle process in terms of the heat absorbed or rejected, and the highest and lowest temperatures.

Illustrate these principles by analogous principles of mechanics. What is the heat weight? Why is it so called? What is the temperature fall? How do you express the work in the simple reversible cycle process in terms of the heat weight and temperature fall? What do you understand by thermodynamic function? What other term has been given to this quantity? Why is it not appropriate?

CHAPTER VII.

GENERAL LAW OF THE RELATION BETWEEN PRESSURE AND VOLUME OF A GAS—GRAPHICAL REPRESENTATION OF THE INNER WORK.

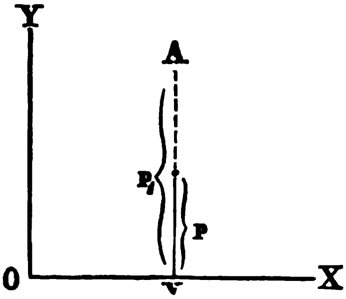

Fig. 20.

Review of the preceding Principles.—We have thus far considered the following cases of the change of pressure and volume of a gas, especially of air.

1. *The volume is constant,* and hence the expansive force increases with the temperature.

If Ov is the specific volume at 0° and p the pressure, then the pressure at 273° is $2p$, etc. The line vA, which gives the relation between pressure and volume, is parallel to the axis of ordinates OY.*

If we denote the inner work at the temperature T_1 by U_1, we have

$$U_1 = \frac{1}{A} T_1 c,$$

where c is the specific heat for constant volume.

If U is the inner work for the temperature T, we have

$$U = \frac{1}{A} Tc;$$

hence the change of inner work is

$$U - U_1 = \frac{c}{A} (T - T_1).$$

---

* [Such a line may be called an "*isometric line*," or line of equal volume.]

Or, since $T = 273 + t$ and $T_1 = 273 + t_1$,

$$U - U_1 = \frac{c}{A}(t - t_1).$$

If Q is the heat imparted during this change,

$$\frac{Q}{A} = U - U_1 = \frac{c}{A}(t - t_1),$$

or

$$Q = c(t - t_1).$$

According to the combined law of Mariotte and Gay-Lussac, we have

$$pv = RT, \quad \text{and} \quad p_1 v_1 = RT_1;$$

hence

$$\frac{pv}{p_1 v_1} = \frac{T}{T_1};$$

or, since $v_1 = v$,

$$\frac{p}{p_1} = \frac{T}{T_1};$$

and hence

$$p = p_1 \frac{T}{T_1} = p_1 \frac{273 + t}{273 + t_1}.$$

2. *The expansive force is constant,* or the expansion takes place under constant pressure.

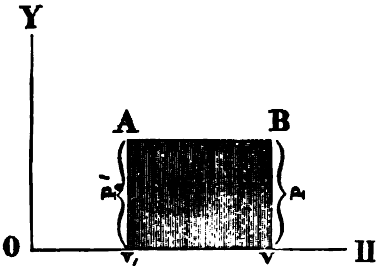

Fig. 21.

If Ov_1 Fig. 21 is the specific volume for the temperature t_1 and pressure p_1, and Ov the volume for the same pressure p and the temperature t, then the change of inner work is

$$U = \frac{c}{A}(t - t_1),$$

and the line AB parallel to the axis of abscissas gives the relation between volume and pressure.*

---

* [Such a line may be called an "*isopiestic line*," or line of equal pressure.]

The outer work performed, which is represented by the rectangle v_1ABv, has the value

$$L_1 = p\,(v - v_1),$$

or

$$L_1 = R\,(t - t_1).$$

Since here both outer and inner work is performed, we have the equation

$$L = U + L_1,$$

or

$$L = \frac{c}{A}\,(t - t_1) + R\,(t - t_1) = \left(\frac{c}{A} + R\right)(t - t_1).$$

The heat imparted is

$$Q = kc\,(t - t_1),$$

where kc is the specific heat by constant pressure.

Further, we have

$$p_1v_1 = RT_1$$

$$pv = RT, \quad \text{hence,} \quad \frac{pv}{p_1v_1} = \frac{RT}{RT_1},$$

or since $p_1 = p$,

$$\frac{v}{v_1} = \frac{T}{T_1} = \frac{273 + t}{273 + t_1}.$$

3. *The air expands under constant temperature*, according to Mariotte's law:

$$pv = p_1v_1 = p_2v_2 = p_3v_3, \text{ etc.}$$

In this case the end of the ordinate p, which gives the pressure for the volume v and temperature t, describes the isothermal, or, in the case of a gas, the isodynamic curve* AB, and the area of the space v_1ABv, gives the outer work performed.

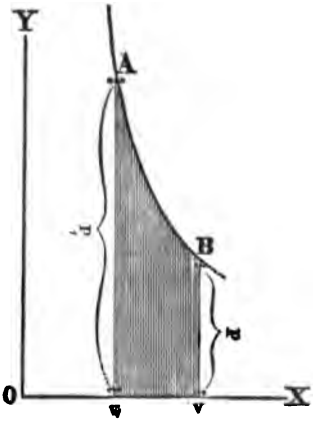

Fig. 22.

This we have found to be

$$L = 2.3026\; v_1p_1 \log \frac{v}{v_1},$$

or

$$L = 2.3026\; v_1p_1 \log \frac{p_1}{p}.$$

---

* [Sometimes called the "*isenergic curve.*"]

The quantity of heat imparted is

$$Q = 2.3026\, Av_1p_1 \log \frac{v}{v_1},$$

or

$$Q = 2.3026\, Avp \log \frac{p_1}{p}.$$

We have also found

$$Q = 2.3026\, ART_1 \log \frac{v}{v_1},\ \text{etc.},$$

and

$$L = 2.3026\, RT \log \frac{v}{v_1},\ \text{etc.}$$

While, therefore, in (1) and (2) the heat Q imparted may be found directly from the initial and final temperatures, or, as we say, is a function of t_1 and t, here it is determined by three quantities, *viz.*, the temperature T and the initial and end volumes, or by T and the initial and end pressures. *

4. *The air expands, performing work, at the expense of its own heat,* according to the law

$$pv^{1.41} = p_1v_1^{1.41} = ,\ \text{etc.},$$

or

$$\frac{p_1}{p} = \left(\frac{v}{v_1}\right)^{1.41}.$$

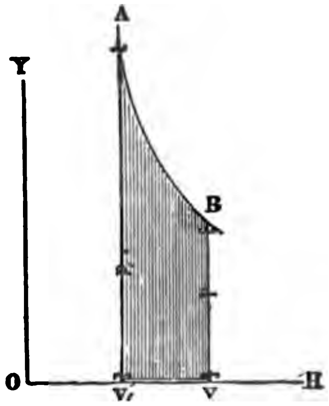

FIG. 23.

The end of the ordinate p_1, Fig. 23, which gives the initial pressure for the specific volume v_1 and the temperature t_1, describes during expansion the abiabatic curve* AB, which approaches the axis of abscissas OX more quickly than the isothermal.

Further we have found

$$\left(\frac{v}{v_1}\right)^{0.41} = \frac{273 + t_1}{273 + t},$$

or

$$\left(\frac{v}{v_1}\right)^{k-1} = \frac{273 + t_1}{273 + t}.$$

---

* [Sometimes called the "*isentropic curve*," or curve of equal entropy.]

The inner work which disappears is

$$U = \frac{c}{A}(t - t_1),$$

where t_1 is the initial and t the final temperature. Since this is equal to the outer work performed, we have also

$$L = \frac{c}{A}(t_1 - t),$$

and this work (or the disappearing inner work), is represented by the area v_1ABv.

For Q we have

$$Q = c(t_1 - t).$$

Finally, we have also found

$$L = \frac{c}{AR} v_1 p_1 \left[1 - \left(\frac{p}{p_1}\right)^{0.2907}\right],$$

or

$$L = \frac{c}{AR} v_1 p_1 \left[1 - \left(\frac{v_1}{v}\right)^{0.41}\right].$$

Since we have found $\frac{c}{AR} = \frac{1}{k-1}$, we have also

$$L = \frac{1}{k-1} v_1 p_1 \left[1 - \left(\frac{v_1}{v}\right)^{k-1}\right].$$

General Law as to the Relation between Volume and Pressure.—In view of what has preceded, it will now be easy to include all these special cases under one general law.

In case (4) when the air expanded from the volume v_1 to v, there was no heat imparted, the pressure diminished rapidly as the volume increased, and the relation between pressure and volume was given by the equation

$$p_1 v_1^k = p_2 v_2^k \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

or, in the case of air,

$$p_1 v_1^{1.41} = p_2 v_2^{1.41}.$$

In the following Fig. let the curve ac represent the law of change of pressure and volume for this case.

In case (3) as the volume increased the decrease of pressure was less than in case (4). The law of change was given by the equation

$$p_1 v_1 = p_2 v_2, \text{ etc.}$$

This equation is at once obtained if we insert $k = 1$ in the first equation above. Let the curve *ab*, the isothermal curve, represent in this case the law of relation between the pressure and volume.

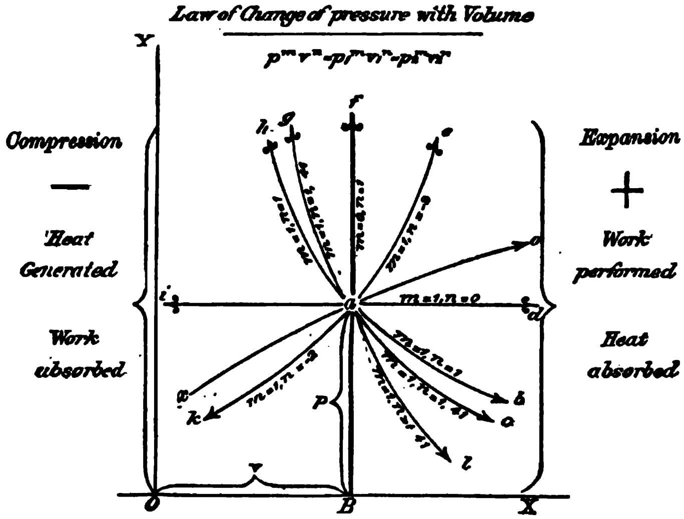

FIG. 24.

In case (2) the pressure was constant as the volume increased, and we had

$$p = p_1 = p_2,$$

and this obtained from (1) by making $k = 0$. The line *ad* or *ai* gives the law for this case.

In the first case, the volume remained constant while the pressure changed, and we had

$$v = v_1 = v_2.$$

This is obtained from Eq. (1) by putting 0 for the exponent of p and making $k = 1$. The line *af* gives the law for this case.

We see, therefore, that the laws connecting pressure and volume are given by the exponents of the factors v and p, and for different exponents we may thus have a number of laws and curves, all of which are based upon the same general law. If, then, $\frac{n}{m}$ is any number, whole or fractional, positive or negative, we have,

$$p v^{\frac{n}{m}} = p_1 v_1^{\frac{n}{m}} = p_2 v_2^{\frac{n}{m}}. \quad . \quad . \quad \text{(XXXIV.)}$$

or

$$p^m v^n = p_1^m v_1^n = p_2^m v_2^n$$

is the general law between pressure and volume.

Thus, if we have $m = 1$ and $n = 1$, we have the law of the isothermal curve *ab*. If $m = 1$ and $n = 1.41$, we have the adiabatic curve *ac*. If $m = 1$ and $n > 1.41$, we have a curve which approaches the axis of abscissas more rapidly than the adiabatic. Such a curve is *al*. If $m = 1$ and $n < 1$ but greater than 0, the corresponding curve will lie between *ab* and *ad*. If n has a negative value, if, for example, $m = 1$ and $n = -2$, the corresponding curve lies between *ad* and *af*. Such a curve is represented by *ae*.

All these curves are convex to the axis of abscissas, but we may have curves which are concave also, as *ao* or *ax*.

We assume, in perfect accordance with the preceding, that curves to the right of the line *Baf*, parallel to the axis of ordinates, apply to expansion. In such case, work is performed and heat is absorbed, as indicated on the Figure. This heat may be at the expense of the heat in the gas itself, or it may be imparted from without. For all curves between *ac* and *af*, on the right, heat is imparted during the expansion. On the other hand, for all below *ac* heat is abstracted. The curves left of *Baf* apply to compression. In such cases work is performed in compressing the gas, or work is absorbed by the intermediate body, and heat is generated. In the Figure *ah* is an isothermal and *ag* an adiabatic line. The sign + indicates work performed by the body, the sign *minus* indicates work absorbed, or performed upon the body.

The above considerations find a very interesting application in the discussion of gas engines.

The question arises, what is the general expression which gives the general law for the *temperature* when the relation between pressure and volume for expansion or compression are known? and how can we determine the work done during expansion by the intermediate body, or absorbed by it during compression?

We can put $$p_1 v_1^{\frac{n}{m}} = \frac{p_1 v_1 v_1^{\frac{n}{m}}}{v_1} = p_1 v_1 v_1^{\frac{n}{m}-1}$$

and $$pv^{\frac{n}{m}} = \frac{pvv^{\frac{n}{m}}}{v} = pvv^{\frac{n}{m}-1},$$

hence $$p_1 v_1 v_1^{\frac{n}{m}-1} = pvv^{\frac{n}{m}-1}.$$

Since, however, $p_1v_1 = RT_1$ and $pv = RT$, we have

$$RT_1 v_1^{\frac{n}{m}-1} = RTv^{\frac{n}{m}-1},$$

that is,

$$\frac{a+t_1}{a+t} = \left(\frac{v}{v_1}\right)^{\frac{n}{m}-1} = \left(\frac{v}{v_1}\right)^{\frac{n-m}{m}} \quad . \quad . \quad \text{(XXXV.)}$$

or

$$\left(\frac{v}{v_1}\right)^{\frac{n-m}{m}} = \frac{T}{T_1}.$$

EXAMPLE.

The specific volume of air $v_1 = 1$ has the temperature $t_1 = 30°$, and expands performing work up to the volume $v = 2v_1$. What will be the final temperature when the law of expansion is $p_1 v_1^{-2} = pv^{-2}$?

We have

$$\frac{a+t_1}{a+t} = \frac{273+30}{273+t}, \quad \text{and} \quad \left(\frac{v_1}{v}\right)^{\frac{n}{m}-1} = \left(\frac{v}{v_1}\right)^{-3} = 2^{-3},$$

hence

$$\frac{303}{273+t} = \frac{1}{2^3} \quad \text{and} \quad 273 + t = 303 \times 8 = 2424, \quad \text{or}$$

$$t = 2424 - 273 = 2151°.$$

[For the same ratio of expansion and $t_1 = 86°$ Fah., we have $t = 3903.8°$ Fah.]

If, on the other hand, the law of expansion had been

$$p_1 v_1^2 = pv^2,$$

we should have had

$$\frac{a+t_1}{a+t} = \left(\frac{v'}{v_1}\right) \quad \text{or} \quad \frac{303}{273+t} = 2,$$

hence $$2t = -546 + 303 = -243, \quad \text{or}$$

$$t = -121.5°.$$

[For the same ratio of expansion and $t_1 = 86°$ Fah., we have $t = -186.7°$ Fah.]

The curve which gives for this case the relation between volume and pressure approaches the axis of abscissas more rapidly than the adiabatic, because the exponent 2 is greater than 1.41, and as already explained, we must therefore abstract from the intermediate body a certain amount of heat. (See the line *al* in the Fig.)

If in the formula already found in the case of the adiabatic curve,

$$L = \frac{1}{k-1} p_1 v_1 \left[1 - \left(\frac{v_1}{v}\right)^{k-1}\right],$$

we put $k = \frac{n}{m}$, we have

$$L = \frac{m}{n-m} p_1 v_1 \left[1 - \left(\frac{v_1}{v}\right)^{\frac{n-m}{m}}\right]. \quad . \quad \text{(XXXVI.)}$$

and this is the general expression for the outer work, when the relation between the volume and tension is given by the general formula.

$$p_1 v_1^{\frac{n}{m}} = p v^{\frac{n}{m}}, \quad \text{or} \quad p_1^m v_1^n = p^m v^n.$$

Since now $\left(\frac{v_1}{v}\right)^{\frac{n-m}{m}} = \frac{T}{T_1}$, and $p_1 v_1 = R T_1$, we have also

$$L = \frac{m}{n-m} R T_1 \left[1 - \frac{T}{T_1}\right] = \frac{m}{n-m} R (T_1 - T)$$

$$= \frac{m}{n-m} R (t_1 - t).$$

Also, since

$$R = \frac{c(k-1)}{A},$$

$$L = \frac{m}{n-m} \frac{c(k-1)}{A} (t_1 - t). \quad . \quad \text{(XXXVII.)}$$

The change of inner work is as always,

$$U - U_1 = \frac{c}{A}(t - t_1).$$

If Q is the amount of heat equivalent to these two works, we have

$$\frac{Q}{A} = U - U_1 + L,$$

or

$$Q = A(U - U_1) + AL, \quad . \quad \text{(XXXVIII.)}$$

and by inserting the values above,

$$Q = c(t - t_1) + \frac{m}{n-m} c(k-1)(t_1 - t).$$

This may be written

$$Q = c(t - t_1) + \frac{m}{n - m} c(k - 1)(t_1 - t)$$

$$= c(t - t_1)\left[1 - \frac{m}{n - m}(k - 1)\right]$$

$$= \frac{mk - n}{m - n} c(t - t_1) \quad . \quad . \quad . \quad . \quad . \quad \text{(XXXIX.)}$$

This formula shows that the heat, Q, imparted or abstracted, *is directly proportional to the difference of the temperatures.*

But the amount of heat Q can be calculated from the specific heat and the temperature difference $t_1 - t$. This specific heat is, however, unknown in the present case. It is evident that it must be different from that for constant pressure or for constant volume. Let us denote it by s. Then

$$Q = s(t - t_1). \quad . \quad . \quad . \quad . \quad . \quad \text{(XL.)}$$

If, now, we put these two values of Q equal, we can easily find s. Thus we have

$$s(t - t_1) = \frac{mk - n}{m - n} c(t - t_1),^*$$

or

$$s = \frac{mk - n}{m - n} c. \quad . \quad . \quad . \quad . \quad . \quad \text{(XLI.)}$$

If, for example, the law of change of pressure with volume is

$$p_1 v_1^{-2} = pv^{-2},$$

we have $m = 1$ and $n = -2$, and hence the specific heat for this law of expansion is

$$s = \frac{k + 2}{1 + 2} c, \quad \text{or for air} \quad \frac{3.41}{3} \times 0.16847 = 0.1915,$$

that is, this is the quantity of heat in heat units which must be imparted to 1 unit of weight of air when expanding according to the assumed law, in order to raise its temperature 1°.

The curve *abc*, Fig. 25, represents the law in the present case, and the shaded area *acde* represents the outer work performed by the gas during expansion, when the specific volume increases from v_1 to $\frac{3}{2}v_1$. We see that the curve departs rapidly from the axis of X as the volume increases. It is, evident, also, that the outer pressure gradually increases from p_1 to p, as we always assume the outer pressure at any moment as less than the inner pressure at that moment by an infinitely small amount.

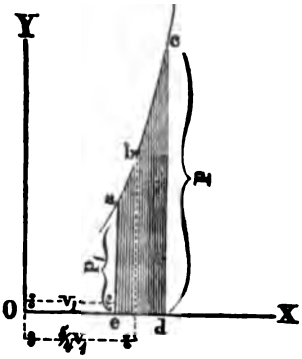

FIG. 25.

Graphical Representation of the Inner Work.—We can now represent the increase or diminution of the *inner* work in a manner similar to that which has been employed for the outer.

Let the specific volume have the pressure p_1 and the volume v_1, and let the inner work be U_1. Then let the air expand while heat is imparted to it, so that the inner work U_1 remains constant. The expansion takes place along the isothermal line *age*. When, then, the air has expanded to the condition pv, the inner work is still therefore U_1.

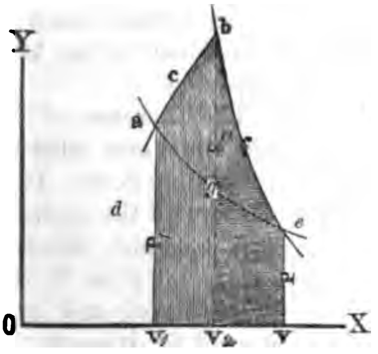

FIG. 26.

If we assume, again, heat imparted so that the volume v_1 and pressure p_1 become v_2 and p_2, the change taking place according to any law or curve, as *acb*, then the outer work performed is v_1abv_2. The inner work at v_2p_2 is then no longer U_1. Suppose it is U_2. Let us denote the heat required for this change by Q_1, then we have

$$\frac{Q_1}{A} = U_2 - U_1 + L_1,$$

where L_1 is the outer work performed.

Let us now assume that the air expands at the expense of its heat, until its volume is v and pressure p. The expansion follows then the adiabatic curve *bfe*, and the area $bfevv_2$ gives the outer work, or what is the same thing, the disappearing *inner* work. Since now the point e of the adiabatic curve falls upon the isothermal, the air has at this point the same temperature as at any point of *age*. The inner work is therefore U_1. While, then, the air passes from the condition v_2p_2 to the condition vp, the inner work changes from U_2 to U_1, or the inner work which disappears is $U_2 - U_1$. Just the same inner work has been imparted on the way from v_1p_1 to v_2p_2. Hence the *increase of the inner work* $U_2 - U_1$, is given by the area $bfevv_2$.

While then the area v_1abv_2, shaded vertically in the Figure, gives the outer work which the air performs in passing from v_1p_1 to v_2p_2 the area v_2bgev, shaded horizontally in the Figure, gives the increase of inner work.

If, therefore, the specific air volume in the condition v_1p_1 expands under the addition of heat, performing work along the curve *acb*, until the volume is v_2 and the pressure p_2, we may find the increase of inner work as follows:

Construct through a, the isothermal *age*, and the adiabatic *bfe* through b, and from their intersection e, let fall the vertical *ve*. The area v_2bfev gives the increase of inner work. The area v_1abev gives the sum of the inner and outer works, which is equivalent to the heat imparted.

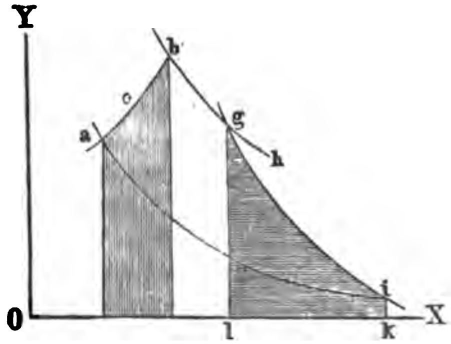

FIG. 27.

The increase of inner work may also be found as follows: Pass through b the isothermal *bgh*, and through any point g on it, the adiabatic *gi*, and produce it till it meets the isothermal through a in i. Let fall the vertical *ki*. Then it is at once evident that the area *gikl* gives the inner work imparted.

QUESTIONS FOR EXAMINATION.

What is an isometric line? Illustrate for air. What is the expression for the change of inner work? For the heat imparted? What is the outer work? What is the relation between pressure and temperature?

What is an isopiestic line? Illustrate for air. What is the change of inner work? What is the outer work? What is the heat imparted? What is the relation between volume and temperature?

Give Mariotte's law. Illustrate. What is an isothermal line? An isodynamic? An isenergic? What is the outer work for air? For any gas? The heat imparted? What is the change of inner work?

Give the adiabatic law for air. For any gas. What is an isentropic curve? Why may we call the adiabatic law the exponential law of Mariotte? What is the inner work which disappears? Why? What is the outer work? What relation does this bear to the inner work? Why? Give other expressions for the outer work.

State the general law between volume and pressure. Are all others special cases of this law? What changes give the adiabatic? The isothermal? The isopiestic? The isometric? Illustrate by a diagram. What is the general law for the relation between volume and temperature for any perfect gas? Deduce. Between pressure and temperature? Deduce.

What is the general expression for the outer work? Deduce it. Give it again in terms of temperature. What is the general expression for the heat imparted?

Deduce a general expression for the specific heat, whatever may be the law of variation of volume with pressure.

Give and explain graphical representations of the inner work.

If 10 cubic meters of air are heated under atmospheric pressure from 0° to 100° C., what is the new volume? What is the new density? What is the weight of each cubic meter of the new volume? What is the work of expansion?

If heated under constant volume, what is the new pressure?

If, while the air is heated and expands, the temperature is kept constant, and the tension at any instant is equal to the outer pressure, what will be the pressure when the volume is 12 cubic meters? What will be the work performed? What the heat imparted?

If no heat is imparted, what will be the pressure when the volume is 20 cubic meters? What will be the work performed? What amount of heat will disappear?

NOTATION OF MOST FREQUENT USE,

COMPILED FOR

CONVENIENCE OF REFERENCE.

$A = \frac{1}{424}$ of a heat unit = thermal equivalent of one unit of work.

$\frac{1}{A} = 424$ meter-kilograms = mechanical equivalent of one unit of heat.

α = co-efficient of expansion = $\frac{1}{273}$ for air and perfect gases.

c = specific heat, generally for constant volume unless otherwise specified.

c_v = specific heat for constant volume.

c_p = specific heat for constant pressure.

D = density.

F = outer work performed by or absorbed in cycle process.

G = weight of a given volume of gas.

J = disgregation work in a heated body.

$k = \frac{c_p}{c_v}$ = ratio of specific heat at constant pressure to that at constant volume. For air = 1.41.

L = outer work performed by expanding body.

$m, n,$ = co-efficients of pressure and volume in general law, for variation of these quantities in gases, viz., $p^m v^n = p_1{}^m v_1{}^n$.

p = specific pressure, *i. e.*, pressure upon unit of surface.

Q = amount of heat measured in heat units.

R = outer work performed by expansion of one unit of weight of gas, when heated under pressure of atmosphere ($760^{m.m.}$ = 10334 kil. per sq. meter) from 0° to 1°C.

s = specific heat in general, whatever be the law of variation of pressure with volume $= \frac{mk - n}{m - n} c$.

T = absolute temperature, reckoned from − 273° C., or — 459.4° Fah.

t = temperature by centigrade thermometer.

U = inner work = vibration work plus disgregation work.

V = volume of any given body.

v = specific volume, *i. e.*, volume of unit of weight.

W = vibration work when any body is heated.

RECAPITULATION OF PRINCIPAL FORMULÆ

FOR

CONVENIENCE OF REFERENCE.

Fundamental Equations :

$Q = A\,(W + J + L)$. . . . . I. (page 124.)

$Q = A\,(U + L)$. . . . . . II. (page 124.)

Expansion of gases—constant pressure :

New volume $= V(1 + 0.00367t)$,

or $V(1 + \alpha t)$. . . . . . III. (page 140.)

$D = \dfrac{1}{1 + \alpha t}$. . . . . . IV. (page 140.)

Weight of air :

$G = \dfrac{1.29318}{1 + 0.00367t}\,V$. . . . . V. (page 141.)

Constant volume :

New pressure $= p\,(1 + \alpha t)$. . . . . VI. (page 143.)

$\alpha = \frac{1}{273} = 0.00367$ for perfect gases.

$\dfrac{1}{A} = \dfrac{R}{c(k - 1)}$. . . . . VII. (page 147.)

R for air $= 29.272$, kilogrs. $k = 1.41$.

Mariotte's law : $v_1 p_1 = v_2 p_2 = v_3 p_3$, etc. . VIII. (page 149.)

Mariotte and Gay Lussac's combined :

$\dfrac{v_1}{v_2} = \dfrac{p_2}{p_1} \cdot \dfrac{1 + \alpha t_1}{1 + \alpha t_2}$. . . . IX. (page 150.)

$\dfrac{D_2}{D_1} = \dfrac{p_2}{p_1} \cdot \dfrac{1 + \alpha t_1}{1 + \alpha t_2}$. . . . X. (page 150.)

$\dfrac{v_1 p_1}{T_1} = \dfrac{v_2 p_2}{T_2} = \dfrac{v_3 p_3}{T_3}$, etc. . XI. (page 152.)

$T_1 = 273 + t_1,\ T_2 = 273 + t_2$, etc.

$p_1 v_1 = RT_1,\ p_2 v_2 = RT_2,\ p_3 v_3 = RT_3$, etc. . XII. (page 153.)

Isothermal curve: $pv = p_1 v_1 = p_2 v_2$, etc.

$L = 2.3026 pv \log \frac{v}{v_1}$

$= 2.3026 p_1 v_1 \log \frac{v}{v_1}$ . . . XIII. (page 157.)

$L = 2.3026 pv \log \frac{p_1}{p}$ . . . . XIV. (page 157.)

$Q = 2.3026 Apv \log \frac{v}{v_1}$ . . . . XV. (page 159.)

Isodynamic curve:

$Q = 2.3026 ART \log \frac{v}{v_1}$

$= 2.3026 ART_1 \log \frac{p_1}{p}$ . . . XVI. (page 160.)

Adiabatic curve:

$p_1 v_1^k = p_2 v_2^k = p_3 v_3^k$, etc. . . XVII. (page 162.)

$k = \frac{c_p}{c_v} = 1.41$ for air.

$\left(\frac{v}{v_1}\right)^{k-1} = \frac{p_1 v_1}{pv}$ . . . . , . XVIII. (page 164.)

$\left(\frac{v}{v_1}\right)^{k-1} = \frac{T_1}{T} = \frac{273 + t_1}{273 + t}$ . . . XIX. (page 164.)

$k - 1 = 0.41$ for air:

$\frac{v}{v_1} = \left(\frac{p_1}{p}\right)^{\frac{1}{k}}$ . . . . . XX. (page 165.)

$\frac{1}{k} = 0.7093$ for air:

$\left(\frac{p}{p_1}\right)^{\frac{k-1}{k}} = \frac{T}{T_1}$ . . . . . XXI. (page 165.)

$\frac{k-1}{k} = 0.2907$ for air:

$L = \frac{c}{A}(t_1 - t) = \frac{c}{A}(T_1 - T)$ . . XXII. (page 167.)

$Q = c(t_1 - t)$. . . . . XXIII. (page 168.)

$L = \frac{c}{AR} p_1 v_1 \left[1 - \left(\frac{p}{p_1}\right)^{\frac{k-1}{k}}\right]$. XXIV*a*. (page 169.)

$L = \frac{c}{AR} p_1 v_1 \left[1 - \left(\frac{v_1}{v}\right)^{k-1}\right]$. XXIV*b*. (page 169.)

Transference from one adiabatic to another:

$$\frac{Q_1}{T_1}=\frac{Q_2}{T_2}=\frac{Q_3}{T_3}, \text{ etc.} \quad . \; . \; . \quad \text{XXV. (page 177.)}$$

Expansion under constant pressure:

$$L=p(v-v_1)=p_1(v-v_1). \quad . \quad \text{XXVI. (page 178.)}$$

$$L=R(t-t_1)=R(T-T_1) \quad . \quad \text{XXVII. (page 178.)}$$

Cycle process:

$$\frac{1}{A}(Q_1-Q_2)=F. \quad . \; . \quad \text{XXVIII. (page 183.)}$$

$$-\frac{1}{A}(Q_1-Q_2)=-F. \quad . \; . \quad \text{XXIX. (page 183.)}$$

$$F=\frac{Q_1}{AT_1}(T_1-T_2) \quad . \; . \; . \; . \; . \quad \text{XXX. (page 186.)}$$

$$F=\frac{Q_2}{AT_2}(T_1-T_2). \quad . \; . \; . \; . \quad \text{XXXI. (page 186.)}$$

$$F=2.3026\,R(T_1-T_2)\log\frac{v}{v_1} \quad . \quad \text{XXXII. (page 187.)}$$

$$F=2.3026\,R(t_1-t_2)\log\frac{v}{v_1} \quad . \quad \text{XXXIII. (page 187.)}$$

Heat weight:

$$\frac{Q_1}{AT_1}=\frac{Q_2}{AT_2}=\frac{Q_3}{AT_3}, \text{ etc.} \quad . \; . \; . \; . \quad \text{(page 189.)}$$

General law of variation of pressure with volume:

$$p^m v^n=p_1{}^m v_1{}^n=p_2{}^m v_2{}^n, \text{ etc.} \quad . \quad \text{XXXIV. (page 197.)}$$

$$\left(\frac{v_1}{v}\right)^{\frac{n-m}{m}}=\frac{T}{T_1}. \quad . \; . \; . \quad \text{XXXV. (page 198.)}$$

$$L=\frac{m}{n-m}p_1v_1\left[1-\left(\frac{v_1}{v}\right)^{\frac{n-m}{m}}\right] \quad . \; . \quad \text{XXXVI. (page 199.)}$$

$$L=\frac{m}{n-m}\,\frac{c(k-1)}{A}(t_1-t) \quad . \; . \; . \; . \quad \text{XXXVII. (page 199.)}$$

$$Q=c(t-t_1)+\frac{m}{n-m}c(k-1)(t_1-t). \quad \text{XXXVIII. (page 199.)}$$

$$Q=\frac{mk-n}{m-n}c(t-t_1) \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{XXXIX. (page 200.)}$$

$$Q=s(t-t_1) \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{XL. (page 200.)}$$

$$s=\frac{mk-n}{m-n}c \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad \text{XLI. (page 200.)}$$

DETERMINATION OF THE DIFFERENTIAL EQUATIONS.

The application of the Calculus to our subject can be briefly illustrated here.

We have from page 124,

$$Q = A(U + L),$$

and by differentiating,

$$dQ = A(dU + dL) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

Since L stands for the outer work, we have $dL = p\,dv$, where p is the specific pressure at any moment and dv is the change of volume.

The inner work U must be a function of p and v, and hence

$$dU = \frac{dU}{dv}dv + \frac{dU}{dp}dp. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

Let us put $X = \frac{dU}{dp}$ and $Z = \frac{dU}{dv}$, then

$$dU = X\,dp + Z\,dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Now if dU is a complete differential, we have, by the rules of the Differential Calculus, the condition

$$\frac{dX}{dv} = \frac{dZ}{dp}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

We have, then, from (1)

$$dQ = A\,[X\,dp + (Z + p)\,dv]. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

We can simplify this expression by putting

$$Z + p = Y, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and hence

$$dQ = A\,(X\,dp + Y\,dv). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7)$$

In this expression X and Y are functions of p and v.

If we differentiate equation (6) with reference to p, we have

$$\frac{dZ}{dv} + 1 = \frac{dY}{dp} \quad \text{or} \quad \frac{dY}{dp} - \frac{dZ}{dp} = 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Now, since the expression for dU is a complete differential, we must have from (4),

$$\frac{dX}{dv} = \frac{dZ}{dp},$$

and hence, from (8),

$$\frac{dY}{dp} - \frac{dX}{dv} = 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (A)$$

This is the first principal differential equation, and shows the connection between the two unknown functions X and Y of equation (7). It also shows that the expression in the parenthesis in equation (7) is not a complete differential, for if it were, the difference of the two differential coefficients would be zero instead of unity. We can therefore conclude that equation (7) is not integrable unless another relation is given between the quantities.

Let, now, S be a new function of p and v, whose form, like that of X and Y, is also at present undetermined. Multiplying and dividing the right side of equation (7) by S, we have

$$dQ = AS\left[\frac{X}{S}dp + \frac{Y}{S}dv\right]. \quad . \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Now we can so choose S as to make the expression in the parenthesis a complete differential, in other words, $\frac{1}{S}$ is the integrating factor.

The connection of X, Y, and S is easily determined. Thus, if the expression in the parenthesis of (9) is a complete differential, we have

$$\frac{d}{dv}\left(\frac{X}{S}\right) = \frac{d}{dp}\left(\frac{Y}{S}\right);$$

or, performing the differentiation,

$$S\frac{dX}{dv} - X\frac{dS}{dv} = S\frac{dY}{dp} - Y\frac{dS}{dp},$$

or

$$S\left[\frac{dY}{dp} - \frac{dX}{dv}\right] = Y\frac{dS}{dp} - X\frac{dS}{dv}.$$

But from equation (A), the expression in the parenthesis on the left is equal to unity, hence

$$S = Y\frac{dS}{dp} - X\frac{dS}{dv}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (B)$$

This is the second principal differential equation. The first equation (A), shows the relation between the functions X and Y, in equation (7). The second equation (B), shows the relation between X, Y, and the integrating divisor S of equation (7).

Now since this divisor S is a function of p and v, we have

$$dS = \frac{dS}{dp}dp + \frac{dS}{dv}dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

Eliminating from equation (7), first Y and then X, by means of equation (B), and substituting dS for its value as given by (10), we have

$$\left.\begin{aligned} dQ &= A\,[X\,dp + Y\,dv] \\ dQ &= \frac{A}{\left(\frac{dS}{dp}\right)}[X\,dS + S\,dv] \\ dQ &= \frac{A}{\left(\frac{dS}{dv}\right)}[Y\,dS - S\,dp] \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (C)$$

These three equations are all identical, and in the solution of various problems, one or the other may be used according as one or the other leads most directly to the desired result. The three functions X, Y, and S, are functions of p and v, the relations between them being given in general by equations (A) and (B). These quantities must be known before we can make use of (C) in any special cases.

If it were known by what relation the inner work U and the quantities p and v are connected, then X and Y would be at once known, because we have

$$X = \frac{dU}{dp},$$

and

$$Y = p + \frac{dU}{dv},$$

and the substitution of these values in equation (B) would give the value of S.

THE PROPERTIES OF THE FUNCTION S.

Since in general $U = F(p,v)$ is not known, we must seek a method of determining this function. Since, if U were known we could determine S, it follows that if we can find S we can determine U.

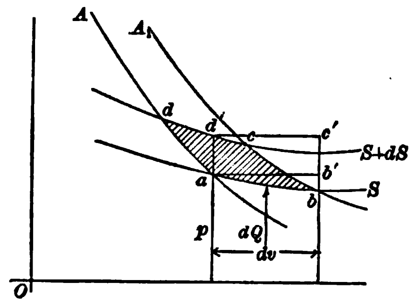

Fig. 1.

Let A and A_1, Fig. 1, be two adiabatic curves lying indefinitely near each other, and let the state of a body be given by the pressure p and the volume v.

For the curve ab let the variation of p with v be given by the law

$$S = f(p,v) = \text{constant},$$

and for the curve bc by

$$S + dS = f(p,v) = \text{constant}.$$

If the body passes through a cycle $abcda$, then the shaded surface represents the work. The area of this surface, which we denote by d^2F, is easily determined, since it may be regarded as an indefinitely small parallelogram whose area is equal to the rectangle $ab'c'd'$, hence

$$d^2F = \text{area} = ab' \times ad'.$$

But ab' is the expansion of the body dv, during the change from a to b, and ad' is the increase of p, when for a constant volume, the value S passes to $S + dS$.

We have, since S is a function of p and v,

$$dS = \frac{dS}{dp}dp + \frac{dS}{dv}dv,$$

and when $dv = 0$, that is, for constant volume,

$$dp = \frac{dS}{\frac{dS}{dp}}.$$

Substituting this for ad', we have

$$d^2F = \text{area} = \frac{dS\,dv}{\frac{dS}{dp}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

We can now determine the heat dQ to be added to the body in the change from a to b, in order that the condition of the body shall, as assumed, answer to S constant. We have only in equation (C), *viz.*,

$$dQ = \frac{A}{\frac{dS}{dp}}[X\,dS + S\,dv],$$

to make $dS = 0$, and we have, accordingly,

$$dQ = \frac{AS\,dv}{\frac{dS}{dp}}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

Now, if the S in equations (11) and (12) is assumed identical, we have at once,

$$d^2F = \text{area} = \frac{dS}{AS}\,dQ. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

This equation can be at once integrated, since S and dS are constant. The integration gives

$$dF = \frac{dS}{AS}\,Q. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

where Q is the amount of heat which is to be added to the body from a to b for a constant value of S.

Conceive now the cycle process performed. Then from c to d a quantity of heat is to be abstracted, which must be greater than Q by an amount equal to the work dF. Representing this by $Q + dQ$, we have

$$dQ = A\,dF,$$

and substituting in the equation (14),

$$\frac{dQ}{Q} = \frac{dS}{S},$$

or integrating

$$\log Q = \log S + \text{constant},$$

or

$$\log \frac{Q}{S} = \text{constant},$$

or

$$\frac{Q}{S} = \text{constant}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

The result expressed by (15) may be stated as follows:

If, in Fig. 16, page 175, we consider a body in various initial states, such that each state is on the adiabatic A, each of these states answers to a certain value of the function S, as S_1, S_2, S_3, etc. If, now, we impart heat to the body in such a manner that during the expansion the value of S for each of the initial positions is constant, and carry the expansion in each case to a second common adiabatic, then the quantities of heat in each case are given by

$$\frac{Q_1}{S_1} = \frac{Q_2}{S_2} = \frac{Q_3}{S_3} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

In any cycle process, then, consisting of two adiabatics and two curves given by $S = f(p_1v) = \text{const.}$, and $S_1 = f(p_1v) = \text{const.}$, like Fig. 1, if the heat Q is added from a to b, and Q_1 subtracted from c to d, we have

$$AF = Q_1 - Q,$$

and since

$$\frac{Q}{S} = \frac{Q_1}{S_1},$$

$$F = \frac{Q}{AS}(S_1 - S), \quad \text{or} \quad F = \frac{Q_1}{AS_1}(S_1 - S). \quad . \quad . \quad . \quad (17)$$

Referring to page 186, we see that this is precisely the same, provided we put for S_1 and S, the absolute temperatures T_1 and T. Our factor S, therefore, stands for the absolute temperature T. The form of the function S is then known. It is the combined law of Mariotte and Gay Lussac, p. 158,

$$pv = R(a + t).$$

Our fundamental formulæ, therefore, become

$$\frac{Yd}{dp} - \frac{dX}{dv} = 1. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{I.})$$

$$a + t = Y\frac{dt}{dp} - X\frac{dt}{dv}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{II.})$$

$$\left.\begin{aligned} dQ &= A\,[X\,dp + Y\,dv] \\ dQ &= \frac{A}{\left(\frac{dt}{dp}\right)}[X\,dt + (a + t)\,dv] \\ dQ &= \frac{A}{\left(\frac{dt}{dv}\right)}[Y\,dt - (a + t)\,dp] \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (\text{III.})$$

APPLICATION OF THE PRINCIPAL EQUATIONS TO GASES.

If the unit of weight of a gas of volume v, pressure p, and temperature t, is raised under constant pressure till the increase of temperature is dt, the heat required is

$$dQ_p = c_p dt.$$

For constant volume, the heat required is

$$dQ_v = c_v dt.$$

But from (III.), if the pressure is constant, $dp = 0$, and by the third equation,

$$dQ_p = \frac{AY}{\frac{dt}{dv}} dt.$$

while for volume constant $dv = 0$, and from the second of equations (III.),

$$dQ_v = \frac{AX}{\frac{dt}{dp}} dt.$$

We have, therefore, for gases,

$$c_p = \frac{AY}{\frac{dt}{dv}} \quad \text{and} \quad c_v = \frac{AX}{\frac{dt}{dp}},$$

and hence we determine at once the functions X and Y in our general equations, for permanent gases,

$$\left.\begin{aligned} X &= \frac{c_v}{A} \frac{dt}{dp} \\ Y &= \frac{c_p}{A} \frac{dt}{dv} \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (18)$$

But for permanent gases $pv = R(a + t)$, hence

$$\left.\begin{aligned} \frac{dt}{dp} &= \frac{v}{R} \\ \frac{dt}{dv} &= \frac{p}{R} \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (19)$$

and by substitution in (18),

$$\left.\begin{aligned} X &= \frac{c_v v}{AR} \\ Y &= \frac{c_p p}{AR} \end{aligned}\right\} \quad \dots\dots\dots\dots \quad (20)$$

We have then determined for gases, the values of the functions X, Y, and S.

Since
$$\frac{dY}{dp} = \frac{c_p}{AR} \quad \text{and} \quad \frac{dX}{dv} = \frac{c_v}{AR},$$
our first principal equation (I.) reduces to
$$c_p - c_v = AR. \qquad \ldots\ldots\ldots \quad (21)$$

That is, *the difference of the specific heats for equal weights, is a constant quantity for every gas.*

If we put R as determined by experiment, for air 29.272, and $c_p = 0.23751$, $c_v = 0.16844$, we find
$$\frac{1}{A} = \frac{R}{c_p - c_v} = 423.8 \text{ meter-kilog.}$$

The experimental value of Joule was 423.55.

Our second principal equation (II.), reduces to
$$a + t = c_p \frac{p}{AR} \cdot \frac{v}{R} - c_v \frac{v}{AR} \cdot \frac{p}{R} = \frac{c_p - c_v}{AR} \cdot \frac{pv}{R}.$$

But from (21), we have $c_p - c_v = AR$, hence $pv = R(a + t)$, and our second principal equation expresses the combined law of Mariotte and Gay Lussac.

Finally equations III. take the form
$$\left.\begin{aligned} dQ &= \frac{c}{R}\,[v\,dp + kp\,dv] \\ dQ &= c\,dt + \frac{AR(a+t)}{v}\,dv \\ dQ &= kc\,dt - \frac{AR(a+t)}{p}\,dp \end{aligned}\right\} \qquad \ldots\ldots \quad (22)$$

where c is the specific heat for constant volume and $c_p = kc_v$, or $k = \frac{c_p}{c_v}$.

ISOTHERMAL AND ISODYNAMIC CURVES FOR PERMANENT GASES.

By isothermal curve we designate that curve according to which the volume v and pressure p change when the temperature is constant.

The equation of this curve is
$$pv = RT = \text{constant}.$$

It is therefore an hyperbola whose asymptotes coincide with the co-ordinate axes.

If the initial condition is given by p_1 and v_1, then the constant p_1v_1 is known, and the curve can be described. Since, however, the constant is also $R(a + t)$, the curve may be determined by the temperature alone, and described without knowing the initial values p_1 and v_1.

FORM OF THE FUNCTION U FOR PERMANENT GASES.

We can easily determine the relation of U to p and v for permanent gases, or gases so far removed from the point of liquefaction that the disgregation work is neglected.

We have the general equation

$$dQ = A[dU + dL],$$

or when, during the addition of the heat dQ, the gas overcomes a resistance equal to its own tension,

$$dQ = A(dU + p\,dv).$$

For permanent gases this equation takes the form given by the second of equations (22),

$$dQ = c\,dt + \frac{AR(a+t)}{v}\,dv.$$

Now, since $pv = R(a + t)$,

$$dQ = c\,dt + Ap\,dv. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Hence, by comparison with the general equation, we have

$$dU = \frac{c}{A}\,dt. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

Integrating between the initial and final temperatures t_1 and t, we have for the change of inner work,

$$U - U_1 = \frac{c}{A}(t - t_1); \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

or denoting initial and final pressure and volume by p_1v_1, p and v, and remembering that $pv = R(a + t)$, we have

$$U - U_1 = \frac{c}{AR}(pv - p_1v_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

Thus for permanent gases the form of the function $U = F(pv)$ is completely determined, and we see from (25) that the change of inner work is determined simply by the difference of temperature, provided that the outer pressure is always equal to the tension of the gas.

If the gas, during the change of state, has performed the outer work L, and the quantity of heat Q has been added, then we have

$$Q = c(t - t_1) + AL; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

or if the gas overcomes a resistance equal to its own pressure,

$$Q = c(t - t_1) + A\int_{v_1}^{v} p\,dv.$$

ISODYNAMIC CURVE.

By isodynamic curve we designate that curve according to which the pressure and volume change when the inner work U is constant, and hence $dU = 0$.

From (24) we see that when $dU = 0$, dt is also zero. Hence no change of temperature occurs. For permanent gases, then, the isodynamic and isothermal curves are identical.

As to the heat Q which must be added, when the initial and final temperatures are equal, or $t_1 = t$, we have from (27),

$$Q = AL.$$

Now L is the work performed by the gas, and AL the corresponding heat which disappears. For isothermal change, then, of a permanent gas, the entire heat added goes to outer work. Also, the same holds true when the gas passes from one condition of equilibrium to another, *no matter according to what law the change of state takes place*, provided only the initial and final temperatures are the same.

For the special case that during the change the resistance is equal to the tension, we have

$$Q = A\int_{v_1}^{v_2} p\, dv.$$

If during the entire change the temperature is constant, we have $p = \frac{R(a+t)}{v}$, and hence

$$Q = AR(a+t)\int_{v_1}^{v_2} \frac{dv}{v}.$$

Hence, since $p_1 v_1 = p_2 v_2 = R(a+t)$,

$$Q = AR(a+t)\log\frac{v_2}{v_1} = Ap_1v_1\log\frac{v_2}{v_1} = Ap_2v_2\log\frac{v_2}{v_1}, \quad . \quad (28)$$

or instead of $\frac{v_2}{v_1}$ we may put $\frac{p_1}{p_2}$.

The outer work $L = \frac{Q}{A}$, or

$$L = R(a+t)\log\frac{v_2}{v_1} = p_1v_1\log\frac{v_2}{v_1} = p_2v_2\log\frac{v_2}{v_1}. \quad . \quad . \quad (29)$$

These last two expressions for the work L for isothermal change have long been known, and can be deduced directly from Mariotte's law. From the standpoint of thermodynamics we see also, however, that the heat is proportional to the work. We see also from (28) and (29) that the amount of heat and work are independent of the kind of gas, for the same initial and final states.

APPARENT AND TRUE SPECIFIC HEAT.

According to (24) and (25) the inner work appears as a function of the temperature only, so that the inner work is the same when the temperature is the same, whatever the kind of gas.

Now we assume that the heat added has, in general, a three-fold action. A part goes to vibration work, and is indicated by temperature. A part goes to disgregation work, and is indicated by change of volume. A part goes to outer work.

For a *perfect* gas, we assume that the disgregation work is zero. In the equation

$$dQ = A\,(dW + dJ + dL),$$

we have, then, $dJ = 0$.

In reality there is no such gas, but dJ is smaller as the gas approaches the perfect condition. For the so-called "permanent" gases, such as air, it is indefinitely small.

If, now, the *unit of volume* of an actual gas is heated under *constant volume*, $dL = 0$. If we assume that the increase of temperature is a measure of the increase of vibration work, we have

$$dQ_v = \omega\,dt + A\,dJ.$$

For a perfect gas, dJ is zero, and ω would be the specific heat for constant volume of the unit of volume (volume capacity). If, however, dJ is not zero, it is undoubtedly always positive, since the particles act attractively. Rankine calls the value of ω in the above equation the "real specific heat." The total heat necessary to raise the unit of volume or weight one degree under constant volume or pressure, as determined by experiment, we may call the "apparent specific heat." If from this be subtracted all the heat required for disgregation and outer work, the remainder is the real specific heat.

We conclude, then, that the identity of the isothermal and isodynamic curves holds good accurately only for perfect gases. For actual gases they deviate more the more the gases depart from the perfect condition. For the so-called permanent gases the deviation is of no significance.

THE ADIABATIC CURVE FOR PERMANENT GASES.

The adiabatic curve gives the law of variation of pressure and volume when no heat is imparted or abstracted during the change of state.

CHAPTER VIII.

COMPARISON OF THE HOT-AIR ENGINE AND STEAM ENGINE.—VARIOUS KINDS OF HOT-AIR ENGINES.

Efficiency of the Steam Engine.—It has been proved that one unit of heat is equivalent to a mechanical work of 424 meter-kilograms. Now one kilogram of good anthracite furnishes about 7,500 heat units, that is, it will heat 7,500 kilograms of water one degree, or will raise 750 kilograms of water 10°. If therefore, all the heat furnished by the combustion of one kilogram of coal could be transformed into mechanical work, we should have $7{,}500 \times 424 = 3{,}180{,}000$ meter-kilograms. This work would be obtained in one second if the combustion occupied one second, in one hour if the combustion occupied one hour. In the latter case, the delivery would be $\frac{3180000}{3600} = 883$ meter-kilograms per second, or $\frac{883}{75} =$ about 12 horse-power.

Experiments upon the steam engine have shown that, even in the best, about 2 kilograms of coal are necessary for one horse-power, and therefore only $\frac{1}{24}$ of the work in the fuel is obtained. Most engines use from $1\frac{1}{2}$ to 2 times as much coal, so that their efficiency is only $\frac{1}{24} \times \frac{1}{1.5} = \frac{1}{36}$ or $\frac{1}{24} \times \frac{1}{2} = \frac{1}{48}$.

At first sight it would seem that the reason of this is to be sought in defective boiler construction, setting, etc. But in all these respects but little room for improvement now remains. Hence Redtenbacher concluded that better results could only be obtained by an entire change in the method of conversion of heat into work. What sort of change is necessary, he has not informed us. In the present condition of science it would be hard to find any one competent to give us such information. Meanwhile Zeuner has shown that our best steam engines, as well as hot-air engines, utilize the fuel exceedingly well, and that we demand an impossibility when we require these machines to utilize all or even the greatest part of the heat contained in the fuel. It would be as reasonable to expect a water wheel to utilize the entire fall, from the source to the sea, of the river which moves it.

Still, the views held as to the imperfect utilization of the fuel by the steam engine, and perhaps also the danger of explosions, which unfortunately still remains, led to the construction of the hot-air engine.

We know that water requires to convert it into steam about 540 heat units. This enormous amount of heat is required simply to convert the liquid water into a gas. Since the air is already a gas, no heat is needed for such a transformation, and hence it would seem to be much cheaper than steam. This however is not the case, as will be seen hereafter.

Work which One Kilogram of Water Performs in Evaporation.—Let *ABCD* be a cylinder whose cross-section is exactly one square meter, Fig. 28. In the bottom is one cubic decimeter, or one kilogram, of water at 0°. Upon the surface of the water rests the air-tight piston *KK*. Since the cross-section of the cylinder is one square meter, the depth of the water is one millimeter; for $\frac{1}{1000}$ of a cubic meter = 1 cubic decimeter.

Suppose the piston *KK* loaded with 10,334 kilograms, and that there is a vacuum above it. The pressure of 10,334 kilograms corresponds then to that of the atmosphere at 0° and $760^{mm.}$ of barometer.

If now we heat the water up to 100°, any further addition of heat generates steam of 10,334 kilograms pressure. It is known to be a physical fact, to which we shall return when we come to speak of steam and the steam engine, that steam generation will not commence until the temperature of 100° is attained. If it is desired to make

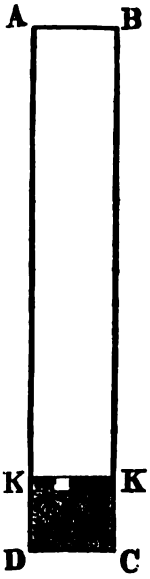

FIG. 28.

it occur earlier, the pressure on the piston must be diminished. The more heat we impart, the more steam is generated and the piston is raised ever higher, while the temperature remains at 100°. When all the water is converted into steam, the piston will be about 1734 millimeters or 1.734 meters above the bottom, since the one cubic decimeter of water will give about 1734 cubic decimeters of steam. The pressure 10,334 kilograms has thus been raised 1.734 meters, which corresponds to a mechanical work of 17,919 meter-kilograms.

Since we must impart about 540 heat units to change the water at 100° into steam at 100°, and also 100 heat units to raise the water from 0° to 100°, we have imparted altogether about 640 heat units in obtaining the above work.

If the piston is loaded with 2 × 10334 kilograms, steam generation commences at 121°, and the water must be heated to this temperature before the piston is raised. If all the water is converted into steam, the height to which *KK* is raised is 914 millimeters = 0.914 meters, and the one cubic decimeter of water furnishes 914 cubic decimeters of steam of 2 atmospheres pressure. Since the steam occupies a space of 914 cubic decimeters while before it occupied 1734, we see that its density is nearly double as great as before.

But now the quantity of heat imparted in this case, in order to completely vaporize the water is only about 640 heat units. The work performed however is $2 \times 10334 \times 0.914 = 18890$ meter-kilograms, or greater than before.

If we load the piston with 3 × 10334 kilograms, vaporization takes place at about 135°. The steam occupies the space of 620 cubic decimeters, and the piston is raised through 620 millimeters. The work performed is $3 \times 10334 \times 0.621 = 19252$ meter-kilograms, while the heat imparted is still only about about 640 heat units, and so on.

We see that the work of the steam is greater the more the piston is loaded, that is, the more the water is heated, or the greater the expansive force.

In all cases, however, the heat required for the outer work is much less than that required for the vaporization. In the 1st case the latter is $540 \times 424 = 228960$ meter-kilograms; in the 2d, $(640 - 121)\ 424 = 220056$, and in the 3d, $(640 - 135)$ $424 = 214120$ meter-kilograms.

This circumstance, that most of the heat serves only to vaporize the water—to separate the molecules—has, as we have remarked, led to the idea of using some naturally gaseous body, and of these there are none more suitable than the air.

Historical Note upon Hot-Air Engines.—The first to apply the idea of using hot air appears to have been John Stirling, of Glasgow. In the year 1827 he devised an air engine, the construction and efficiency of which are not now known. Six years later John Ericsson constructed a similar engine in London, which he called a "*caloric engine.*" The invention made little progress in England, although such men as Faraday and Ure were interested in it, and Ericsson removed to America, where his activity in many directions has been so marked. Here he worked at the perfecting of his caloric engine, and in 1848 he succeeded in introducing his first engine on an improved system, in the Delamater iron foundry in New York. It was only 5 horse power, but in the following year one of 60 horse power was set up, and in 1851 a caloric engine was exhibited at the London exposition. The engine thus became more widely known, but was regarded in Europe more as an interesting toy, incapable of competing in practice with the steam engine.

This view was by no means contradicted by the experiments made in America, in the following years. The invention of Ericsson was followed up with great zeal, and a company was formed with John B. Kitching at its head, to which also the Secretary of the Navy, Kennedy, belonged, to build a large vessel with caloric engines called the "Ericsson." This vessel made its first voyage on the 15th of February, 1853.

The vessel was 2200 tons, and had four caloric engines, which set in motion two paddle-wheels. The cylinder was 14 feet in diameter and 6 feet stroke, and the air was heated to 195° C. In order to prevent loss of heat and reduce the expenditure of fuel, there were four remarkable contrivances near the cylinder, called "regenerators." Each of these consisted of a network of wires, whose combined length was nearly fifty miles. Before the air escaped it passed through these regenerators, and gave up its heat to the wires, which then were ready to impart heat to the fresh charge of air. The engine, according to Ericsson's calculation, was 600 horse power, and since it consumed in twenty-four hours eight tons of coal, each horse power per hour required the extraordinarily small quantity of 1.11 pounds, or about ½ kilogram. According to the report of Prof. Wm. A. Norton,* however, the power of the engine on the trial trip was only 300 horse power, which gives 2.2 pounds of coal per horse power per hour, an amount which, in comparison with the best marine engines, which require about 3.11 pounds, shows still a noticeable economy.

Although as regards economy of fuel, therefore, the engine was preferable to an ordinary steam engine, it had the disadvantage of much greater weight and space. Prof. Norton came therefore to the conclusion, that, in its present condition, the engine was not suited for marine use or for locomotives, but that where weight was of less account, and economy of fuel was desired, it possessed many good qualities.

Ericsson himself noticed during the test many defects, which upon his return to New York he sought to remedy. Instead of four cylinders he used two of somewhat less diameter, but longer stroke. But the test with this new apparatus gave more unfavorable results than the first, and so in the beginning of 1854, the engines were taken out and replaced by ordinary marine steam engines.

Ericsson now busied himself with the construction of smaller machines, for the purposes of the lesser industries. In 1860 he succeeded in producing an engine which found general acceptance not only in America, but also in France, Germany, and Sweden. In Germany the Director of the workshops of the Hamburg Magdeburg Steamship Company, Andrea, specially interested himself in the introduction of the new invention. But it also has not realized the hopes which were placed in it. After a few years it was removed from many establishments, and replaced by the steam engine. We shall have occasion later on to describe the Ericsson engine in detail.

Comparison of the Work performed by Hot Air and Steam.—Let us see now whether hot air is in fact a cheaper motor than steam.

We suppose again under the piston *KK*, Fig. 29, whose cross-section is one square meter, one kilogram of air at 0° and atmospheric pressure. This occupies a space of 0.7733 cubic meters, and hence the piston *KK* is at a distance 0.7733 meters from the bottom. Let there be a vacuum above the piston, and let it be loaded with 10,334 kilograms.

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* American Journal of Sciences and Arts, 2d Series, vol. xv., May, 1853.

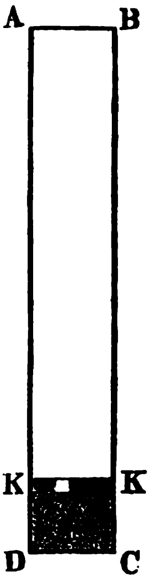

Fig. 29.

The air cannot expand because its pressure is in equilibrium with the piston pressure.

If now we heat the inclosed air gradually, up to 273°, it expands, as we know, to double its volume, and the piston is raised 0.7733 meters. The work performed is hence 10334 × 0.7733 = 7991 meter-kilograms, and the heat imparted is 0.2375 × 273 = 64.84 heat units.

When we converted 1 kilogram of water into steam of one atmosphere, we imparted 640 heat units, and obtained a work of 17,919 meter-kilograms. For a work of only 7,791 kilograms, we should need to evaporate only $\frac{7791}{17919} = 0.45$ kilograms of water, for which we should impart only 640 × 0.45 = 288 heat units. We obtain, therefore, in fact, by means of air, as much work by the expenditure of 64.94 heat units as by the expenditure of 288 heat units with water; or inversely, if we wish to obtain the same work with water as with air, we must use $\frac{288}{64.84} = 4.44$ times as much fuel.

But now, by the use of steam we can cause a good vacuum above the piston. Thus, in a condensing engine, the back pressure upon the piston is only $\frac{1}{6}$ to $\frac{1}{7}$ of an atmosphere. In our illustration with steam then, already discussed, after the piston has moved through 1.734 meters, we can gradually remove the pressure from it down to $\frac{1}{7} \times 10334$ or $\frac{1}{6} \times 10334$ kilograms, during which the steam expands, performing work at the expense of its inner work, and thus a considerable amount of mechanical work can be obtained without further expenditure of heat. In the hot-air engine, on the other hand, we cannot make the back pressure less than one atmosphere. On this account the difference in fuel consumption will be less than as above computed.

To this we may add that with steam the engine can be much smaller than with air. For with steam, as we have seen, for a work of 7,991 kilograms we need to vaporize only 0.45 kilograms of water. The piston then will be raised only 1.734 × 0.45 = 0.7803 meters, and the entire space required is 0.7813 cubic meters. With air, on the contrary, we require 2 × 0.7733 = 1.5466 cubic meters, or almost twice as much space. (It is worth remarking that with the steam engine we must keep on hand 20 or 30 times as much steam as is used per stroke, so that the *entire* apparatus, boiler and all, may occupy more space than the hot-air engine.)

If we compress the air under the piston to 3 atmospheres, it occupies only $\frac{1}{3}$d. of its volume for one atmosphere. We must then load the piston with 3 × 10334 kilograms. Its height will then be $\frac{0.7733}{3} = 0.2578$ meters. Upon heating to 273° it rises to double this height, and the work done is $3 \times 10334 \times \frac{0.7733}{3}$ $= 10334 \times 0.7733 = 7991$ meter-kilograms, while, as before, 64.8 heat-units are required. By gradually diminishing the pressure upon the piston down to 10,334 kilograms, we can now obtain work by the expansion of the air. The work thus obtained is, however, in part lost when the air is again compressed to three atmospheres, and thus brought back to its original condition.

Let us compare with this the example already given, where 1 kilogram of water was converted into steam of 3 atmospheres. If here we wish a work of only 7,991 meter-kilograms, we have to vaporize only $\frac{7791}{19252} = 0.405$ kilograms of water, and therefore expend $640 \times 0.405 = 259.2$ heat-units, or still $\frac{259.2}{64.8} = 4$ times as much as for the air. But now, by expansion down to $\frac{1}{6}$th to $\frac{1}{7}$th of an atmosphere, we can obtain considerable work, almost none of which is lost, because the water mass of 0.405 kilograms which we have to force back into the boiler, occupies a very small space, so that the work required to force it in is very small. For such reasons, then, scarcely double as much fuel may be required as for air.

From the above it follows, that hot air is, in general, a cheaper motor than steam, but the difference is less the greater the tension of the steam and the greater the expansion. To this we must add that the engine proper for the same power is less in size for the steam than air, and that the steam requires to be heated to a much less temperature than the air. This allows the steam piston and stuffing boxes to be better lubricated, and materially reduces the friction. On the other hand, the hot-air engine is free from danger of explosion, and requires little or no water.

Let us now examine the construction and theory of these engines a little more closely.

I. OPEN HOT-AIR ENGINE WITH OPEN FIREPLACE, IN WHICH THE HOT AIR IS EXPELLED AT EACH STROKE. CALORIC ENGINE OF ERICSSON—SYSTEM OF 1860.

In Fig. 30 we have an ideal section of such an engine. The cylinder is *abcd*. In the right half of it are two pistons *pq* and *rs*, of which one is the working piston and the other the feed piston.

In the back part of the cylinder is an iron fire-box *efgh*; *ik* is the grate, *l* the ash-pit. The hot gases pass from the fire space through the pipe *mn* into the annular space *vw* which surrounds the cylinder *abcd*. In this way as much heat as possible is imparted to the cylinder. The hot air escapes from *vw* by the chimney *oo* into the outer air.

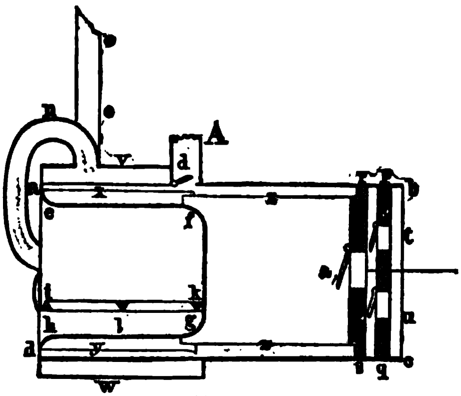

Fig. 30.

The working piston *pq* has valves, two of which are shown at *t* and *u*. These valves open toward the left. The motion of this piston is transferred, by means of two rods and lever work, to a fly-wheel, one half of which is heavier than the other. From this, by means of mechanism, motion is imparted to the feed piston *rs*, so that it has a greater velocity than the working piston.

To the left of the feed piston is fastened a cylinder, *zz*, of thin cast-steel plate, which has a somewhat greater diameter than the fire-box. The fire-box is surrounded by another cylinder of cast-steel, *xy*, which receives the heat radiated from the cylinder and fire-box.

Finally we have at *A*, a pipe with a valve *d*, through which the hot air escapes after acting in the engine. As *A* communicates with the chimney *oo*, the hot air and products of combustion are discharged together into the outer air, or may be discharged into a closed space.

Method of Action.—Let us now consider the method of action of the engine. First, the fly-wheel is turned by a simple apparatus, so that the centre of gravity of the heavy half lies a little to one side of the highest point. The centre of gravity then sinks of itself, and the fly-wheel turns 180°. This motion is imparted by means of link work, to the pistons *pq* and *rs*. But since the feed piston *rs* moves more rapidly than the working piston, *pq*, a partial vacuum is caused between. In consequence of this, the valves *t* and *u* in the working piston open, and air enters between the pistons. The valve *a*, in the feed piston, which possesses a very different form from that shown in the Figure, and is applied at a different place—closes. The hot air remaining in the cylinder departs through *d*, which remains open during the entire motion of the feed piston from right to left.

When the feed piston has reached its extreme position on the left, the working piston has not completed its stroke, but still moves toward the left, while the feed piston now moves toward the right. The air between is thus compressed, and the valve *a* opens, and admits a portion of the air into the hot space left of the feed piston.

This air, which enters cold, is now rapidly heated, especially by the hot plates *xx* and *zz*, and thus has a greater expansive force. This pressure is distributed according to the law of distribution of pressure in a fluid, over the *entire volume* of air, that is, it acts not only upon the feed piston, but also upon the working piston, which it forces toward the right.

But when this motion towards the right begins, the feed piston is already traveling in the same direction, and since it moves faster than the working piston, more cold air is being continually forced through *a* into the hot space. The tension of the inclosed air thus increases, and reaches its maximum when the feed piston has passed through half its stroke, and has its greatest velocity.

From this point it diminishes gradually down to about 1.16 atmospheres, when the feed piston is at the end of its stroke.

At this moment the exhaust valve *d* opens, and the hot air escapes rapidly, while the feed piston returns towards the left. While then the tension of the hot air is sinking to one atmosphere, the working piston returns a certain amount, about $\frac{1}{17}$th of its entire stroke, so that we can assume during this period the pressure equal upon both sides of the working piston.

It should be especially remarked, that the valve *a*, during forward motion of the feed piston, remains constantly open, that, therefore, the increased tension of the hot air is transferred to the cold air between the pistons, and that hence, only the working piston is impelled by the increased tension. For this reason it is called the "*working* piston." The rear piston, by means of its greater velocity, forces the cold air into the hot space, and is hence called the "*feed* piston." By this arrangement the working piston is shielded from the radiant heat of the hot portion of the cylinder, and can be lubricated and kept in good condition. In fact, in this lies a great part of the ingenuity of the whole invention. During the backward motion of the feed piston, the valve *a* is closed.

In order that the cold air, when it arrives in the hot space behind the feed piston may be heated quickly, it must enter more readily than the valve *a* would allow, and the following arrangement is adopted. Upon the circumference of the feed piston *rs*, rectangular notches are cut, from $\frac{1}{2}$ to 1 inch broad, by $\frac{1}{4}$ deep. In the Figure 31 only two can be seen at *oo*. Behind the piston is a steel ring, *tt*, which fits the cylinder snug, but whose inner circumference is about $\frac{1}{4}$ inch from the feed piston. Thus in the position shown in the Figure, communication is open

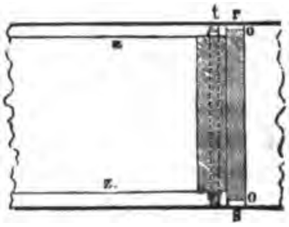

FIG. 31.

between the space right and left of the feed piston, and cold air can pass through. The ring is held in place on the left by several pins, upon the circumference of the back extension of the piston. When the piston moves toward the left there is a partial vacuum between the pistons, and one atmosphere pressure on the left, hence the steel ring is pressed close up to the piston face, and the holes *oo* are covered by it and closed.

In order to make the action of the machine still clearer, we have given in Figures 32, 33, 34, 35 and 36, the principal positions of the pistons.

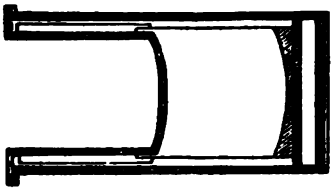
Fig 32.

In Fig. 32 the working piston has arrived at the end of its forward stroke, and the feed piston is already started on the back stroke. The tension of the air in the engine is one atmosphere, both between the pistons and back of the feed piston.

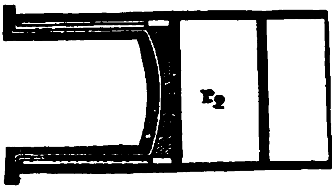

Fig. 33.

In Fig. 33 the feed piston is at the end of its back stroke, while the working piston has moved only a part of its way toward the left. Between the two is cold air of one atmosphere tension.

In Fig. 34 the feed piston has commenced its forward stroke while the working piston has arrived at the left end of its stroke. The air between is compressed, the valve in the feed piston is open, and in the working piston shut, and cold air is passing into the hot space behind. This air when heated communicates its higher pressure, according to the laws of fluid pressure, through the entire volume of air, and thus acts upon the working piston.

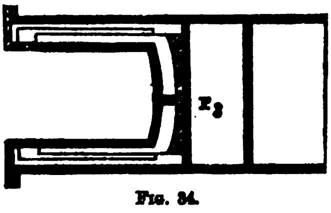

Fig. 34.

In Fig. 35 the feed piston is at the middle of its forward stroke, where it has its greatest velocity, and where the greater part of the air has been forced through and heated. Here then must be the maximum tension. Both pistons now travel nearly together, and but little more air enters the heated space, so that now the pressure diminishes as the volume increases, and the air works *expansively*.

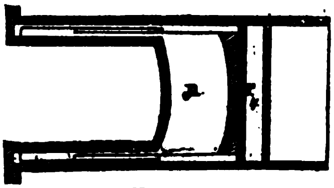

Fig. 35.

Finally, in Fig. 36 the feed piston has reached the end of its forward stroke. At this moment the exhaust valve opens, and the tension of the air falls to one atmosphere. The working piston has still a portion of its stroke to go, while the feed piston moves back. When the working piston arrives at the end of its stroke we have the position of Fig. 32.

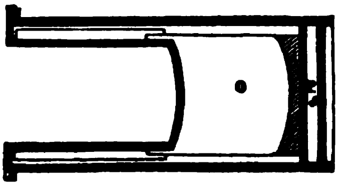

Fig. 36.

Variation of Pressure.—Let us now determine the air tension at the various positions indicated in Figs. 32, 33, 34, 35 and 36.

In Figs. 32, 33, the air right and left of the feed piston has the pressure of the atmosphere, which we denote by p. Let the volume of air in the space r_2 be A cubic meters, and let its absolute temperature be $T_1°$. Behind the feed piston let there be confined B cubic meters at $T_2°$.

If now the specific volume (volume of one kilogram) of the A cubic meters be v_a, and that of B be v_b, then we have from Equation XII.,

$$pv_a = RT_1 \quad \text{or} \quad v_a = \frac{RT_1}{p} \quad . \quad . \quad . \quad . \quad (1).$$

and

$$pv_b = RT_2 \quad \text{or} \quad v_b = \frac{RT_2}{p} \quad . \quad . \quad . \quad . \quad (2).$$

If we denote the weight of the A cubic meters by G_a, and that of B by G_b, we have

$$G_a = \frac{A}{v_a} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3).$$

$$G_b = \frac{B}{v_b} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

If we insert in (3) and (4) the values of v_a and v_b given by (1) and (2) we have

$$G_a = \frac{Ap}{RT_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

$$G_b = \frac{Bp}{RT_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

Therefore, from (5) we can find the weight of cold air inclosed, and from (6) that of the warm air. The entire weight of air is then

$$G = G_a + G_b = \frac{p}{R}\left(\frac{A}{T_1} + \frac{B}{T_2}\right) \quad . \quad . \quad . \quad . \quad (7).$$

and this is the weight of air contained in the engine for all the other positions during the forward feed stroke.

In the space r_3, Fig. 34, let there be C cubic meters of air of the temperature $T_1°$, and in m D cubic meters with the temperature $T_2°$, and let the pressure be p_1. We find as in (5) for the weight of C,

$$\frac{Cp_1}{RT_1}$$

and for the weight of D,

$$\frac{Dp_1}{RT_2}.$$

Both weights must together be equal to G, hence

$$G = \frac{p_1}{R}\left(\frac{C}{T_1} + \frac{D}{T_2}\right) \quad . \quad . \quad . \quad . \quad . \quad (8).$$

and putting (7) and (8) equal and reducing

$$\frac{p_1}{p} = \frac{AT_2 + BT_1}{CT_2 + DT_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

From this we can determine the ratio of the pressures for the position in Fig. 33 to that in Fig. 34.

In like manner, in Fig. 35 let us have in r_4, E cubic meters at $T_1°$, and in n, H cubic meters at $T_2°$, and the tension in both spaces p_2. Then we have for the weight in E as before $\frac{Ep_2}{RT_1}$

and for the weight of *H*, $\frac{Hp_2}{RT_2}$, and hence

$$G = \frac{p_2}{R}\left(\frac{E}{T_1} + \frac{H}{T_2}\right) \quad . \quad . \quad . \quad . \quad . \quad (10).$$

From (10) and (7)

$$\frac{p_2}{p} = \frac{AT_2 + BT_1}{ET_2 + HT_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (11).$$

This gives then the maximum tension of the air.

If in Fig. 36 the volume in r_5 is *I* cubic meters and that in *o*, *K* cubic meters, and if the tension in both spaces is p_3, we shall have in similar manner,

$$\frac{p_3}{p} = \frac{AT_2 + BT_1}{IT_2 + KT_1} \quad . \quad . \quad . \quad . \quad . \quad . \quad (12).$$

Let now the cross-section of the cylinder be *F* square meters, and the distance between the pistons in Fig. 33, be r_2, in Fig. 34, r_3, in Fig. 35, r_4, in Fig. 36, r_5; then

$$A = Fr_2, \quad C = Fr_3, \quad E = Fr_4, \quad \text{and} \quad I = Fr_5.$$

If also, instead of the annular space behind the feed piston in Fig. 33, we suppose a cylindrical space of equal volume, whose length is *b*, we have

$$B = Fb \quad \text{and} \quad b = \frac{B}{F}.$$

If in Fig. 34 the distance of the feed piston from the fire-box is *m*, in Fig. 35, *n*, in Fig. 36, *o*, we can put

$$B = Fb; \; D = Fm + Fb = (b + m)F; \; H = (b + n)F$$
$$K = (b + o)F.$$

Substituting these values in (7), (9), (11), and (12), we obtain the following formulæ:

$$G = \frac{Fp}{R}\left(\frac{r_2}{T_1} + \frac{b}{T_2}\right) \quad . \quad . \quad . \quad . \quad . \quad (a).$$

$$\frac{p_1}{p} = \frac{r_2T_2 + bT_1}{r_3T_2 + (b + m)T_1} \quad . \quad . \quad . \quad . \quad . \quad (b).$$

$$\frac{p_2}{p} = \frac{r_2 T_2 + b T_1}{r_4 T_2 + (b + n) T_1} \quad . \quad . \quad . \quad . \quad . \quad (c).$$

$$\frac{p_3}{p} = \frac{r_3 T_2 + b T_1}{r_5 T_2 + (b + o) T_1} \quad . \quad . \quad . \quad . \quad . \quad (d).$$

According to Boetius we have for a one horse-power caloric engine,

o = stroke of feed piston = 0.418 meters.
l = stroke of working piston = 0.22 meters.
$r_2 = 0.275$ meters. $m = 0.054$ "
$r_3 = 0.180$ " $n = 0.209$ "
$r_4 = 0.063$ " $r_5 = 0.025$ "
$F = 0.165$ square meters.

Further, $B = Fb = 0.2$ of the entire space Fo.

Hence

$$Fb = 0.2 \times 0.165 \times 0.418 = 0.013794 \text{ cubic meters,}$$

and

$$b = 0.0836 \text{ meters.}$$

If we assume that the cold air between the pistons preserves in all positions the constant absolute temperature $T_1 = 273 + 10 = 283°$, and the hot air behind the feed piston has the temperature always of $T_2 = 273 + 300 = 573°$, we have from (*a*), (*b*), (*c*), etc., the following results:

$$G = \frac{0.165 \times 10334}{29.272}\left(\frac{0.275}{283} + \frac{0.0836}{573}\right),$$

or

$$G = 0.06524 \text{ kilograms.}$$

The weight of cold air drawn in at each stroke is

$$G_a = \frac{Ap}{RT_1} = \frac{Fpr_2}{RT_1} = \frac{0.165 \times 10334 \times 0.275}{29.272 \times 283} = 0.0565 \text{ kilograms.}$$

Since the machine made 45 revolutions in one minute, the weight of air used per second is $\frac{45}{60} \times 0.0565 = 0.0424$ kilograms.

Also from (*b*) we obtain

$$\frac{p_1}{p} = \frac{0.275 \times 573 + 0.0836 \times 283}{0.180 \times 573 + (0.0836 + 0.054)\,283} = \frac{181.064}{141.911} = 1.276.$$

From (c)

$$\frac{p_2}{p}=\frac{181.064}{0.063\times 573+(0.0836+0.209)\,283}=\frac{181.064}{118.735}=1.525.$$

From (d)

$$\frac{p_3}{p}=\frac{181.064}{0.025\times 573+(0.0836+0.418)\,283}=\frac{181.064}{156.108}=1.160.$$

Let us now find the delivery of the engine under the assumption, which is in fact very nearly correct, that the tension varies as the ordinates to a straight line.

In Fig. 34, the working piston has its extreme position on the left. The air behind it has a tension of 1.276 atmospheres, or is 0.276 atmospheres in excess of the outer-air pressure.

In Fig. 35, the maximum tension is 1.525, or 0.525 atmospheres in excess of the outer air.

The mean effective pressure upon the working piston, while passing from the position in Fig. 34 to that in Fig. 35, is, if the pressure varies as the ordinates to a straight line, $\frac{0.276+0.525}{2}$ $= 0.4005$ atmospheres. Since the distance passed through by the working piston is 0.038 meters, we have for the work performed

$$0.4005\times 10334\times 0.165\times 0.038=25.983 \text{ meter-kilograms.}$$

Since the effective pressure in Fig. 36, is $1.16-1=0.160$ atmospheres, we have for the mean pressure, while the working piston moves from the position in Fig. 35 to that in Fig. 36, $\frac{0.525+0.160}{2}=0.343$ atmospheres. The distance passed through is 0.172 meters, and hence the work done is

$$0.343\times 10334\times 0.165\times 0.172=100.594 \text{ meter-kilograms.}$$

The total work is therefore

$$25.983+100.594=126.577 \text{ meter-kilograms.}$$

From this we must subtract the work done in compressing the air, which compression commences at the position of Fig. 33 and lasts to Fig. 34. The final pressure is 0.276 effective, and hence the mean pressure is $\frac{0.276}{2}$, and the work done is

$$\frac{0.276}{2} \times 10334 \times 0.165 \times 0.041 = 9.668 \text{ meter-kilograms.}$$

Hence the effective work done per revolution is

$$126.577 - 9.668 = 116.909 \text{ meter-kilograms.}$$

and the work per second, since there are 45 revolutions per minute, is

$$\frac{45}{60} \times 116.909 = \frac{3}{4} \times 116.909 = 87.682 \text{ meter-kilograms.}$$

According to experiment, about 60 per cent. of the theoretical work is expended in overcoming friction, etc., so that only 40 per cent. remains effective. The effective delivery therefore is

$$87.682 \times 0.40 = 35.07 \text{ meter-kilograms per second.}$$

We see then that an Ericsson engine which is rated at one horse power, gives in reality a mechanical effect of not quite one-half of one horse power.

In place of the preceding lengthy calculations, we may make use of Equation XXXIII., by using the expansion ratio $\frac{v_2}{v_1} = 1.28$, and multiplying the result by the weight of air acting per second. In that equation, t_1 and t_2 are the highest and lowest temperatures, and we have

$$L = 2.3026\, RG\,(t_1 - t_2) \log 1.28.$$

$$L = 67.4017\, G\,(t_1 - t_2) \log 1.28.$$

Since $\log 1.28 = 0.1072$, we have

$$L = 7.2255\,(t_1 - t_2)\, G.$$

If the temperature t_2 of the outside air is 10°, and the highest temperature is $t_1 = 300°$, we have $t_1 - t_2 = 290°$. Since now the air drawn in per second is $G = 0.0424$ kilograms, we have

$$L = 7.2255 \times 290 \times 0.0424 = 88.84 \text{ meter-kilograms;}$$

a value very closely agreeing with the above.

II. OPEN HOT-AIR ENGINES WITH INCLOSED FIREPLACE.—THE HOT AIR, AS BEFORE, IS EXPELLED INTO THE AIR.

In the machine already described, air is necessary for combustion of the fuel as well as for the action of the engine. The air of combustion serves only to heat the air in the cylinder, without coming into direct contact with it. The fire is "*exterior.*" A large part of the heat of combustion is thus lost. As the hot air is expelled into the air after performing its work, the engine is "*open.*" It is then an "open" engine with "exterior" fire.

To avoid loss of heat, the air of combustion and the gases of combustion may be used directly in the working cylinder, instead of only giving up a portion of their heat to the working air. We have then "*interior*" fire, and as the gases are expelled after working, the engine is still "*open.*"

In the "open" engine then, with interior fire, the air is first compressed to a certain degree. It then passes into an inclosed fireplace where its tension is of course still further increased, and then, together with the products of combustion, it passes into the working cylinder and acts directly upon the working piston. Such an engine must evidently be an "open" one, in order to allow the products of combustion to escape.

When the air comes in contact with the fuel, a part of its oxygen unites with the elements of the fuel to form carbonic acid gas, carbonic oxide gas and water. These products of combustion together with the heated air remaining uncombined, pass into the working cylinder. It is evident that the heat of the fuel is thus much better utilized. The gases of combustion have little or no injurious effect upon the piston or cylinder walls, and as the air enters the fire-box at a greater pressure than that of the atmosphere, the combustion is very rapid. It is difficult however to prevent particles of coal, soot, and ashes from being carried into the working cylinder and injuring the working parts.

A description of a large engine of this kind will be found in *Dingler's Polytechnisches Journal,* Band CLXXXV., Heft 6. It was designed by the engineer Mazeline, and runs a paper-mill. The principal parts are represented in Fig. 37. *D* is the working cylinder, with the piston D_1 and piston rod P. This cylinder is surrounded by another of somewhat greater diameter, and the cold air coming from the feed cylinder F passes through the annular space between, and is thus warmed while keeping the working cylinder D tolerably cool. The feed cylinder has the piston F_1 and two pair of valves a, a, and b, b, of which the first allow air to be drawn in at every stroke, and the others admit the compressed air to the jacket of the working cylinder. After the air is here warmed and has thus abstracted heat from the working cylinder, it passes through the pipe A_2A_2 indicated by dotted lines, into an inclosed fireplace AA, which is simply indicated in Fig. 37.

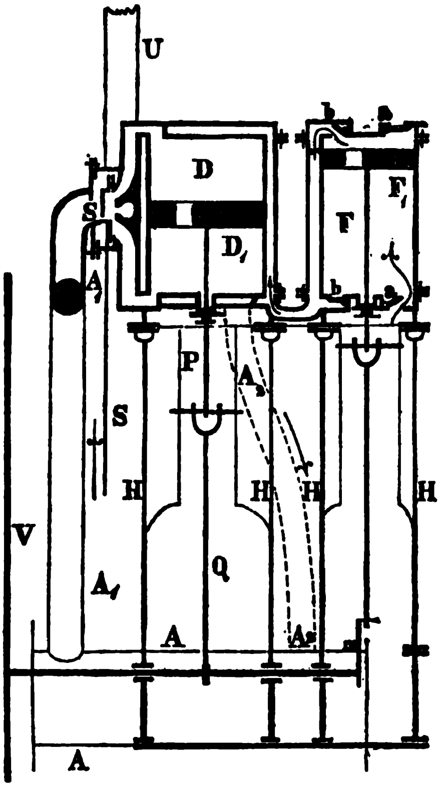

FIG. 37.

This consists of an iron cylinder inclosing another of fire-brick, in which is the grate and fire space. Above the grate is a funnel closed above, air tight, by an iron cover. In this is a cock which when turned by special mechanism, allows the fuel to be uniformly spread over the grate. There is also an arrangement for stirring and shaking the grate.

After the air is heated by the fire, and its tension thus greatly increased, it passes, together with the products of combustion, through the space between the fire-brick and the iron cylinder, through pipe A_1, to the working cylinder D, where it is admitted, as shown in Fig. 37, above and below the piston by the slide SS, precisely as in the case of a steam engine. In the actual engine, valves are used.

The piston rod P is attached to the connecting rod Q which works the crank of the main shaft. This latter works the connecting rod and piston of the feed or air pump F. $H, H, \ldots$ are supporting pillars of the frame, which carry the cylinders and the guides for the cross-heads of the connecting rods. Upon the main shaft we have also the fly-wheel V.

The working cylinder has a diameter of 1.40 meters and a stroke of 1.50 meters. The diameter of the feed cylinder is 1 meter, and the stroke the same as that of the working piston, or 1.50 meters. The cross-section of the latter is therefore

$$\frac{\pi \times 1.40 \times 1.40}{4} = 1.5386 \text{ sq. m.}$$

and its volume

$$1.5386 \times 1.50 = 2.309 \text{ cub. m.}$$

while the cross-section of the feed cylinder is

$$\frac{\pi \times 1.00 \times 1.00}{4} = 0.7854 \text{ sq. m.}$$

and its volume

$$0.7854 \times 1.50 = 1.178 \text{ cub. m.}$$

The volume of the latter is therefore only little more than half of the former.

The pistons and piston rods are kept lubricated by a special arrangement with soap-water, as oil or other fatty matter would be decomposed by the high temperature of the air in the working cylinder.

In order to set the engine in action, a special reservoir of compressed air is required, from which air is admitted to the feed cylinder. In the engine described, a turbine, which is also used in the same mill, is made use of for starting.

The action of the engine is now as follows:

The air drawn into the feed cylinder by the first stroke is by the next stroke at first compressed. This compression is of course adiabatic, since no heat is imparted or abstracted during the compression. After compression to a certain point, and after the air has thus been heated by the compression to a certain temperature, the proper valve opens and allows the compressed air to pass round the working cylinder to the fireplace. From this point the pressure in the feed pump is constant, and the valve remains open until all the compressed air has been forced into the fireplace. As soon as the first air particles enter the fireplace they occupy a larger space, and hence the working piston begins its stroke. Neglecting the resistances in the conducting pipes, this is moved by the same pressure as that in the feed cylinder itself. The working piston then begins its stroke when the compression in the feed cylinder attains its maximum, or what is the same thing, when the valve is forced open. The pressure in the working cylinder also remains constant until the feed piston has completed its stroke. From this point on, the working piston is driven by the expansion of the heated air, whose pressure at the end of the stroke must be about one atmosphere. This expansion is also adiabatic. As soon as the working piston has completed its stroke, the second stroke of the feed pump compresses the previously sucked-in air before it, then the other pressure valve opens, the air enters the fireplace, and so on.

In this engine, which has been examined by Tresca, of the *Conservatoire des Arts et Métiers*, the maximum pressure of the air in the feed cylinder was 1.94 atmospheres, and the compression lasted up to about one half (0.515) of the entire stroke. The maximum pressure in the working cylinder, on the other hand, was only 1.68 atmospheres, which is to be attributed to the fact that the pipes conducting the air from the feed pump to the furnace were too narrow. At 0.611 of the stroke expansion began, and the pressure fell gradually to the end of the stroke, where it was only 1 atmosphere.

We see from the preceding, that the maximum pressure in this engine exceeds that in Ericsson's only by a small amount, although according to the views of the constructor, it should be 5 to 7 atmospheres. This pressure was most probably not attained, because the resistances of the driven machines, reduced to the circumference of the crank circle, were not great enough. Certainly the engine could have produced a greater effect than it did during the experiments.

III. CLOSED HOT-AIR ENGINE WITH EXTERIOR FIRE.

The hot-air engine may be also "closed," in which case the same air acts over and over. Thus the air, after acting upon the working piston, instead of being discharged into the atmosphere, is cooled down to its original temperature, and then used over. Such an engine must necessarily have "exterior" fire. It is analogous to a steam engine with surface condenser, in which the condensed steam enters the boiler again, and is used over. To this class belong the engines of Laubereau and of Lehmann, which will be discussed hereafter. We may also consider the Belou engine as of this class. It is at least converted into such, when we conduct the still warm air which departs at each stroke from the working cylinder through a pipe surrounded by water, where it cools down to its original temperature before it enters the feed cylinder again.

Open hot-air engines are analogous to non-condensing steam engines. The air is discharged and a fresh supply taken in.

CHAPTER IX.

THEORY OF THOSE OPEN AND CLOSED HOT-AIR ENGINES, IN WHICH, DURING EACH PERIOD, THE AIR GOES THROUGH A SIMPLE REVERSIBLE CYCLE PROCESS.

We shall now give the general theory for all hot-air engines, whether open or closed, provided only, that in each period the air goes through a complete reversible cycle process.

There are closed engines, such as those of Laubereau and Lehmann, in which only a part of the air is compressed or rarefied, while another part is in another condition. To such engines the formulæ which we are about to deduce are not applicable. For all others in which the *entire* volume of air is either compressed or rarefied, our discussion holds good.

Let the area of the piston in the feed cylinder be f, and H the length of stroke. Then the volume of air drawn in per stroke is fH.

If t is the temperature of this volume, measured by the Centigrade thermometer, and T the absolute temperature, then the weight is

$$G = \frac{fHp}{RT} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

where p is the tension.

Let this weight of air be forced at every stroke into the hot space, and then with an increased volume, due to the heating, enter the working cylinder and drive the piston there with constant pressure through a certain distance, and then act expansively for the rest of the way. If the engine is open it is then discharged with tolerably high temperature into the air. If the engine is closed heat is abstracted, until its temperature is the same as its original temperature, at the beginning of the cycle process.

Let the air in the feed cylinder be compressed until its tension is p_1, and its temperature therefore rises from T to T_1°. We have then, from Equations XXI*a* and XXI*b*,

$$\frac{T_1}{T} = \left(\frac{p_1}{p}\right)^{\frac{k-1}{k}} = \left(\frac{p_1}{p_1}\right)^{0.2907}$$

or

$$T_1 = T\left(\frac{p_1}{p}\right)^{0.2907} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

If T, p and p_1 are known, T_1 can be found.

Now the work L, which must be performed in raising 1 kilogram of air by compression from the temperature T to T_1, is by Equation XXII*b*,

$$\frac{c}{A}\left(T_1 - T\right),$$

therefore the work necessary to raise G kilograms from T to T_1° is

$$L_1 = \frac{c}{A}\left(T_1 - T\right)G \quad . \quad . \quad . \quad . \quad . \quad (3).$$

As soon as the air in the feed pump has reached the tension p_1, the valves open, and the compressed air is gradually forced into the heating apparatus. Here it receives the temperature T_2, at which, under tolerably constant pressure p_1, it moves this working piston until the entire amount of compressed air has been heated, and has passed into the working cylinder. Then, as already described, the air acts expansively during the rest of the way, until its pressure p_1 has sunk to p.

The work L_2, performed by the feed pump in forcing the compressed air into the heating apparatus, is, if H_1 is that portion of its stroke during which the air is forced out:

$$L_2 = fH_1p_1.$$

The weight of air is then also given by

$$G = \frac{fH_1p_1}{RT_1},$$

hence we have

$$L_2 = RT_1G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

Now, while the feed piston is compressing the air and forcing it out, the atmosphere helps it, and its work is

$$L_3 = fHp.$$

But since $G = \frac{fHp}{RT}$, we have

$$L_3 = RTG \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

Hence we have for the work performed by the feed piston per stroke

$$L_1 + L_2 - L_3 = G\left(\frac{c}{A} + R\right)(T_1 - T) \quad . \quad . \quad (6).$$

Since c, A and R are constant, we see that the work is greater, the greater T_1, or the more the air is compressed, the greater G, and the less T.

Now let us determine the work which the hot air performs in the working cylinder.

The working piston, acted upon by the constant pressure p_1, passes through the distance H_1.

If F is the area of the working piston, we have the work done

$$L_4 = FH_1p_1.$$

But we also have

$$G = \frac{FH_1p_1}{RT_2},$$

hence

$$L_4 = RT_2G \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

During the expansion, the pressure p_1 sinks to p, and the temperature T_2 to T_3. Since p_1, p and T_2 may be considered as known, T_3 is given by the equation

$$\frac{T_3}{T_2} = \left(\frac{p}{p_1}\right)^{0.2907}$$

or

$$T_3 = T_2\left(\frac{p}{p_1}\right)^{0.2907} \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

Accordingly the work during expansion is

$$L_5 = \frac{c}{A}(T_2 - T_3)\,G. \quad . \quad . \quad . \quad . \quad . \quad (9).$$

The overcoming of the atmospheric pressure p through the distance H is

$$L_6 = FHp.$$

But since

$$G = \frac{FHp}{RT_3},$$

we have

$$L_6 = RT_3G \quad . \; . \; . \; . \; . \; . \; . \quad (10).$$

The effective work of the hot air in the working cylinder is therefore

$$L_4 + L_5 - L_6 = G\left(\frac{c}{A} + R\right)(T_2 - T_3). \quad . \quad (11).$$

This is greater, the greater G and T_2 and the less T_3. If we subtract (6) from (11), we have for the effective delivery per stroke

$$L = G\left(\frac{c}{A} + R\right)(T_2 - T_3 - T_1 + T). \quad . \quad (12).$$

This very simple formula shows *that the delivery* L *of a hot-air engine, whether open or closed, in which the air makes a reversible cycle process, depends upon the temperatures in the engine and upon the weight of air* G. This weight of air is, from (1)

$$G = \frac{fHp}{RT},$$

and hence for the same pressure p and temperature T_1, is greater, the greater fH, or the contents of the feed pump. It increases also with the tension p of the air drawn in. If the atmospheric pressure were 3, 4, or 5 times greater than it really is, then for the same cylinder volume the delivery of the machine would be 3, 4, or 5 times greater, and inversely, for the same delivery the volume of the feed pump, and hence of the working cylinder, can be 3, 4, and 5 times smaller. In the *closed* engine we may generate such an artificial pressure. We have only to compress the air in the feed pump to the required degree before starting, and by the completion of the cycle process bring it back to this condition. In the open engine this is not possible. For this reason it would seem that the hot-air engine of the future must be of this kind, viz., *closed*.

We shall now illustrate the foregoing considerations and calculations graphically.

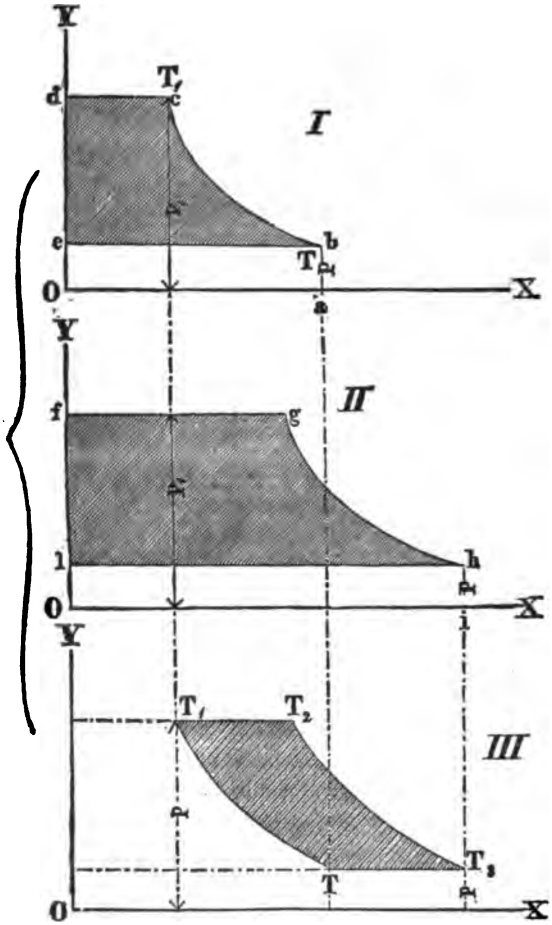

FIG. 38.

Let Oa, Fig. 38, I, be the volume of air drawn in per stroke by the feed pump, of pressure p and temperature T. Let this volume be compressed adiabatically from b to c, the pressure rising to p_1 and the temperature to T_1. When the pressure p_1 is reached, the valve opens and the air is forced out under constant pressure p_1. The work performed during this operation is evidently given by the area $abcdO$. But the outer air has performed the work $abeO$. Hence the shaded area $bcde$ gives the work of the feed pump in compressing the air and forcing it out into the heating apparatus.

In the heating apparatus the temperature rises to T_2 and the volume is increased. The increased volume is given by fg in Fig. 38, II. As soon as this volume is reached, the air expands adiabatically from g to h and its volume is now Oi. During expansion the pressure p_1 sinks to p, and the absolute temperature T_2 to T_3. The entire work performed by the working piston is given by the area *fghiO*. But this piston has had to overcome the pressure of the atmosphere p, which requires the work *lhiO*. Hence the shaded area *fghl* gives the effective work of the working piston. If from this we subtract the work of the feed piston, we obtain the effective work of the machine, or that work, part of which goes to useful effect and part to overcome the prejudicial resistances. This work is then given by the shaded area $T_1T_2T_3T$ (III.), inclosed by two adiabatics and two straight lines. In Chapter VI. the cycle process consisted of two adiabatics and two isothermals. But there, during expansion the air sank to its original temperature, while here it sinks only to a higher temperature T_3.

Let us now deduce formulæ for the dimensions of the hot-air engine.

If we substitute in (12) $\frac{FHp}{RT_3}$ in place of G, we have

$$L = \frac{FHp}{RT_3}\left(\frac{c}{A} + R\right)(T_2 - T_3 - T_1 + T),$$

or

$$L = \frac{FHp}{T_3}\left(\frac{c}{AR} + 1\right)(T_2 - T_3 - T_1 + T).$$

Since, according to Equation VII.,

$$\frac{c}{AR} = \frac{1}{k-1}, \text{ we have } \frac{c}{AR} + 1 = \frac{k}{k-1} = \frac{1.41}{0.41} = 3.4390,$$

and hence

$$L = 3.439\frac{FHp}{T_3}(T_2 - T_3 - T_1 + T).$$

If the engine makes n strokes per minute, the delivery per second is

$$L_s = 3.439\frac{FHp}{T_3}\cdot\frac{n}{60}(T_2 - T_3 - T_1 + T).$$

The delivery in horse powers, is then

$$N = \frac{3.439}{60 \times 75}\ \frac{FHp}{T_3}n(T_2 - T_3 - T_1 + T)\ . \quad \text{(XLII.)}$$

where p is the atmospheric pressure in kilograms per square meter. If p is given in atmospheres

$$N = 10334 \frac{3.439}{60 \times 75} \cdot \frac{FHp}{T_3} n (T_2 - T_3 - T_1 + T),$$

or

$$N = 7.8972 \frac{FHpn}{T_3} (T_2 - T_3 - T_1 + T) \text{ horse powers.} \quad \text{(XLIII.)}$$

If we consider N as given, we have for the volume FH of the working cylinder

$$FH = \frac{T_3 N}{7.8972 pn (T_2 - T_3 - T_1 + T)} \text{ cubic meters. . } \quad \text{(XLIV.)}$$

Hence we see that the contents of the working cylinder are not only less as the pressure p increases, but also the greater the number of revolutions.

Let us now determine the relation between the area of the working and feed pistons.

The weight G of air drawn into the feed cylinder at every stroke, of T° temperature and p atmospheres' pressure, is

$$G = \frac{fHp}{RT}.$$

This same weight of air fills, after expansion, the entire working cylinder. Hence we have

$$G = \frac{FHp}{RT_3}.$$

If the two weights are equal, we have

$$\frac{F}{f} = \frac{T_3}{T} \quad \text{or} \quad f = F\frac{T}{T_3} \quad . \ . \ . \quad \text{(XLV.)}$$

We have also

$$G = \frac{fH_1 p_1}{RT_1} \quad \text{and} \quad G = \frac{FH_1 p_1}{RT_2},$$

hence

$$\frac{F}{f} = \frac{T_2}{T_1} \quad \text{or} \quad f = F\frac{T_1}{T_2}.$$

Since f is the same in both cases,

$$\frac{T}{T_3} = \frac{T_1}{T_2} \quad \text{or} \quad T_2 : T_1 : : T_3 : T,$$

or

$$T_2 \times T = T_3 \times T_1. \quad . \quad . \quad . \quad \text{(XLVI.)}$$

Since T_2 is the highest, and T the lowest temperature of the air during the process, and T_3 and T_1 are the intermediate temperatures, we have the important principle,—*when a given quantity of air goes through a cycle of the kind in question, the product of the extreme temperatures is equal to the product of the intermediate temperatures.*

In deducing this principle, we have assumed that the air is compressed in the feed cylinder, and forced into the heating apparatus under the constant pressure p_1, and that the heated air drives the working piston with the same pressure p_1 through the distance H_1. In short, that the air is heated under constant pressure. By the aid of the higher mathematics the correctness of this principle may be proved, for any case whatever, where the heating takes place according to the general law,

$$p^m v^n = p_1^m v_1^n = \text{etc.},$$

if only the rise and fall of temperature are proportional to the heat imparted or abstracted. Let us seek to show the correctness of this by a practical example, in such a way that it will be evident that the same proof will hold good for every other special case.

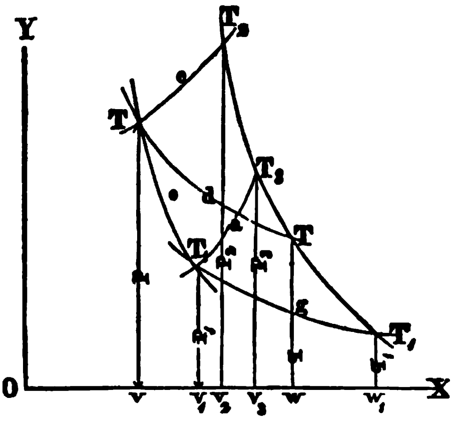

FIG. 39.

Let $Ov = v$, Fig. 39, be the specific air volume, its tension p and temperature T. Suppose heat is imparted, and the air expands while performing work along the curve TcT_2, to the condition $v_2p_2T_2$. Let the heat thus imparted be Q.

Let us assume that the exponent $\frac{n}{m}$ has the value -2. We further assume the volume $v_2 = \frac{5}{4}v$.

We can easily compute the tension p_2, since v_1p_1 are known.

Suppose $v = 1, p = 1$. Then by the law,

$$1 \times 1^{-2} = p_2\left(\frac{5}{4}\right)^{-2} \text{ or } p_2 = \frac{25}{16} = 1.5625.$$

If the absolute temperature T is $273 + 100 = 373°$, we can easily find T_2. Thus by Equation XXXV.,

$$\frac{T_2}{T} = \left(\frac{v}{v_2}\right)^{\frac{n}{m}-1},$$

or

$$T_2 = 373\left(\frac{5}{4}\right)^3 = 373 \times 1.9531 = 728.5°.$$

If more heat had been added, until, for example, the volume $v_1 = \frac{3}{2}v$ or $2v$, the tension p_2 and temperature T_2 would have been greater.

The specific heat is

$$s = \frac{mk - n}{m - n}c,$$

or putting for m, n, k, and c their special values,

$$s = \frac{1.41 - (-2)}{1 - (-2)} 0.16847 = 0.1916.$$

Let us pass through the point T_2, determined by $v_2 = \frac{5}{4}v$ and $p_2 = 1.5625p$, an adiabatic curve, which we shall call the adiabatic curve of the point T_2. We may construct this curve from the formula

$$p_2v_2^k = p_3v_3^k = p_4v_4^k, \text{ etc.},$$

by substituting various values for v_3, v_4, etc., and finding the corresponding pressures. In Fig. 39, this curve is $T_2T_3TT_1$.

Now let us suppose the air of volume v_2 and pressure p_2 and absolute temperature T_2, to expand adiabatically until its pressure falls to p_3, which we may suppose, for example, to be $= p$. We can then easily find the volume v_3 and temperature

T_3, which the air possesses after expansion. (We might have supposed v_3 given, and then found p_3 and T_3.)

We find p_3 from the equation

$$p_2v_2^k = p_3v_3^k,$$

or

$$1.5625\left(\tfrac{5}{4}\right)^{1.41} = 1 \times v_3^{1.41} \quad \text{or} \quad \log 1.5625 + 141 \log \tfrac{5}{4} = 1.41 \log v_3,$$

hence

$$v_3 = 1.7155.$$

As soon as we know v_3, we can find T_3 from the formula

$$\frac{T_3}{T_2} = \left(\frac{1.25}{1.7155}\right)^{0.41} T_3 = 469.8°.$$

During expansion, therefore, the absolute temperature sinks from 728.5 to 469.8°.

Let now the air of volume v_3 and pressure p_3 and absolute temperature T_3 be compressed to the volume of v_1, pressure p_1, and temperature T_1, such that the point T_1 lies on the adiabatic through T, and let this compression take place *according to the same law* as the expansion. It is evident that a certain quantity of heat must be abstracted, in precisely the same manner as before heat was imparted. That there is abstraction of heat and expenditure of work, is seen from the curve T_3aT_1 approaching the axis from above from right to left.

Let us determine now the pressure p_1, the volume v_1, and the temperature T_1 of the air when it reaches the adiabatic ToT_1.

We have for the curve T_3aT_1

$$p_3v_3^{-2} = p_1v_1^{-2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

and for ToT_1

$$pv^k = p_1v_1^k \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

From (2) we have

$$p_1 = p\left(\frac{v}{v_1}\right)^k$$

and putting this value in (1)

$$p_3v_3^{-2} = p\frac{v^{1.41}}{v_1^{3.41}} \quad \text{or} \quad v_1^{3.41} = \frac{pv^{1.41}v_3^2}{p_3}.$$

If in our special case, $p_3 = p = 1$ and $v = 1$ and $v_3 = 1.7155$

$$v_1^{3.41} = (1.7155)^2 = 2.743 \quad \text{or} \quad v_1 = 1.3725.$$

Since now we know v_3 and v_1 we can find T_1. Thus

$$\frac{T_1}{T_3} = \left(\frac{v_3}{v_1}\right)^{-3} = \left(\frac{1.7155}{1.3725}\right)^{-3}$$

or

$$T_1 = 240.2^\circ.$$

Thus the absolute temperatures are

$$373, \quad 728.5, \quad 469.8, \quad 240.2.$$

Hence

$$\frac{T_2}{T} = \frac{728.5}{373} = 1.95 \quad \text{and} \quad \frac{T_3}{T_1} = \frac{469.8}{240.2} = 1.95,$$

or

$$\frac{T_2}{T} = \frac{T_3}{T_1}.$$

This is the same result which we have already obtained for heat addition and subtraction according to a straight line or according to the law

$$pv^0 = p_1v_1^0 = p_2v_2^0.$$

The heat Q imparted on the path TcT_2 is

$$0.1916\,(728.5 - 373),$$

or generally

$$s\,(T_2 - T) \text{ heat units.}$$

The heat abstracted on the path T_3aT_1, is

$$Q_1 = 0.1916\,(469.8 - 240.2),$$

or generally

$$s\,(T_3 - T_1) \text{ heat units.}$$

Since now from the proportion

$$T_2 : T :: T_3 : T_1,$$

we have

$$T_2 - T : T_3 - T_1 :: T : T_1,$$

we at once obtain

$$s(T_2 - T) : s(T_3 - T_1) :: T : T_1,$$

that is

$$Q : Q_1 = T : T_1.$$

Which is the same principle proved in Chapter V., with isothermal lines and adiabatics. We should obtain the same result if we take $\frac{n}{m}$ equal to -3, -4, -5, or generally equal to any number, positive or negative, whole or fractional.

It is now easy to determine the points T or T_1 in which the isothermal lines TdT and T_1gT_1 intersect the adiabatic line T_2T_3T.

To determine the point T we have

$$pv = qw. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

and

$$p_2v_2^k = qw^k. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

From (1) we find $q = \frac{pv}{w}$, and inserting this in (2)

$$w^{0.41} = \frac{p_2v_2^{1.41}}{pv}.$$

Thus in the case of our special example,

$$w^{0.41} = 1.5625 \left(\frac{5}{4}\right)^{1.41} \quad \text{or} \quad w = 6.384,$$

v, of course, being 1. We have also

$$q = \frac{1}{6.384} = 0.1566,$$

when $p = 1$.

In like manner for w_1 we have

$$w_1^{0.41} = \frac{p_3v_3^{1.41}}{p_1v_1} \quad \text{and} \quad q_1 = \frac{p_1v_1}{w_1}.$$

If, then, in any closed or open hot-air engine, in which the air makes a reversible cycle, the air compressed in the feed cylinder is not heated under constant pressure; whatever may be the law of heating—whether under *constant volume*, as in Sterling's engine, or according to any law, as $p_1v_1^{-2} = pv^{-2}$, as shown by the curve TcT_2 in the Figure—*the product of the two extreme temperatures is always equal to the product of the mean, if heat is abstracted according to the same law as it is imparted.*

The formulæ which we have developed thus far, therefore, apply to all kinds of open and closed hot-air engines, provided that in the latter the expansion is such that by the subsequent compression and cooling, the air returns to its original condition; and provided that in the former the air escapes into the atmosphere under the assumed conditions.

Transformation of our Formulæ.—Since in the formula already deduced

$$L = \frac{FHp}{T_3}\left(\frac{c}{AR} + 1\right)(T_2 - T_3 - T_1 + T),$$

we may put $\frac{1}{k-1}$ in place of $\frac{c}{AR}$, the equation can be written

$$L = \frac{FHp}{T_3}\left(\frac{ck}{c(k-1)}\right)(T_2 - T_3 - T_1 + T)$$

$$= \frac{FHp}{c(k-1)\,T_3}\left[ck\,(T_2 - T_1) - ck\,(T_3 - T)\right].$$

Now $ck\,(T_2 - T_1)$ is the amount of heat added, and $ck\,(T_3 - T)$ that abstracted—in case of an open engine, that which is given up to the air. Also ck is the specific heat for *constant* pressure, or for the law

$$pv^0 = p_1v_1^0 = \text{etc.}$$

But, as we have seen, our formulæ also apply when the change of volume and pressure is given generally by

$$p^mv^n = p_1^mv_1^n = \text{etc.}$$

If s is the specific heat generally, we have then

$$L = \frac{FHp}{c(k-1)\,T_3}\,s\,(T_2 - T_1 - T_3 + T). \quad . \quad . \quad (\text{L}b.)$$

whereas we know

$$s = \frac{mk - n}{m - n}c.$$

If this engine makes n strokes per minute, we have the horse-power per second

$$N = \frac{n}{60 \times 75} \cdot \frac{FHps}{c(k-1)T_3}(T_2 - T_1 - T_3 + T),$$

where p is the pressure in kilograms per square meter.

If p is given in atmospheres, we have

$$N = \frac{10334n}{60 \times 75} \cdot \frac{FHps}{c(k-1)T_3}(T_2 - T_1 - T_3 + T),$$

or

$$N = 2.296\,\frac{FHpns}{c(k-1)T_3}(T_2 - T_1 - T_3 + T) \text{ horse-power.} \quad \text{(LI.)}$$

Inversely, the volume FH of the working cylinder is

$$FH = \frac{c(k-1)T_3N}{2.296pns(T_2 - T_1 - T_3 + T)} \text{ cubic meters,}$$

or since

$$c(k-1) = 0.06904,$$

$$FH = \frac{0.03007NT_3}{pns(T_2 - T_1 - T_3 + T)} \text{ cubic meters.} \quad . \quad \text{(LII.)}$$

Maximum Delivery of the Hot-Air Engine.—If it were possible in practice to raise at will the temperature of the air in a hot-air engine as high as we please, we might construct such engines with smaller dimensions and relatively higher delivery, and then probably, since they are not liable to explosion, they might replace the steam engine. But since the temperature is practically limited, this is not the case.

We may consider the extreme temperature to which the air may be raised, as about 300° C., although this has been occasionally exceeded by as much as 30° without injury.

If we assume, then, 300° as the maximum temperature, and further assume that the temperature of the air on entrance, or in case of a closed engine, at the beginning of a cycle, is 0°, then in our formulæ we have $T_2 = 273 + 300 = 573$ and $T = 273 + 0 = 273$. Now for the maximum delivery, or the least volume of cylinder for given delivery, the intermediate temperatures T_1 and T_3 must have definite values. Thus, it is clear that the compression in the feed-pump cannot be carried so far as to raise the temperature to 300° C., because then no heat could be imparted by the heating apparatus, and therefore no mechanical work could be obtained. Thus, if we had $T_1 = T_2$, we would also have, according to L., $T = T_3$, and the expression

$$T_2 - T_3 - T_1 + T$$

would be zero. For the same reason we cannot have $T_1 = T$, for then from L., $T_2 = T_3$, and the above sum is zero. The value of T_1 must then lie somewhere between T and T_2. Now the expression $T_2 - T_3 - T_1 + T$ will be a maximum, when $T_1 + T_3$ is a minimum. Also, the product of T_1 and T_3 is constant and equal to 573×273, or $T_2 \times T$. We can easily find then, by calculus, that $T_1 + T_3$ is a minimum when $T_1 = T_3$.

If now $T_1 = T_3$, we have from L.,

$T_2 T = T_1^2 = T_3^2$, hence $T_3 = T_1 = \sqrt{T_2 T} = \sqrt{573 \times 273} = 395.51$. The intermediate temperatures Centigrade must then be, $t_1 = t_3 = 395.51 - 273 = 122.51°$. If we insert these values in LII., we have for the volume of cylinder FH when the delivery is a maximum,

$$FH = \frac{0.2163 N}{pns} \quad . \quad . \quad . \quad . \quad . \quad \text{(LIII.)}$$

Inversely

$$N = \frac{FHpns}{0.2163} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(LIV.)}$$

If the heat is added or abstracted under constant pressure, then $s = 0.2375$ and

$$FH = 0.9107 \frac{N}{pn} \quad . \quad . \quad . \quad . \quad . \quad \text{(LV.)}$$

hence

$$N = \frac{FHpn}{0.9107} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(LVI.)}$$

If, as in Sterling's engine, heat is imparted or abstracted under constant volume, $s = 0.1685$. In this case, the volume at the end of expansion is equal to that at the beginning of the cycle process. The air of volume V and pressure p, as in Fig. 40, is first compressed adiabatically to V_1 and p_1. Then the volume V_1 is heated from T_1 to T_2, and its pressure rises from p_1 to p_2. The volume V_1 now expands adiabatically to its original volume V, the temperature falling from T_2 to T_3, and pressure from p_2 to p_1, or for the maximum delivery T_2 falls to T_1. Finally heat is abstracted from v till T_1 or T_3 falls to T, and p_1 to p.

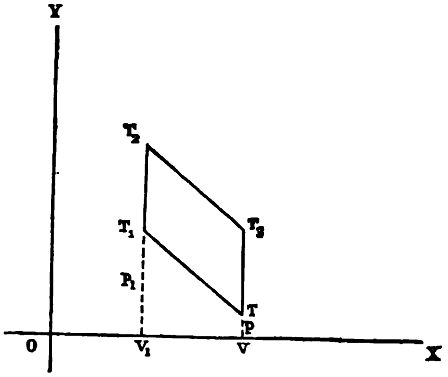

Fig. 40.

If the heating and cooling takes place according to the law

$$p_1 v_1^{-2} = p v^{-2}$$

we have

$$s = 0.1916.$$

If in a Sterling engine the original volume is fH, we have

$$G = \frac{fHp}{RT}, \text{ and in LII. we have } T \text{ in place of } T_3.$$

Hence

$$fH = \frac{0.03007 \times 273 \times N}{pn \times 0.1685 \times 54.98} = 0.886 \frac{N}{pn} \; . \quad \text{(LVII.)}$$

Inversely

$$N = \frac{fHpn}{0.886} \; . \; . \; . \; . \; . \quad \text{(LVIII.)}$$

We see that the delivery in all cases is greater, and hence the final volume of the air at the end of the cycle is less, the greater the initial pressure p and the number of strokes n per minute.

Heat Imparted and Abstracted—Consumption of Fuel.—If in formula L*b*, viz.,

$$L = \frac{FHp}{c(k-1)T_3} s(T_2 - T_1 - T_3 + T)$$

we put AR in place of $c(k-1)$, we have

$$L = \frac{FHp}{ART_3} [s(T_2 - T_1) - s(T_3 - T)],$$

or since

$$\frac{FHp}{RT_3} = G$$

$$L = \frac{G}{A} [s(T_2 - T_1) - s(T_3 - T)] \text{ meter-kilograms.}$$

Here $Gs(T_2 - T_1)$ is the heat added to the air per stroke, and $Gs(T_3 - T)$ is the heat abstracted, or, if the engine is open, the heat given up to the outside air. If we denote the first by Q and the second by Q_1 we have

$$Q = s(T_2 - T_1)G \quad . \quad . \quad . \quad . \quad . \quad \text{(LIX.)}$$

$$Q_1 = s(T_3 - T)G \quad . \quad . \quad . \quad . \quad . \quad \text{(LX.)}$$

If there are n strokes per minute, or $60n$ per hour, the heat imparted per hour is

$$Q_h = 60ns(T_2 - T_1) G \text{ heat units} \quad . \quad . \quad \text{(LXI.)}$$

and that abstracted is

$$Q_{1h} = 60ns(T_3 - T) G \quad . \quad . \quad . \quad . \quad \text{(LXII.)}$$

For the maximum delivery, $T_2 - T_1 = 573 - 395.51 = 177.49$ and $T_3 - T = 395.51 - 273 = 122.51$, hence

$$Q_h = 10649.4nsG = 9509\, nspFH \text{ heat units.}$$

$$Q_{1h} = 7350.6nsG = 6563nspFH \quad \text{"} \quad \text{"}$$

where p is given in atmospheres.

We have therefore *per horse-power per hour*

$$\frac{Q_h}{N} = \frac{9509nspFH}{N} \text{ heat units.}$$

$$\frac{Q_{1h}}{N} = \frac{6563nspFH}{N} \text{ heat units.}$$

If one kilogram of coal furnishes 7,500 heat units, the consumption of fuel per hour for every horse-power is

$$\frac{Q_h}{7500\,N} \text{ kilograms,}$$

if all the heat is utilized. If, however, half is lost by radiation, the consumption per horse-power per hour is

$$\frac{Q_h}{3750\,N} \text{ kilograms} \quad . \quad . \quad . \quad . \quad \text{(LXIII.)}$$

EXAMPLE I.—What is the delivery of a hot-air engine in which $FH = 2.309$ cubic meters, $n = 46$, $p_1 = 1.94$ atmospheres, and the working pressure $p_2 = 1.68$ atmospheres, when $T = 273 + 10 = 283°$?

According to XXI*a* and XXI*b*,

$$T_1 = 283 \left(\frac{1.94}{1}\right)^{0.2907} = 343°.$$

From experiments upon this engine, the escaping air was found to have a temperature $t_3 = 250°$, or an absolute temperature $T_3 = 273 + 250 = 523°$. Hence the temperature T_2 in the working cylinder is

$$T_2 = 523\,(1.68)^{0.2907} = 608.25°.$$

Substitute in XLIII., and we have

$$N = 7.8972 \frac{2.309 \times 46}{523} (608.25 - 523 - 343 + 283)$$

$$= 40.5 \text{ horse-power.}$$

The horse-power, as actually found by Tresca by means of the indicator, was about 40.

If we assume that 30 per cent. of this theoretical delivery is consumed by friction, we should have $0.70 \times 40.5 = 28.35$ effective horse-power. The work of a steam engine of the same dimensions with 3 or 4 atmospheres' pressure would be much greater. But if we take in the boiler and all, the space occupied is in favor of the hot-air engine.

The weight of air G used per stroke is

$$\frac{FHp}{RT_3} = \frac{2.309 \times 10334}{29.272 \times 523} = \frac{23861}{15309} = 1.558 \text{ kilograms.}$$

The heat imparted per stroke is therefore by LIX.

$$Q = 0.23751\,(608.25 - 343)\,1.558 = 98.149 \text{ heat units,}$$

and the heat imparted per horse-power per hour is

$$\frac{98.149 \times 46 \times 60}{40.5} = \frac{270894}{40.5} = 6689 \text{ heat units.}$$

Hence the expenditure of coal per hour per horse-power, if all the heat were given up to the air, would be $\frac{6245}{7000} = 0.892$ kilograms. According to Tresca's experiments, the consumption was 1.44 kilograms, and therefore about ⅓ of the heat was lost.

The best steam engines use as low as 1, and at most 2 kilograms of coal per hour per effective horse-power. The consumption for the hot-air engine is thus somewhat greater than for the *best* steam engines.

EXAMPLE 2.—Required to construct a hot-air engine, so that the cylinder volume, for a given delivery, shall be a minimum. What should be the volume if the theoretical delivery, for 48 strokes per minute, is to be 100 horse-power?

According to LV., we have

$$FH = 0.9107 \frac{100}{1 \times 48} = \frac{91.07}{48} = 1.9 \text{ cubic meters.}$$

A steam engine working without expansion or condensation, under 8½ atmospheres, with 48 strokes per minute, would require only one cylinder of 0.368 cubic meters contents.

If the cylinder of our hot-air engine is not to be greater, we must have a closed engine, and make the initial pressure of the air somewhat more than 5 atmospheres. The compression in the feed cylinder would then raise the pressure to about 16 atmospheres. These are pressures which certainly can hardly be recommended in practice. A cylinder whose volume is twice or three times 0.368 cubic meters, can hardly be called excessively large, however. The cylinder of the early condensation engines of Watt had for an effective pressure of 1½ atmospheres, and a theoretical delivery of 120 horse-power, a volume of 1 cubic meter. To this was added the immense boiler, which is wanting in the hot-air engine.

If the air in our machine is compressed before admission to 2 atmospheres, the working cylinder for the same delivery will need to be only half the size, or about 0.95 cubic meters, that is, about 3 × 0.368 cubic meters, and the pressures in the engine will not be excessive.

From all this, it appears that even such hot-air engines as have a theoretical delivery of 100 horse-power, and therefore an effective power of 50 to 60 horse-power, are not without a probable future, *only they must be closed engines, whose initial pressure is 2 or 3 atmospheres, and they must be constructed for the maximum delivery.* Especially to be recommended are smaller engines (8 to 20 horse-power), because they make in the same time more revolutions per minute, and occupy a relatively less space than the larger. It is worth noticing that Zeuner, in his "Wärme Theorie," has found for the cylinder volume results double of those here given. The reason is, that he considers only single-acting engines. If in his formulæ we substitute $2u$ in place of u, which denotes the number of revolutions per minute, the results coincide.

Let us now compute for our engine above, of 100 horse-power, the volume of the feed cylinder, as well the consumption of fuel, and the amount of cooling water, assuming that the machine is closed.

We have already found

$$fH = FH \frac{T_1}{T_2} = 1.9 \frac{395.51}{573} = 1.9 \times 0.6902 = 1.311$$

cubic meters = the volume of the feed-pump.

The amount of heat added per hour is

$$Q_h = 9509nspFH,$$

or inserting numerical values

$$Q_h = 9509 \times 48 \times 0.2875 \times 1.9 = 205851 \text{ heat units.}$$

Hence the consumption of coal per hour for each horse-power is

$$\frac{Q_h}{8750N} = \frac{205851}{8750 \times 100} = 0.549 \text{ kilograms.}$$

Since the effective delivery is perhaps at most 60 horse-power, each effective horse-power per hour would require

$$\frac{0.549 \times 100}{60} = 0.915 \text{ kilograms,}$$

or a less quantity than the best steam engines.

The heat abstracted per hour is found from the proportion

$$\frac{Q_h}{Q_{1h}} = 9509 : 6568, \text{ or since } Q_h = 205851$$

$$20581 : Q_{1h} = 9509 : 6568, \quad \text{or}$$

$$Q_{1h} = 142037 \text{ heat units.}$$

If the cooling water is heated from 0 to 80°, we must have per second

$$\frac{142036}{3600 \times 80} = 0.493 \text{ kilograms,}$$

or in English weights, about 1.08 lbs., or $\frac{1.08}{62.5}$ cubic feet. That heat is here not taken into account which is abstracted from the air by partial evaporation of the water.

The Regenerator.—Let us now consider an apparatus applied by Ericsson in his first caloric engine, which he calls the "regenerator."

In the engine already discussed, we saw that the air escaped with a temperature of 260°. It escaped with almost the same temperature from Ericsson's first engine. It was evidently desirable to utilize, at least partially, this heat, and thus economize fuel. He therefore caused the escaping air to pass through a network of wires, which thus became heated. The cold air drawn in by the feed-pump then passed through the apparatus, and was thus heated.

Now from formula 22, page 167, we have seen that the delivery depends especially upon the weight of air *G*. This weight depends, however, whether the air is compressed or not, upon the temperature. The greater the temperature, the less, for equal volume, the weight. Therefore, under similar circumstances, the working cylinder receives a less weight of air and performs less work. For the same delivery the volume of the working cylinder must be increased. For this reason the use of the regenerator gave, in Ericsson's first engine, a less delivery than without, and this may be the reason of its omission in his later engine.

If, however, the value of a hot-air engine is estimated, as is proper, by the ratio of fuel consumption to the dimensions and delivery, the use of the regenerator is advantageous *when the engine does not give the maximum delivery.* If, for example, the expansion is not carried so far that the temperature sinks to $T_3 = 122.51°$ C., but only say to $T_3 = 160°$ C., the heat $160 - 122.51 = 37.49°$ can be added by the regenerator. The delivery will be less than the maximum, but in the same degree less fuel will be needed. If this 37.49° is lost, not only will the delivery be less than the maximum, but also more fuel will be necessary. It follows, then, that *in a hot-air engine which gives the maximum delivery, whether open or closed, the regenerator is of no effect;* but in one which is not so arranged as to give the maximum delivery, it may be advantageous.

Absolute Maximum Delivery.—The absolute maximum delivery of a hot-air engine can only be attained, when the cycle process is of the character described in Chapter VI., in which case the heat imparted for a certain work is least. Therefore the addition and abstraction of heat must be so regulated that the air expands in the working cylinder, at first, according to the isothermal, and then according to the adiabatic curve, and that the compression in the feed-pump should also be similar. The law of addition and abstraction of heat is therefore

$$pv = p_1v_1.$$

If we wish a formula for the delivery of such a machine, we must put $T_1 = T_2$ and $T_3 = T$ in the equation

$$L = \frac{G}{A}[s(T_2 - T_1) - s(T_3 - T)] \quad \text{on page 256,}$$

since the temperature or inner work is constant during the reception and abstraction of heat. If, however, $T_1 = T_2$ and $T_3 =$

T, we cannot find the delivery from the equation. We know, however, that under the given circumstances we can determine the heat imparted and abstracted from the initial and final volumes, or from the initial and final temperatures. For the heat imparted we have, page 160,

$Q = 2.3026ART_1 \log \frac{v_2}{v_1}$, if v_1 is the initial volume at the greater pressure p_1, and v_2 the final volume after isothermal expansion.

For the heat abstracted

$$Q_1 = 2.3026\,ART_2 \log \frac{v_3}{v_4},$$

where v_3 is the greater, and v_4 the less volume.

We can now find the work L according to Equation XXX., page 186. We have

$$L = \frac{Q}{AT_1}(T_1 - T_2),$$

for one kilogram of air. For G kilograms we have

$$L = \frac{Q}{AT_1}(T_1 - T_2)\,G.$$

Putting for Q its value we obtain

$$L = 2.3026RG(T_1 - T_2) \log \frac{v_2}{v_1}.$$

This is, then, the absolute maximum delivery of a hot-air engine. If we denote it by L_a, we have

$$L_a = 2.3026RG\,(T_1 - T_2) \log \frac{v_2}{v_1}.$$

If we express the weight G in terms of the cylinder volume, we have, page 240,

$$G = \frac{FHp_3}{RT_2},$$

where p_3 is the final pressure and T_2 the final temperature after expansion in the working cylinder. We have therefore

$$L_a = 2.3026FHp_3 \frac{T_1 - T_2}{T_2} \log \frac{v_2}{v_1}.$$

But we have $\frac{v_2}{v_1}=\frac{p_1}{p_2}$, and according to the law of the adiabatic curve

$$\frac{p_2}{p_3}=\left(\frac{T_1}{T_2}\right)^{\frac{k}{k-1}},$$

hence

$$p_2=p_3\left(\frac{T_1}{T_2}\right)^{\frac{k}{k-1}}.$$

If we substitute this value of p_2 in the expression for $\frac{v_2}{v_1}$, we have

$$\frac{v_2}{v_1}=\frac{p_1}{p_3}\left(\frac{T_2}{T_1}\right)^{\frac{k}{k-1}}.$$

Hence

$$L_a=2.3026FHp_3\frac{T_1-T_2}{T_2}\log\frac{p_1}{p_3}\left(\frac{T_2}{T_1}\right)^{3.439}\text{ meter-kilograms.}$$

If the strokes per minute are n, the delivery per second is

$$\frac{n}{60}\times 2.3026FHp_3\frac{T_1-T_2}{T_2}\log\frac{p_1}{p_3}\left(\frac{T_2}{T_1}\right)^{3.439}\text{ meter-kilograms,}$$

and the horse-power is

$$N=\frac{n}{60\times 75}\times 2.3026FHp_3\frac{T_1-T_2}{T_2}\log\frac{p_1}{p_3}\left(\frac{T_2}{T_1}\right)^{3.439}.$$

If the pressure is given in atmospheres, we have

$$\frac{10334\times 2.3026}{60\times 75}nFHp_3,\text{ etc., or}$$

$$N=5.288nFHp_3\frac{T_1-T_2}{T_2}\log\frac{p_1}{p_3}\left(\frac{T_2}{T_1}\right)^{3.439}\text{ horse-power.}$$

If we take as outer limits, $T_1=573$ and $T_2=273$, we have

$$N=581.15FHp_3n\log\frac{p_1}{12.804p_3}.$$

If this value is real, we must have

$$p_1>12.804p_3,$$

that is, the pressure for the least volume v_1 must be at least greater than 12.804 times the initial pressure p_3. If, then, we take $p_1 = 20$ and $p_3 = 1$ atmosphere, we have

$$N = 1.127 FHn,$$

and hence for the cylinder volume

$$FH = 0.888 \frac{N}{n}.$$

For an engine in which heat was added under constant pressure the greatest pressure is given by

$$\frac{T_1}{T} = \left(\frac{p_1}{p}\right)^{0.2907}$$

For the maximum delivery $T_1 = 395.51$ and $T = 273$, hence

$$p_1 = \sqrt[0.2907]{\frac{395.51}{273}} = 3.581 \text{ atmospheres.}$$

The cylinder volume in this case is

$$FH = 0.9107 \frac{N}{pn},$$

where N is the horse-power and $p = 1$.

While, then, the volume for the same delivery is only a little greater, the greatest pressure is much less than in the first case.

Since it is difficult to construct engines for 20 atmospheres' pressure, we cannot use the system which gives the absolute maximum of work, that is, gives the greatest effect with the least expenditure of fuel. If we run the pressure up to 14 or 15 atmospheres, the cylinder volume will be much greater than for the other systems.

Formulæ for Hot-Air Engines—Shortest Form.—The formula for the absolute maximum delivery

$$L = \frac{Q}{AT_1}(T_1 - T_2)\,G,$$

which we have found for the simple cycle process, can be deduced in this form for every hot-air engine.

Since in the equation

$$L=\frac{G}{A}\left[s\left(T_2-T_1\right)-s\left(T_3-T\right)\right] \quad \text{page } 244,$$

$s\left(T_2-T_1\right)$ is the heat imparted, and $s\left(T_3-T\right)$ that abstracted, of which the first is denoted by Q and the second by Q_1, we have

$$L=\frac{G}{A}\left(Q-Q_1\right).$$

But now we have proved that for all hot-air engines

$$T_2 : T_3 = T_1 : T$$

when the heat addition or abstraction takes place according to the law

$$p^m v^n = p_1^m v_1^n = \text{etc.}$$

Hence

$$T_2-T_1 : T_3-T = T_2 : T_3 \quad \text{or} \quad = T_1 : T.$$

Therefore

$$s\left(T_2-T_1\right) : s\left(T_3-T\right) = T_2 : T_3 = T_1 : T,$$

or

$$Q : Q_1 = T_2 : T_3 = T_1 : T.$$

Accordingly

$$Q=\frac{Q_1 T_1}{T} \quad Q_1=\frac{QT}{T_1},$$

or

$$Q=\frac{Q_1 T_2}{T_3} \quad Q_1=\frac{QT_3}{T_2}.$$

Inserting the first value of Q_1 in the above equation for L, we have

$$L=\frac{G}{A}\left(Q_1-\frac{QT}{T_1}\right)=\frac{QG}{AT_1}\left(T_1-T\right) . \quad \text{(LXIV.)}$$

Hence we can calculate the delivery of a hot-air engine from the heat imparted QG, the initial temperature T, and the temperature which the air receives by compression according to the adiabatic curve.

We see that for $G = 1$, that is, for one kilogram of air, this equation is identical with that found for the simple cycle process, Chapter VI.

If we insert the second value of Q_1, we have

$$L = \frac{G}{A}\left(Q - \frac{QT_3}{T_2}\right) = \frac{QG}{AT_2}(T_2 - T_3). \quad \text{(LXV.)}$$

Therefore we can find the delivery from the heat imparted QG, the temperature T_2 which the air receives in the heating apparatus, and that which it has after expansion in the working cylinder.

Since the specific heat does not occur in these formulæ, we see that the delivery of a hot-air engine is independent of the specific heat. If, for example, in the two systems already noticed, Ericsson's and Sterling's, the temperatures T_1 and T, or T_2 and T_3, as well as the amount of heat QG, are the same, the engines will all give the same delivery. The weight of air in the Sterling engine must indeed be greater than in the other, because the specific heat s is less. Neither has, then, any advantage over the other, apart from the dimensions of the engine. The delivery of both, as well as of all systems in which there is a cycle of the kind in question, is proportional to the heat imparted and to the temperatures occurring in the engine only.

We can also find the delivery L from the heat Q_1G abstracted.

If we insert in the equation for L, the second value for Q, we have

$$L = \frac{G}{A}\left(\frac{Q_1T_2}{T} - Q_1\right) = \frac{Q_1G}{AT}(T_1 - T) \ . \quad \text{(LXVI.)}$$

If we insert the second value of G, we have

$$L = \frac{G}{A}\left(\frac{Q_1T_2}{T_3} - Q_1\right) = \frac{Q_1G}{AT_3}(T_2 - T_3) \ . \quad \text{(LXVII.)}$$

There only remains to deduce formulæ for hot-air engines in which the compression and expansion do not take place according to the adiabatic curve, but according to some other, for which the general law is

$$pv^{\frac{n}{m}} = p_1v_1^{\frac{n}{m}}$$

where $\frac{n}{m}$ possesses a different volume from that for the other curves of heat addition and subtraction. The deduction of such formulæ is unnecessary, since they are applicable to no existing systems, and it is improbable that in future any engines will be constructed to which they are applicable.

As already remarked, our discussion and formulæ do not apply to the more recent engines of Laubereau or Lehmann, in which the air does not go through a simple cycle process, and which, according to the author's view (particularly the latter), are especially suited for minor industrial uses. We shall therefore seek at the close of this chapter to briefly describe a hot-air engine, the principle of which seems, from a practical standpoint, worthy of notice. We allude to the high-pressure engine of Richard Unger, described in the *Civil-Ingenieur* and *Polytechn. Journal.* In the next chapter we will treat in detail of the engines of Laubereau and Lehmann.

Construction of Unger's Engine.—In the hot-air engines considered in this chapter, the cold air is first compressed adiabatically in the feed-cylinder, and its tension increases, therefore, according to the exponential law of Mariotte. If, however, we compress the air in the feed-pump while we abstract the heat developed, the expansive force increases according to the simple law of Mariotte, and the compressed air posesses at the same pressure p_1, a much less volume than when the compression took place adiabatically. Since, also, the temperature of this air is less, it will expand much more when heated to the same degree, and hence perform more work; or inversely, for the same performance it expands less, and hence the volume of the working cylinder is less. Also, the same weight of air is heated to a less degree for the same amount of heat imparted, and hence the highest temperature in the engine is less than when the air is compressed, according to the exponential law of Mariotte.

This principle has been applied by Richard Unger, in his high-pressure caloric engine. The compressed air in the feed-pump is cooled by injecting cold water in spray, which is thus converted into steam. This water is forced into the feed-cylinder by a small pump, worked by the engine itself. In this way, according to the *Journal,* the air is brought to about 30° C., for a pressure of 6 atmospheres (5 atmospheres effective). The steam formed increases somewhat the expansive force of the air, and probably diminishes the piston friction.

The furnace is a cylindrical space, inclosed by iron plates, spherical above and below. It consists of three annular concentric spaces. In the central space the coal is consumed. This is connected by openings with the inner space, which is closed above, but open below. This latter communicates with the outer space.

Into the fireplace proper, as well as into the inner space, a part of the compressed air is forced by the feed-pump by narrow pipes, thus securing perfect combustion. The hot products of combustion then mix with the larger part of the cold air furnished by the compression-pump, which fills the outer annular space, where it is heated by the central fire space. In this way the air receives a temperature of 250 to 300°.

The hot air is then led by special pipes to the valve chests, and enters first on one, then on the other side of the piston.

Before the compressed air reaches the furnace it enters a receiver, probably in order better to regulate the air necessary for combustion. (*Dingler's Polytechn. Journal,* Bd. clxxxvi., Heft 1.)

CHAPTER X.

THE HOT-AIR ENGINES OF LAUBEREAU AND LEHMANN.

In the preceding chapters we have given the theory of those open and closed hot-air engines in which a definite volume of air makes in the engine a cycle process. If the engine is single-acting, as, for example, Ericsson's, such a cycle is completed during one revolution; in double-acting engines we have also a cycle during the same period, but, other things being the same, we have a double weight of air, so that for the same dimensions we have a double performance. The two engines which we now consider, are in reality closed engines, but the air in them *does not complete a cycle in the way heretofore assumed.* Thus, while heretofore the *entire* inclosed air volume was either compressed or rarefied, now only a part is thus treated, while the other part is in another condition. For this reason the formulæ thus far developed do not apply to these engines. We cannot, therefore, determine their performance from the heat added or abstracted and the temperature fall, according to the fundamental principles of the mechanical theory of heat, but we must rather adopt the method which we have followed in the calculation of the Ericsson engine.

Description of the Laubereau Engine.—This engine consists of a hollow iron cylinder, *abcd* (see Figs. 41 and 42, following), surrounded by a somewhat wider cylinder, *efgh.* The space between is filled with cold water, in order to cool the air in the cylinder. In the lower part of the cylinder is a bell, of cast-iron, whose sides are corrugated, in order to afford a greater surface to the hot air. The sides of this bell, as well as of the cylinder, are shaded black in the Figs. 41 and 42. In

Fig. 42 the bell is heated by gas, brought on through the pipe *o* and burner *l*. To secure the air necessary for active combustion, *l* is surrounded by a tube, *mn*, through which a current of air passes. In Fig. 41 the furnace *A* is supplied with coal by the cast-iron door *p*, the products of combustion pass out through *m*, and thus the bell *ik* is heated. These products collect at *q*, and pass off through the chimney *ss*.

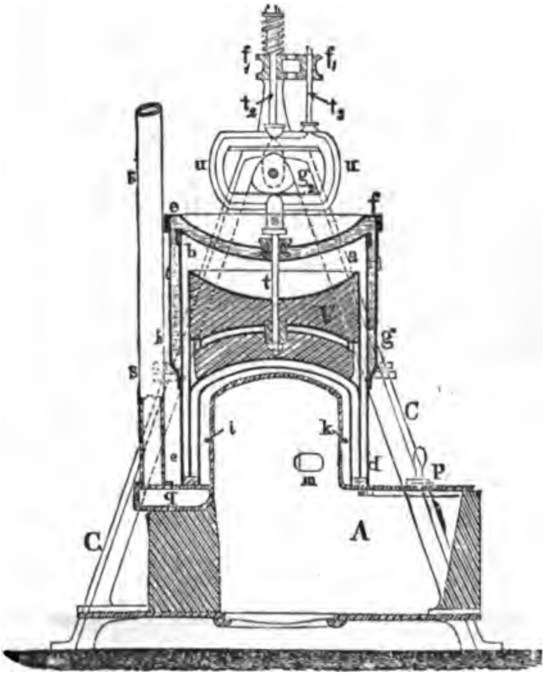

Fig. 41.

The interior of the cylinder *abcd* is partly filled by the *distribution piston V*, the exterior of which is of cast-steel, and which consists of two parts. The interior of both parts, and the space between, is filled with some poor conductor of heat. The piston stock has a rod *t*, which passes air tight through a stuffing-box in the cover of the cylinder *abcd*. The continuation of the distribution piston below forms a thin plate cylinder, as in Ericsson's engine, which, when the piston is in its lowest position, incloses the bell *ik*, and easily absorbs its radiant heat.

The object of the distribution piston is as follows. Suppose a certain amount of air in the cylinder *abcd*, then by far the greatest portion will be above the piston, since there is the greatest free space. But since the upper part of the cylinder *abcd* is surrounded by cold water, the air there must have the temperature of this water. If, now, the piston *V* rises, the air must pass from the upper to the lower space, and come in contact with the hot plates of the piston, and the hot bell *ik*, so that it is immediately heated, and has a great tension. If the piston again descends, the heated air passes into the upper cold space and imparts its heat to the cold water. The temperature thus sinks to its original value. It follows that the water surrounding the upper part of the cylinder must be constantly renewed. This is done by a small pump, *P*, merely indicated in the Figures.

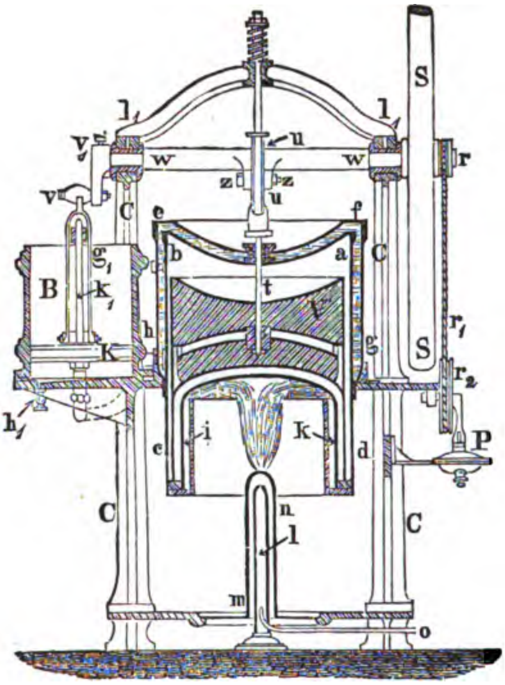

FIG. 42.

Now that we have seen how the air in the engine is alternately heated and cooled, and thus has a greater and less expansive force, let us explain how the increased force of the heated air is converted into mechanical work.

From the lower part of the inner space of *abcd*, in the neighborhood of *h*, a pipe, dotted in Fig. 42, leads to the lower part of the *working* cylinder *B*. This cylinder is open above. In the bottom is a cock, h_1, by which air can be admitted to the cylinder and to the engine. The cylinder is fitted with an air-tight piston, *K*, and rod, k_1, which works vertically up and down through fixed guides above. By the forked-shaped connecting-rod g_1, the crank vv_1 and shaft *ww* are set in motion. The shaft passes through boxes l_1l_1, borne by the frame *CC*. Upon the shaft is the fly-wheel *SS*, and the disc *r*, which by means of the belt r_1 and the pulley r_2 work the pump *P*. The shaft *ww* is divided in the middle, and furnished with two small cranks *zz*, connected by a triangular cam g_2, Fig. 41. This cam moves in the rectangular frame *uu*, which is connected below with the rod *t* of the distribution piston. Above it is connected to two rods, t_2 and t_3, which work vertically in the guides $f_1 f_1$. The cam g_2 thus answers to the eccentric in the steam engine.

When, now, the engine is put in motion, the gas is either kindled or burning coal introduced through the door *p*. After a few minutes, when the bell *ik* is sufficiently heated, the engine is turned by means of the fly-wheel *SS* beyond its dead point, so that the distribution piston *V* is moved upwards. The cold air is thus forced rapidly into the lower warm space, where it is heated up to about 300° C. The tension, before 1 atmosphere, thus becomes about 1¼ or more, according to the construction of the machine. By reason of this increased tension the piston *k* in the working cylinder is raised. When the distribution piston has reached the upper end of its stroke, the working piston has still some distance to go. The first, therefore, descends while the working piston completes its stroke. The tension of the air in the engine sinks immediately below 1 atmosphere, and the working piston is forced down by the outer air pressure.

At the moment when the latter commences to ascend, the distribution piston has already risen part way, a part of the cold air is again heated, the entire air mass has thus a greater tension, and the working piston is again raised, and so on.

While the motion of the working piston is tolerably uniform, that of the distribution piston is more irregular. At the upper and lower points of its stroke, it must linger, while the stroke up and down is quickly accomplished. Indeed, it may even be that the distribution piston makes its entire stroke while the working piston is reversing its motion. The motion of the distribution piston is dependent upon the construction and shape of the triangular cam g_2 and frame *uu*. (Such an arrangement is represented in Fig. 777, Art. 475, of Du Bois' translation of Weisbach's Mechanics of Engineering, vol. 2.)

According to reports, these engines of Laubereau should not make less than 500 revolutions per minute, and just so often the air must be heated and cooled. From a practical standpoint this appears hardly possible. If the data are correct, we see how very rapidly air can receive and give up heat, when made to move over warm or cold plates.

Theory of Laubereau's Engine.—Let us first investigate the tension relations in the engine. We may proceed here in a manner similar to that adopted for the Ericsson engine. We use the following notation.

First, it is assumed that the cylinder *efgh*, as well as the distribution piston V, are closed above and below by plane surfaces. Let

F sq. meters be the cross-section of the distribution piston, or the area of the cylinder *abcd*.

H meters, the length of stroke of the distribution piston.

H_1 meters, that distance which the distribution piston passes over while the working piston moves through h_1.

h meters, the stroke of the working piston.

h_1 meters, the distance passed over while the distribution piston passes through H_1.

G the weight of air in the engine, which is always constant.

T the absolute temperature of the cold air.

t its temperature Centigrade.

T_1 the highest absolute temperature of the air.

p the pressure of the cold air in the cylinder *abcd*, when the distribution piston is below.

p_1 the highest, and

p_2 the lowest pressure during a revolution. Finally

b meters, the height of the prejudicial space or clearance, considered not as annular but as a cylinder.

If the distribution piston is below, the air volume above it is

$$FH$$

of the temperature T and pressure p.

This weighs, according to known principles,

$$\frac{FHp}{RT} \text{ kilograms.}$$

Below the piston we have the air volume

$$Fb$$

of the pressure p and temperature T_1, whose weight is

$$\frac{Fbp}{RT_1}.$$

The entire weight of air inclosed is then

$$G = \frac{FHp}{RT} + \frac{Fbp}{RT_1} = \left(\frac{H}{T} + \frac{b}{T_1}\right)\frac{Fp}{R} \quad . \quad . \quad . \quad (1).$$

The distribution piston has passed through the distance $H - H_1$ when the tension in the entire engine has risen to p_1. Above the piston there is now the air weight

$$\frac{FH_1p_1}{RT},$$

and below

$$\frac{F(H-H_1)p_1}{RT_1} + \frac{Fbp_1}{RT_1},$$

so that for this position the air weight is

$$G = \frac{FH_1p_1}{RT} + \frac{(H - H_1 + b)Fp_1}{RT_1}$$

$$= \left[\frac{H_1}{T} + \frac{H - H_1 + b}{T_1}\right]\frac{Fp_1}{R} \quad . \quad . \quad . \quad . \quad (2).$$

From (1) and (2) we have

$$\frac{p_1}{p} = \frac{HT_1 + bT}{H_1T_1 + (H - H_1 + b)T} \quad . \quad . \quad . \quad . \quad (3).$$

If we put $H - H_1 = H$, or $H_1 = 0$, that is, if we assume the working piston to be first raised when the distribution piston is at the upper end of its stroke, we have

$$\frac{p_1}{p} = \frac{HT_1 + bT}{(H + b)T}.$$

This is evidently the greatest value which p_1 can have.

Since we can also write

$$p_1 = \frac{HT_1 + bT}{(H + b)T}p. \quad . \quad . \quad . \quad . \quad . \quad (4).$$

we see that p_1 depends essentially upon the initial pressure p. If we have $b = 0$, we have

$$p_1 = \frac{T_1}{T}p \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

Therefore the pressure p_1 is directly as the temperature T_1 and the initial pressure p, and inversely as the temperature T.

After the distribution piston has risen the distance H_1, the working piston moves through the distance h_1 under the constant pressure p_1. The air weight is now

$$G = \frac{FHp_1}{RT_1} + \frac{Fbp_1}{RT_1} + \frac{fh_1p_1}{RT_1} \quad . \quad . \quad . \quad . \quad (6).$$

From (6) and (2) we have

$$fh_1 = \frac{FH_1(T_1 - T)}{T} \quad \text{and}$$

$$h_1 = \frac{F}{f}\ \frac{H_1}{T}(T_1 - T) \quad . \quad . \quad . \quad . \quad . \quad (7).$$

If again we put $H_1 = 0$, we shall have $h_1 = 0$, that is, the working piston goes through no distance under constant pressure, but is raised by expansion of the air.

If further we take $f = F$ and take $T_1 = 2 \times 273$ and $T = 273$, that is, if we warm the air from 0 to 273°, we have, as is evident,

$$h_1 = H_1.$$

Let us determine now the height h, which the working piston has to move, in order that the pressure may sink to the original pressure p.

When the working piston has passed through h, the air volume in the engine is

$$FH + Fb + fh.$$

This volume, at the pressure p and temperature T_1, weighs

$$G = \frac{FHp}{RT_1} + \frac{Fbp}{RT_1} + \frac{fhp}{RT_1} = \left(\frac{H}{T_1} + \frac{b}{T_1}\right)\frac{p}{R} + \frac{fhp}{RT_1}.$$

Equating this to (1), we obtain

$$\frac{FH + Fb + fh}{RT_1} = \frac{FH}{RT} + \frac{Fb}{RT_1},$$

and after reduction

$$h = \frac{FH}{fT}(T_1 - T). \quad . \quad . \quad . \quad . \quad . \quad (8).$$

From (7) and (8) we have now

$$h_1 : h :: H_1 : H.$$

While now the working piston is at its highest point, the distribution piston passes through $H - H_1$, and the pressure of the air sinks from p to p_2. The inclosed air weight is now

$$G = \frac{fhp_2}{RT_1} + \frac{FH_1p_2}{RT_1} + \frac{Fbp_2}{RT_1} + \frac{F(H - H_1)p_2}{RT}.$$

Equating this to (1), we have

$$(2H - H_1)p_2T_1 + (H_1 + b - H)p_2T = (HT_1 + bT)p$$

$$\frac{p_2}{p} = \frac{HT_1 + bT}{(2H - H_1)\,T_1 - (H - H_1 - b)\,T} \quad . \quad . \quad . \quad (9).$$

If here we put $H - H_1 = H$, or $H_1 = 0$, we have

$$\frac{p_2}{p} = \frac{HT_1 + bT}{2HT_1 - (H - b)T} \quad . \quad . \quad . \quad . \quad . \quad (10).$$

EXAMPLE.

Let $F = 1$ sq. meter, $f = \frac{1}{2}$ sq. meter, $H = 0.2$ m., and $H_1 = 0.1$ meter. Also, $b = 0.02$ meter, the temperature t of the cold air 0°, or $T = 273$, and of the hot air $t_1 = 273°$, or $T_1 = 2 \times 273$. What are the pressure relations in the engine, and how great are h_1 and h?

From (8),

$$\frac{p_1}{p} = \frac{0.2 \times 2 \times 273 + 0.02 \times 273}{0.1 \times 2 \times 273 + (0.2 - 0.1 + 0.02)\,273}$$

$$= \frac{0.4 + 0.02}{0.2 + 0.12} = \frac{0.42}{0.32} = 1.313 \text{ atmospheres.}$$

If H_1 were 0, we have from (4)

$$\frac{p_1}{p} = \frac{0.2 \times 2 \times 273 + 0.02 \times 273}{0.2 \times 273 + 0.02 \times 273} = \frac{0.42}{0.22} = 1.909 \text{ atmospheres.}$$

This is evidently the highest pressure which can be attained for the temperature 273°, when only air is inclosed in the engine.

If the prejudicial space is zero, we have

$$p_1 = 2 \text{ atmospheres.}$$

The minimum pressure p_2 is from (9)

$$\frac{p_2}{p} = \frac{0.2 \times 2 \times 273 + 0.02 \times 273}{(0.4 - 0.1)\,2 \times 273 - (0.2 - 0.1 - 0.02)\,273} = \frac{0.42}{0.52} = 0.808 \text{ atmosphere.}$$

makes 45 revolutions per minute, and has an efficiency of 0.5, we have

$$N = 2.296 \times 45 \times 0.5 \times 0.5\,(0.4 + 0.2)\,\frac{1.313 - 0.808}{2}$$

$$= 4 \text{ horse-powers.}$$

Tresca has observed in engines of this kind, for cylinder diameter of 0.5 meter, and hence cylinder cross-section of 0.196 square meter, stroke of 0.4 meter, and effective pressure during rise of piston of 0.25 atmosphere, a delivery of 0.8 horse-power. For a cross-section of working cylinder of 0.5 square meter, as in our example, the delivery for this slight pressure would be $\frac{0.5}{0.196} \times 0.8 = 2.04$ horse-power. If we consider that the pressure in our example is 0.313, and that the effective pressure of the atmosphere is also greater, we can conclude that our formula is reliable if the number of revolutions is not far from 45.

The delivery is considerably greater when p_1 and p_2 have the above calculated maximum values, or when the engine is so constructed that the working piston lingers at its upper or lower positions while the distribution piston describes a complete stroke.

Dimensions for a given Delivery.—If we put in the last formula in place of h and h_1 the values from (7) and (8), we have

$$N = 2.296nw\,\frac{F}{T}\,(T_1 - T)\,(H + H_1)\,\frac{p_1 - p_2}{2} \quad . \quad \text{(LXIX.)}$$

whence it appears that the delivery increases not only with the area F and the height H of the distribution cylinder, but also with the temperature T_1.

From this formula we have at once the area F for an engine of given power. Thus,

$$F = \frac{2TN}{2.296nw\,(T_1 - T)\,(H + H_1)\,(p_1 - p_2)} \quad . \quad \text{(LXX.)}$$

Here n, w, T_1, T, and H, H_1 are considered as given. The values of p_1 and p_2 are then found from (3) and (9).

For the maximum delivery, where $H_1 = 0$, we have

$$F = \frac{2TN}{2.396\,nwH(T_1 - T)(p_1 - p_2)} \,. \quad \text{(LXXI.)}$$

EXAMPLE.

What must F and f be, when the actual delivery N of the engine is two horse-power, and $w = 0.30$, $n = 60$, $T_1 = 2 \times 273$, $T = 273$, $H = 0.05$, and $b = 0.01$?

If we require from the engine the maximum of work, we have from the equation $\frac{p_1}{p} = \frac{HT_1 + bT}{(H + b)T}$ by inserting the numerical values.

$$\frac{0.05 \times 2 \times 273 + 0.01 \times 273}{(0.05 + 0.01) \times 273} = 1.833 \text{ atmospheres,}$$

and from Equation 10

$$\frac{p_2}{p} = \frac{HT_1 + bH}{2HT_1 - (H - b)T}$$

$$= \frac{0.05 \times 2 \times 273 + 0.01 \times 273}{2 \times 0.05 \times 2 \times 273 - (0.05 - 0.01) \times 273} = 0.688 \text{ atmosphere.}$$

Hence

$$F = \frac{2 \times 273 \times 2}{2 \times 296 \times 60 \times 0.3 \times 0.05 \times 273 \times 1.145}$$

$$= \frac{4}{2.366} = 1.69 \text{ square meters.*}$$

The entire volume of the air inclosed in the distribution cylinder must therefore be

$$1.69 \times 0.05 = 0.0845 \text{ cubic meters.}$$

From (8) we have for the volume of the working cylinder

$$fh = 0.0845 \frac{2 \times 273 - 273}{273} = 0.0845 \text{ cubic meters.}$$

If we take $h = 0.05$ meters, we have $f = \frac{0.0845}{\frac{1}{2}} = 0.169$ sq. meters.

The Hot-Air Engine of Lehmann.—In the engine of Laubereau we meet with two evils, the direct action of the hot air upon the working piston, and the alternate heating and cooling of a part of the inclosed air which does not contribute to the action of the machine. The first objection is common also to the engines of Unger and Belou, while Ericsson has avoided it in his engine in a very ingenious manner. In fact this invention is specially distinguished in that the working piston always remains cool (the temperature at most 40° to 50°), so that it can be well lubricated, and also in that the motion of the piston is transmitted in a very simple and ingenious manner to the fly-wheel and this to the feed piston.

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* If we take for the stroke 0.5m. instead of 0.05m., we should have for F, 0.169 square meters. Hence the radius of the distribution piston would be only 0.233 meter.

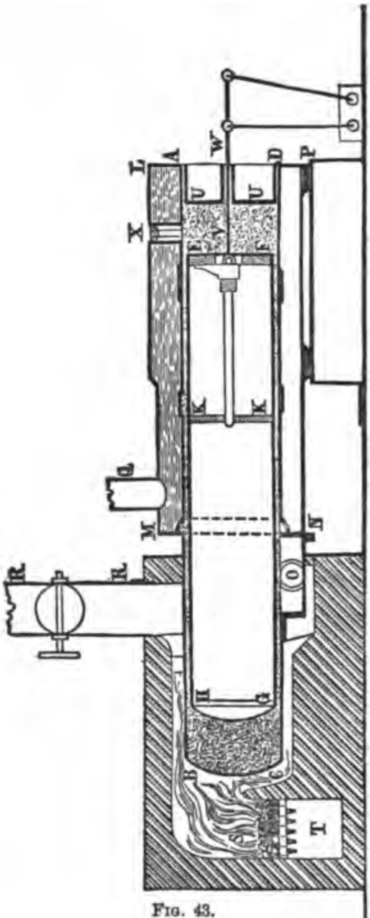


FIG. 43.

The objections to Laubereau's engine have been met by Lehmann, who, with the same idea at bottom, has joined the special advantages of Ericsson's invention. In Lehmann's hot-air engine, also, an inclosed quantity of air is alternately heated and cooled, but it does not act directly upon the working piston. This latter is protected by the feed piston from the heat of radiation and conduction, while still the expansive force of the hot air is applied in the same cylinder in which it is heated and cooled. For this reason, a part of the air is not heated and cooled unnecessarily at every revolution.

Fig. 43 represents a section of Lehmann's engine, a description of which, together with a number of experimental results, have been given by Eckerth in the *Vierteljahrsschrift des deutschen Ingenieur-und Architekten-Vereins*, 1 Jahrgang, 2 Heft.

ABCD, Fig. 43, is a horizontal cylinder of cast-iron, open in front and closed behind. *S* is the fire space, with the ash pit *T*. The hot air plays around the outside of the cylinder at the left end, and then mounts through the pipe *R*, which is furnished with a throttle valve for regulating the draft, into the air. About ⅔ds of the cylinder is surrounded by a wider cylinder, *LMNP*, and the annular space between is kept filled with cold water, which enters below and escapes through the pipe *Q* after it has cooled the air.

In the cylinder *ABCD* two pistons move. One, *UU*, is the working piston, which is made to fit air-tight by a leather washer. This washer is so constructed that it allows air to enter when the outer air pressure is greater than the inner, but hugs only the tighter and prevents exit when the inner air pressure is greater than the outer. The outer piston, called the compressor, consists chiefly of an air-tight riveted plate cylinder *EFGH*, stiffened inside by the piston *KK*, and closed in front by the wooden piston *EF*. The diameter of this cylinder is but little less than that of *ABCD*, so there is only a narrow space between.

The compressor is connected with a rod *VW*, which passes air-tight through the working piston. This latter transmits its motion to the fly-wheel in a manner similar to Ericsson's engine, and the motion of the compressor is similarly effected.

In order to preserve its motion in a right line, guides are fitted to the cylinder *ABCD*, and to diminish the friction we have a roller at *O*.

The cranks and connecting-rods by which the motion of the working piston is communicated to the compressor are so arranged that the crank for the latter is 65° in advance of that for the working piston.

In the vicinity of *X* there is a regulator, which opens a valve when the normal action is exceeded.

The mutual motions of the working piston and compressor are represented in Fig. 44.

The circles above are the crank circles. We have divided each into 12 equal parts. When the crank of the working piston has gone from 1 to 2, the working piston has gone through the distance $2a$, and simultaneously the feed piston has gone through the distance $2b$. When the crank of the working piston reaches 3, the working piston has passed through $3a$, and the feed piston through $3b$, and so on.

We see now from the diagram that when the working piston reaches its extreme position on the right (see Fig. 43), the compressor has gone almost the half of its stroke toward the left. This position we have indicated in the diagram by *I*. The space right and left of the compressor is then equal, but in the one we have cold air and in the other hot air, while the air layer in the narrow annular space between compressor and cylinder forms a poor conductor, by which the propagation of heat from the hot to the cold space is prevented. This air answers, then, the purpose of the ring valve in Ericsson's engine.

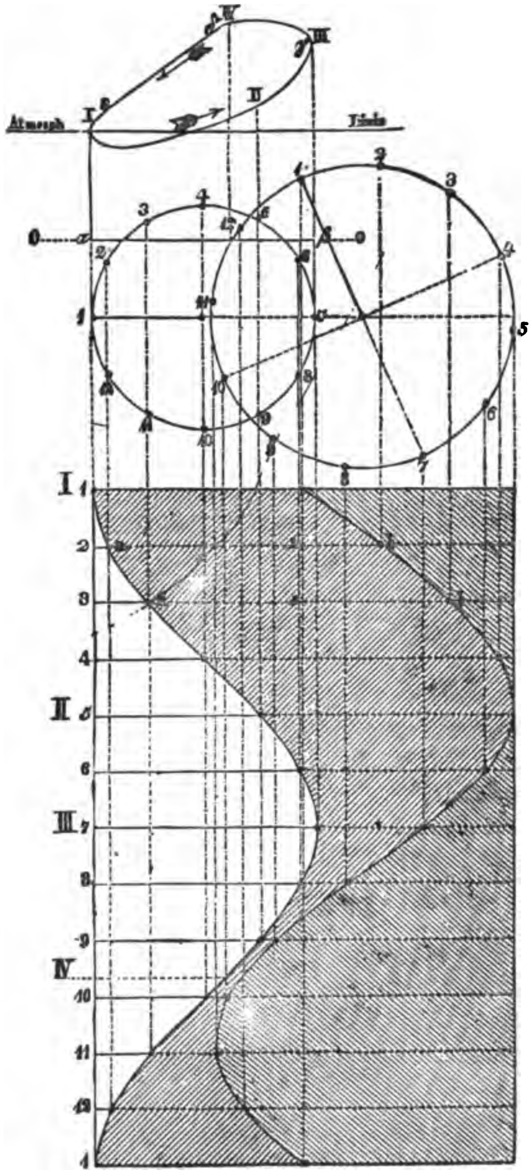


Fig. 44.

When the two pistons have the position in *I*, the compressor goes toward the left somewhat more rapidly than the working piston. The hot air is thus driven out of the hot space behind the compressor into the cold space in front, and its tension is diminished. Since, however, the working piston also moves toward the left, the inclosed air is compressed. But the increase of the tension by the compression is at first less than the decrease by cooling. If, therefore, the entire air volume had at first the pressure of the atmosphere, its pressure will be now somewhat less. In Ericsson's engine the same is true. As soon as the feed piston begins to move backward, the air between the two pistons is rarefied, and cold air enters through the valve in the working piston. Eckerth, however, ascribes this decrease of tension to losses of air. From 3 on (see diagram), the working piston and compressor move with almost equal velocity toward the left, and since at 3 there is but little hot air behind the compressor, the entire inclosed air volume soon takes the pressure of the atmosphere, as the closed curve *I.II.III.IV.* above the crank circle in the diagram represents. This is obtained by laying off from the line $\alpha\beta$ the pressures corresponding to the positions *I.II.III.IV.* From 4 on, where the working piston moves toward the left much more rapidly than the compressor, and the latter is almost at the end of its stroke, the pressure rises above the atmosphere.

At *II.* the compressor has its extreme left position, and all the hot air has passed into the cold space, and since the working piston also has nearly completed its stroke toward the left, the pressure is considerably higher than the atmosphere. The maximum pressure, however, is not yet attained. From *II.* to *III.* it must increase rapidly, for now the compressor returns and forces cold air into the hot space, while the working piston still is going toward the left as before. About at *III.*, then, we have the maximum pressure.

Between *III.* and *IV.* it remains tolerably constant, because both pistons are moving together toward the right with nearly equal velocity, and because the increase by the heating of the cold air is balanced by the increase of volume.

From *IV.* on, the working piston goes more rapidly to the right than the compressor, and the pressure sinks gradually until it is again 1 atmosphere.

Calculation of the Pressures in Lehmann's Engine.—Let

F = the cross-section of the larger cylinder.
F_1 = that of the compressor.
l, the length of compressor.
a, the distance between pistons in the position *I.*
a_1, the distance from end of compressor to that of cylinder.
T, the absolute temperature in the cold space.
T_1, that in the annular space between compressor and cylinder.
T_2, that in the hot space.

For the inclosed air weight G we have, when p is the pressure for the position *I,*

$$G = \frac{Fap}{RT} + \frac{(F - F_1)\,lp}{RT_1} + \frac{Fa_1p}{RT_2}.$$

If we put $(F - F_1)\,l = \frac{1}{n}Fa$, we have

$$G = \frac{Fp}{R}\left(\frac{a}{T} + \frac{a}{nT_1} + \frac{a_1}{T_2}\right) \quad . \quad . \quad . \quad . \quad (1).$$

Again for the position *II.* (see diagram), let b be the distance between pistons, b_1 that between cylinder end and compressor end, p_1 the pressure for this position, then

$$G = \frac{Fp_1}{R}\left(\frac{b}{T} + \frac{a}{nT_1} + \frac{b_1}{T_2}\right) \quad . \quad . \quad . \quad . \quad (2).$$

From (1) and (2) we obtain

$$\frac{p_1}{p} = \frac{\dfrac{a}{T} + \dfrac{a}{nT_1} + \dfrac{a_1}{T_2}}{\dfrac{b}{T} + \dfrac{a}{nT_1} + \dfrac{b_1}{T_2}}.$$

If for position *III.* we have c, c_1, and p_2 instead of b, b_1 and p_1, we obtain

$$\frac{p_2}{p} = \frac{\dfrac{a}{T} + \dfrac{a}{nT_1} + \dfrac{a_1}{T_2}}{\dfrac{c}{T} + \dfrac{a}{nT_1} + \dfrac{c_1}{T_2}}.$$

For position *IV.*, taking p_3, d, and d_1 in the same signification as p_2, c, and c_1 for *III.*, we have

$$\frac{p_3}{p}=\frac{\frac{a}{T}+\frac{a}{nT_1}+\frac{a_1}{T_2}}{\frac{d}{T}+\frac{a}{nT_1}+\frac{d_1}{T_2}}.$$

The engine described by Eckerth was of 1 horse-power, and had the following dimensions:

Diameter of working piston.........	0.349	meters.
Stroke " "	0.175	"
Diameter of compressor...........	0.342	"
Stroke " "	0.244	"
Length " "	1.527	"
Area of working piston........	$F = 0.0957$	sq. meters.

Further, if we take the dimensions of the drawings given by Eckerth in his description of Lehmann's engine as approximately correct,

$a = 6.5''$ (Vienna) $a_1 = 6.375''$ $b = 7.5''$ $b_1 = 0.25''$
$c = 3.5''$ $c_1 = 2.8''$ $d = 0.25''$ $d_1 = 8.25''$.

In the experiments the cold water entered with a temperature of 30° to 60°, and left at 40° to 70°. The cold air had then a somewhat higher temperature than $\frac{70+40}{2}=55°$ on the average. If we take it at 60°, $T = 273 + 60 = 333°$. If we assume the mean temperature of the air in the annular space at 100°, and that in the hot space at 300°, we have $T_1 = 373$, and $T_2 = 573$.

According to Eckerth, n is about $= 3$. We have taken for n 2.5.

Hence we have

$$\frac{p_1}{p}=\frac{\frac{6.5}{333}+\frac{6.5}{2.5\times 373}+\frac{6.375}{573}}{\frac{7.5}{333}+\frac{6.5}{2.5\times 373}+\frac{0.25}{573}}=\frac{0.03762}{0.02993}=1.2502.$$

Also for position *III.*

$$\frac{p_2}{p} = \frac{\frac{6.5}{333} + \frac{6.5}{2.5 \times 373} + \frac{6.375}{573}}{\frac{3.5}{333} + \frac{6.5}{2.5 \times 373} + \frac{2.8}{573}} = \frac{0.03762}{0.02237} = 1.6816.$$

For *IV.*

$$\frac{p_3}{p} = \frac{\frac{6.5}{333} + \frac{6.5}{2.5 \times 373} + \frac{6.375}{573}}{\frac{0.25}{333} + \frac{6.5}{2.5 \times 373} + \frac{8.25}{573}} = \frac{0.03762}{0.02212} = 1.7007.$$

Experiments with the indicator gave as the maximum pressure an excess of 10 pounds, or an effective pressure of $10 + 14.1 = 24.1$. Hence

$$\frac{p_3}{p} = \frac{24.1}{14.1} = 1.709,$$

a value which agrees very closely with our calculation. The pressure relations are here, therefore, more favorable than in Ericsson's engine, and considerably more so than in Laubereau's engine, as might have been expected from a consideration of the construction.

Delivery of the Engine.—According to the diagram, the pressure of the air, while the working piston passes from *I.* to 4, is tolerably constant, and somewhat less than that of the atmosphere. From 4 to *III.*, it increases about as the volume diminishes, and the increase follows the isothermal curve therefore. From *III.* to *IV.* again, the pressure remains tolerably constant, and then from *IV.* to 1 decreases almost exactly according to the ordinates of a straight line, that is, the decrease of pressure is proportional to the distance traversed by the working piston.

We have denoted the area of the piston by F. Let the distance passed over from *III.* to *IV.* be w, then the work of the air during this motion of the piston is

$$L_1 = Fwp_3.$$

From *IV.* to 1 the pressure is $\frac{p_3+p}{2}$ kilograms. If the distance is w_1, the work is

$$L_2 = Fw_1\frac{p_3+p}{2}.$$

The work of the heated air is therefore

$$L_1 + L_2 = F\left(wp_3 + w_1\frac{p_3+p}{2}\right).$$

Further, let the pressure from *I.* to 4 be γ_0 on the average, and the distance w_2, then, since $p_0 < p$, that is, is less than the atmospheric pressure, the work of this latter is

$$L_3 = Fw_2(p - p_0).$$

So that the entire delivery is

$$L_1 + L_2 + L_3 = F\left(wp_3 + w_1\frac{p_3+p}{2} + w_2(p - p_0)\right).$$

The work of compression from 4 to *III.* is

$$L_4 = 2.3026 V_2 p_2 \log \frac{p_2}{p}$$

if V_2 is the least volume and p_2 the greatest pressure.

The first occurs when the working piston reaches the back end of its stroke. Instead of V_2 we can put Fx, and have then,

$$L_4 = 2.3026 Fxp_2 \log \frac{p_2}{p},$$

so that the effective delivery per stroke is

$$L_1 + L_2 + L_3 - L_4 = L = F\left[wp_3 + w_1\frac{p_3+p}{2} + w_2(p - p_0)\right.$$
$$\left. - 2.3026xp_2 \log \frac{p_2}{p}\right].$$

If there are n strokes per minute, the delivery per second in horse-powers is

$$N = \frac{n}{60 \times 75} F \left[wp_3 + w_1 \frac{p_3 + p}{2} + w_2 (p - p_0) - 2.3026 x p_3 \log \frac{p_3}{p} \right],$$

where p_3, p, etc., are expressed in kilograms. If we express the pressures in atmospheres, we have

$$N = \frac{10334 n}{60 \times 75} F \left[wp_3 + w_1 \frac{p_3 + p}{2} + w_2 (p - p_0) - 2.3026 x p_3 \log \frac{p_3}{p} \right],$$

or

$$N = 2.296 Fn \left[wp_3 + w_1 \frac{p_3 + p}{2} + w_2 (p - p_0) \right.$$

$$\left. - 2.3026 x p_3 \log \frac{p_3}{p} \right] \quad . \quad . \quad . \quad . \quad (\text{LXXII.})$$

EXAMPLE.

For the engine already discussed, $F = 0.0957$ sq. meters, and n, on an average, was 97. If the distances $w_1 w_1$, etc., traversed by the working piston, are taken from the diagram, we have $w = 0.06^{\text{m.}}$, $w_1 = 0.115$, and $w_2 =$ about $0.06^{\text{m.}}$ What is N, when we take $p_3 = 1.71$, and $p_0 = 0.97$ atmospheres?

Since the least volume $v_3 = 0.02125$ cubic meters, $x = 0.222^{\text{m.}}$, hence

$$N = 2.296 \times 0.0957 \times 97 \left[0.06 \times 1.71 + 0.115 \frac{1.71 + 1}{2} + 0.06 \times 0.03 - 2.3026 \right.$$

$$\left. \times 0.222 \times 1682 \log 1.682 \right],$$

$$N = 21.314 \, [0.1026 + 0.1558 + 0.0018 - 0.1941]$$

$$= 21.314 \times 0.0661 = 1.4088 \text{ horse-power.}$$

A mean of several experiments with the friction brake gave the actual delivery 73.8 meter-kilograms, or $\frac{73.8}{75} = 0.984$ horse-power, so that the efficiency is about $\frac{0.984}{1.4088} = 0.69$. According to Eckerth, who reckoned the theoretical delivery from diagrams, the efficiency was about 0.66. Both seem somewhat too great. For a ½ horse-power caloric engine of Ericsson, the efficiency, according to experiments by the author (Dingler's *Polytechn. Journal*, Bd. clxxix.) was 0.40, a result with which Tresca's experiments closely agreed. It does not seem likely that the efficiency of the present construction is greater by 26 to 29 per cent.

The above formulæ for the calculation of the power of a Lehmann engine may, according to the determinations and views of the author, be replaced, as long as the proportions are unchanged, by the following simple formula:

$$L_s = 0.163^n (t_1 - t_2)\,G \text{ meter-kilograms,}$$

where L_s is the delivery per second, n the number of revolutions per minute, t_1 the highest and t_2 the lowest temperatures of the inclosed air, and G the weight of air.. This last can be easily calculated from (1). We have thus $G = 0.02784$ kilograms. This formula rests upon similar reasoning to that on page 234.

Eckerth states that the machine required 13.5 cubic feet, or a weight of 426.4 kilograms of cooling water. This was furnished by a small pump worked by the engine, and forced into the cold space, from whence, after being heated, it flowed back to the reservoir. In 2 hours this water showed an increase of 61 − 35 = 26°, or 13° per hour. The heat absorbed by the water was hence

$$426.4 \times 13 = 5543.2 \text{ heat units per hour.}$$

The hourly delivery was

$$73.8 \times 60 \times 60 = 265680 \text{ meter-kilograms,}$$

corresponding to $\frac{265680}{424} = 626.6$ heat units.

Hence the heat units per hour imparted to the air are

$$5543.2 + 626.6 = 6170.$$

The ratio of the heat transformed into work to that absorbed by the cooling water is therefore

$$\frac{626.6}{5543.2} = 0.113.$$

The engine required per hour 4.585 kilograms of coal, whose heating value was estimated at 3,500 heat units. The coal furnished therefore

$$4.585 \times 3500 \text{ heat units.}$$

Since only 626.6 units were utilized, the ratio of the heat transformed into mechanical work to that furnished by the coal, or the *thermal effect*, is

$$\frac{626.6}{4.585 \times 3500} = 0.039, \text{ or about 4 per cent.}$$

The efficiency of the furnace was

$$\frac{6170}{4.585 \times 3500} = 0.38.$$

The difference in temperature of the cooling water at entrance and exit was, while the engine was working at about 1 horse-power, about constant and 9° Centigrade.

The heat units per hour imparted to the water was, as above, 5543.2, hence per minute 92.39. The amount of water used per minute is then

$$\frac{92.39}{9} = 10.266 \text{ kilograms} = 10.266 \text{ cubic decimeters.}$$

The diameter of the pump, as also its stroke, was about 2 inches.

The following table, taken also from the *Vierteljahresschrift*, gives a comparison between the hot-air engines of Ericsson, Laubereau, and Lehmann:

System.	Diameter of Working Piston.	Useful Delivery per Second.	Efficiency.	Consumption of Fuel per Horse-power per Hour.	Cooling Water per Horse-power per Hour.
Ericsson....	0.414 m.	1.77 H. P.	0.46	4.13 kil. of coal of heating value 7,000–7,500.	
Laubereau..	0.342 m.	0.8	0.40	4.5–5 kil. coke of heating power 7,000–7,500.	20–30 cubic feet for a rise of temperature of 17°.
Lehmann..	0.239 m.	1	0.66 (?)	4.6 kil. hard coal of heating power about 8,500.	6¾ cub. ft. for rise of temperature of 26°.

According to the views of the author, the engines described are especially suited for the lesser industries. They are, for reasons cited, more durable than Laubereau's, and the annoying pounding of Ericsson's is entirely avoided. Moreover, the consumption of fuel, as shown in the table, is only the half of that for the other two systems. In this respect it is as economical as the best steam engines.

Caloric engines are also perfectly free from danger, and their attendance and management is much simpler than for the steam engine.

CHAPTER XI.

GAS ENGINES, ESPECIALLY THOSE OF OTTO AND LANGEN.

ALMOST at the same time as the introduction of the Ericsson engine in Europe, the gas engine was invented by Lenoir in Paris. Let us consider the circumstances which gave rise to the idea.

It is known that when the electric spark is led through a mixture of air and hydrogen, there is a combustion of the latter as it unites with a part of the oxygen of the air to form steam. By this combustion the air, as well as the products of combustion, are heated, and, if the sides of the containing vessel are movable, there will be expansion and increase of volume; if not, there will be a considerable increase of pressure. The same is the case when gas is kindled. Such facts, which were undoubtedly known to Lenoir, may have suggested to him the gas engine.

This machine is of simple construction, and in exterior appearance very similar to a steam engine with horizontal cylinder. The principal part is thus a hollow cylinder closed at both ends, in which moves an air-tight piston. This is connected by means of piston and connecting-rods with the crank of a fly-wheel, from which by a belt the power can be taken off.

Let the piston be at the right end of its stroke. By turning the fly-wheel it moves from right to left. While it moves in this direction, a mixture of illuminating gas or of hydrogen and air enters through a slide valve, worked of course by the engine. When a certain quantity of this mixture has entered, the slide shuts in the mixture in the cylinder, and at this moment a spark from an induction coil kindles the gas and causes an increase of pressure. This drives the piston to the other end of its stroke. Arrived at the left end, a second slide allows the products of combustion, together with the accompanying air, to escape, while the fly-wheel, by reason of its living force, goes on and moves the piston in the opposite direction, from left to right. While it moves in this direction the mixture again enters, and is kindled as before, and thus the piston is driven toward the right.

The action is therefore very similar to that of a non-condensing expansion steam engine. But while the greatest work is performed by the entering steam, the work of the entering gas mixture is here zero. During this interval the friction of the engine and the resistances must be overcome by the living force of the fly-wheel.

As the explosion of the gas is instantaneous, the increase of pressure is also immediate. But since the piston can only gradually yield, there is at the moment of firing a shock which is not only hurtful to the moving parts, but which cannot contribute to the motion of the piston or the performance of work. Hence an amount of heat corresponding to this impact is abstracted from the gas and transferred to the cylinder and piston. These parts would soon become very hot if heat were not abstracted. The cylinder is therefore surrounded with a jacket filled with cold water. In this way the temperature of the cylinder sides is kept down to 80° or 90°. It is, however, evident that the performance can by no means be as great as the consumption of gas would warrant.

This objection to the Lenoir engine, viz., that a large part of the power is lost by impact, and yet, in spite of cooling, the piston and stuffing boxes, etc., are considerably heated, has been met by Otto and Langen, of Cologne, in a very ingenious way, and thus a machine produced whose power for the same gas consumption is much greater. We shall discuss it more at length later on.

In order to diminish the heat, Hugon introduced with the gas a quantity of water, which absorbed in vaporizing considerable heat. The expansive force is thus indeed somewhat diminished, but the moving parts suffer less, and the machine requires less repair and is more durable.

In Germany the Lenoir gas engine has dropped out of sight, although in the beginning various establishments sought its improvement, and not indeed without practical results. In

France, however, especially in Paris, it has been applied in the building of houses to the raising of building materials, and in England also it has been used for stone sawing, polishing, pumping, printing, etc.

Gas engines are, as is evident from the foregoing, properly nothing more than hot-air engines with inner fire. But while in the latter the air is heated by coal or coke, in the former it is heated by the combustion of illuminating or hydrogen gas, kindled either by an electric spark as in Lenoir's engine, or by a gas jet as in Langen's and Hugon's. The calculation of the delivery of these engines is therefore similar to that for hot-air engines. Before we pass on to it we shall give a few general considerations which find their application in all gas engines, and which we borrow in large part from the work of Prof. Grashof—"*Resultate der Mechanischen Wärmetheorie,*"—upon which we shall base our future calculations.

The degree of heating of the air in a gas engine depends evidently upon the ratio of illuminating gas and atmospheric air in the mixture, as also upon the composition of the gas. The illuminating gas is a mixture of several others, more especially the following: 1, bicarbureted hydrogen or olefiant gas; 2, carbureted hydrogen or fire-damp; 3, hydrogen; 4, carbonic oxide; and 5, nitrogen. While the two first are combinations of carbon with hydrogen, the carbonic oxide consists of carbon and oxygen. During complete combustion, the carbon forms with the oxygen in the air carbonic acid, while the hydrogen and oxygen unite to form steam. These products are therefore, after the combustion, mixed with the heated air.

One cubic meter of illuminating gas (especially from hard coal) consists on the average of

0.42	cubic	meters of	carbureted hydrogen gas (CH_4),
0.08	"	"	bicarbureted " " (C_2H_4),
0.40	"	"	hydrogen,
0.07	"	"	carbonic oxide, and
0.03	"	"	nitrogen.

The weight of this cubic meter at 0.76 meters of barometer and 15° C. is

$$S_0 = 0.535 \text{ kilograms},$$

while that of 1 cubic meter of air under the same circumstances is

$$s_0 = 1.225 \text{ kilograms.}$$

The density of the gas is therefore

$$d_0 = \frac{S_0}{s_0} = \frac{0.535}{1.225} = 0.4367.$$

1 *kilogram* of gas gives, by complete combustion,

$$K_1 = 10430 \text{ heat units,}$$

and 1 *cubic meter* of gas therefore gives

$$K = S_0K_1 = 0.535 \times 10430 = 5580 \text{ heat units.}$$

The weight of air required for the complete combustion of 1 kilogram of gas is

$$L_1 = 14.5 \text{ kilograms,}$$

and 1 cubic meter of gas requires for complete combustion an air volume,

$$L = \frac{L_1S_0}{s_0} = \frac{14.5 \times 0.535}{1.225} = 6.3 \text{ cubic meters.}$$

If we mix 1 cubic meter of gas with a cubic meters of air, the weight of 1 cubic meter of the mixture is evidently

$$s = \frac{1.225a + 0.535}{a+1} \text{ kilograms.}$$

For example, for $a = 10$

$$s = \frac{12.25 + 0.535}{11} = 1.162 \text{ kilograms.}$$

The density of the mixture is then

$$d = \frac{s}{s_0} = \frac{1.225a + 0.535}{1.225(a + 1)} = \frac{1.225(a + 0.4367)}{1.225(a + 1)} = \frac{a + 0.4367}{a + 1}.$$

For the preceding example

$$d = \frac{10 + 0.4367}{11} = 0.9488.$$

After complete combustion of the gas, the density of the resulting mixture of carbonic acid, steam, nitrogen, and air at 15° C. is

$$D = \frac{a + 0.48}{a + 0.83}.$$

Since here the numerator is greater and the denominator less than in the preceding expression for d, the density has increased.

For example, for $a = 10$ we have

$$D = \frac{10 + 0.48}{10.83} = \frac{10.48}{10.83} = 0.9677,$$

while before combustion it was 0.9488.

In general, then, the increase of density is

$$\frac{D}{d} = \frac{(a + 0.48)(a + 1)}{(a + 0.83)(a + 0.4367)} = \frac{a^2 + 1.48a + 0.48}{a^2 + 1.2667a + 0.3625}.$$

$$\text{For } a = 8, \quad 10, \quad 12, \quad 14, \quad \text{we have}$$

$$\frac{D}{d} = 1.024, \ 1.020, \ 1.017, \ 1.014.$$

The specific heat of the mixture is—

for constant volume

$$c_v = \frac{0.1684a + 0.286}{a + 0.48},$$

for constant pressure

$$c_p = \frac{0.2375a + 0.343}{a + 0.48}.$$

Let us seek now how many heat units are imparted to each kilogram of the mixture of $a + 1$ cubic meters when the gas is kindled.

From experiments we can conclude that during combustion about ¼ to ⅓ of the heat is conducted away by the cooling water, and therefore only ⅔ to ¾ of the gas is used for the heating of the air.

If we assume generally that the αth portion of 1 cubic meter of gas furnishes the heat required for the increase of expansion, this last is

$$\alpha K \text{ heat units.}$$

This is distributed over $(a + 1)$ cubic meters weighing $(a + 1)\,s$ kilograms. Hence each kilogram contains

$$\frac{\alpha K}{(a + 1)\,s} \text{ heat units.}$$

If now, originally, the absolute temperature of the gas mixture is T_0, and immediately after firing T_1, we have

$$T_1 - T_0 = \frac{1}{c}\,\frac{\alpha K}{(a + 1)\,s}.$$

If we denote further, the original pressure at $T_0{}^\circ$ by p_0 (kilograms per square meter), and that after the firing by p_1, we should have, if there were no increase of density,

$$\frac{p_1}{p_0} = \frac{T_1}{T_0}.$$

Since, however, the mixture is increased in density, in the ratio of D to d, we have

$$\frac{D}{d}\,\frac{p_1}{p_0} = \frac{T_1}{T_0},$$

or putting D_1 for $\frac{D}{d}$,

$$D_1\,\frac{p_1}{p_0} = \frac{T_1}{T_0}, \text{ or}$$

$$p_1 = \frac{1}{D_1}\,\frac{T_1}{T_0}\,p_0.$$

EXAMPLE.

According to the experiments of Tresca, there were mixed in a Lenoir gas engine, on the average, 13 parts of air with 1 part of illuminating gas. What is—1, the specific heat c, for constant volume; 2, the increase of temperature $T_1 - T_0$; and 3, what is the expansive force of the mixture after firing?

We have in our formulæ $a = 13$, hence

$$c_v = \frac{0.1684 \times 13 + 0.286}{13 + 0.48} = 0.1836.$$

If we take $\alpha = \frac{2}{3}$,

$$T_1 - T_0 = t_1 - t_0 = \frac{1}{0.1836} \; \frac{\frac{2}{3} \times 5580}{1.225 \times 13 + 0.535} = 1231^\circ.$$

If we take $T_0 = 273 + 15^\circ = 288^\circ$, we have

$$T_1 = 1231 + 288 = 1519^\circ.$$

For $a = 13$

$$D_1 = \frac{D}{d} = \frac{13^2 + 1.48 \times 13 + 0.48}{13^2 + 1.2667 \times 13 + 0.3625} = 1.016.$$

Hence the pressure after firing is

$$p_1 = \frac{1}{1.016} \times \frac{1519}{288} = 5.19 \text{ atmospheres},$$

if p_0 is 1 atmosphere.

Delivery and Gas Consumption of the Lenoir Engine.—The gas behind the piston of this engine drives it forward after firing, by reason of its increased tension, and thus performs work. If the cylinder were not surrounded by cold water, and the gas could therefore expand adiabatically, and if we had to do only with atmospheric air, we should have

$$pv^k = p_1v_1^k = \quad . \quad . \quad .$$

Since, however, heat is abstracted during expansion, and since, also, the value of k (1.41), or the number by which the specific heat of air at constant volume must be multiplied in order to obtain that at constant pressure, is also different, because we have to do with a mixture of air, carbonic acid, steam, etc., the law of change of p and v cannot be represented by this expression. It can, however, be given by the general expression

$$p^mv^n = p_1^mv_1^n \quad \text{(page 197)}.$$

For this case the specific heat, under the assumption that we have to do with air, is

$$\frac{mk - n}{m - n} c_v,$$

where therefore $k = 1.41$ and $c_v = 0.1684$.

Now we have found in the present case, for c_v the value $\frac{0.1684a + 0.286}{a + 0.48}$ (for $a = 13$ this gives $c = 0.1836$) and for the specific heat for constant pressure or c_p we have $\frac{0.2375a + 0.343}{a + 0.48}$, hence

$$k = \frac{c_p}{c_v} = \frac{0.2375a + 0.343}{0.1684a + 0.286} = 1.386.$$

The specific heat for the law

$$p^m v^n = p_1^m v_1^n$$

is therefore, in our case,

$$\frac{m\frac{c_p}{c_v} - n}{m - n} c_v = \frac{mc_p - nc_v}{m - n}.$$

We can now find the delivery during expansion by the formula

$$L = \frac{m}{n - m} p_1 V_1 \left[1 - \left(\frac{V_1}{V}\right)^{\frac{n-m}{m}}\right]$$

if V_1 is the volume of the mixture before, and V that after the expansion, or at the end of the stroke of the piston.

If, now, the cross-section of the cylinder or the area of the piston is F sq. meters, the entire stroke of the piston s, and the distance passed over during entrance of the gas $e_1 s$, where therefore e_1 is a proper fraction, we have

$$\frac{V_1}{V} = \frac{Fe_1 s}{Fs} = e_1.$$

The value of e_1 we call the "*degree of fill.*"

If we insert it in the above formula we have

$$L = \frac{m}{n - m} p_1 F e_1 s \left[1 - e_1^{\frac{n-m}{m}}\right],$$

or

$$L = Fs\left[mp_1 \frac{e_1 - e_1^{\frac{n}{m}}}{n - m}\right].$$

The work of the back pressure of the air is

$$Fp_0(s - e_1 s) = Fp_0 s(1 - e_1).$$

Hence the effective delivery per stroke is

$$Fs\left[mp_1\frac{e_1 - e_1^{\frac{n}{m}}}{n - m} - p_0(1 - e_1)\right].$$

If there are u revolutions per minute, we have for the delivery per second

$$L_s = \frac{u}{30}Fs\left[mp_1\frac{e_1 - e_1^{\frac{n}{m}}}{n - m} - p_0(1 - e_1)\right] \text{ meter-kils. . (LXXIII.)}$$

If we denote the efficiency by y, we have for the actual or useful delivery per second

$$L_u = y\frac{u}{30}Fs\left[mp_1\frac{e_1 - e_1^{\frac{n}{m}}}{n - m} - p_0(1 - e_1)\right] \text{ meter-kilograms.}$$

If we put

$$l = \frac{y}{30}\left[mp_1\frac{e_1 - e_1^{\frac{n}{m}}}{n - m} - p_0(1 - e_1)\right]. \quad . \quad . \quad \text{(LXXIV.)}$$

we have

$$L_u = uFsl \text{ meter-kilograms.}$$

If the horse-power corresponding is N, we have

$$N = \frac{uFsl}{75} \quad . \quad . \quad . \quad . \quad . \quad \text{(LXXV.)}$$

The consumption of gas per revolution is found from the proportion

$$a + 1 : 1 = 2Fe_1 s : x \quad \text{or} \quad x = \frac{2Fe_1 s}{a + 1}.$$

The consumption per second is then

$$\frac{uFe_1 s}{30(a + 1)}.$$

Hence the gas consumption for each horse-power per second is

$$\frac{uFe_1s}{30(a+1)N},$$

and per horse-power per hour

$$\frac{120uFe_1s}{(a+1)N}.$$

Substituting the value of N

$$G = \frac{9000}{a+1}\frac{e_1}{l} \text{ cubic meters. } \quad . \quad (\text{LXXVI.})$$

According to Grashof, the above formulæ give results agreeing well with experiment, when we put $m = 1$ and $n = 2$. Hence the law of pressure variation is given by the simple equation

$$pv^2 = p_1v_1^2 = \cdot \cdot \cdot \cdot$$

The curve representing this law approaches the axis of abscissas more rapidly than the adiabatic (page 194). If we insert these values of m and n in Equation LXXIV., we have

$$l = \frac{y}{30}[p_1(e_1 - e_1^2) - p_0(1 - e_1)],$$

or putting $p_0 = 10334$,

$$l = 344.4y(1 - e_1)\left(\frac{p_1}{p_0}e_1 - 1\right) \quad . \quad (\text{LXXVII.})$$

If we determine, according to this formula, the value of l for different values of e_1, and then find the corresponding values of G, it will appear that only for a certain value of e_1 will the economy, or the ratio of the delivery to the consumption of gas, be a maximum.

Thus for $a = 13$, we have, as already found,

$$\frac{p_1}{p_0} = 5.19.$$

The efficiency y is on the average 0.57. If we take it at 0.55, we have, putting for e_1 the values

$$e_1 = 0.40 \qquad 0.45 \qquad 0.50 \qquad 0.55 \qquad 0.60,$$

for l

$$l = 122.4 \qquad 139.1 \qquad 151.1 \qquad 158.1 \qquad 160.2,$$

and for G

$$G = 2.12 \qquad 2.08 \qquad 2.13 \qquad 2.24 \qquad 2.41.$$

Hence we see that the economy is greatest for e_1 about 0.50. Under the above assumptions, then, the cylinder must be half filled with the mixture before it is fired.

EXAMPLE 1.—What is the delivery of a gas engine, the diameter of cylinder being 2 decimeters, and stroke 4 decimeters, when $e_1 = 0.50$, and the number u of revolutions per minute is 60, the efficiency being taken at 0.25?

The cross-section of the cylinder, or the area F of the piston, is

$$F = 3.1416 \times (0.1)^2 = 0.0314 \text{ sq. meters.}$$

Since in the present case

$$l = 151.1,$$

we have for N

$$N = \frac{60 \times 0.0314 \times 0.4 \times 151.1}{75} = 1.52 \text{ horse-power.}$$

The gas consumption per hour would therefore be

$$1.52 \times 2.13 = 3.2376 \text{ cubic meters.}$$

EXAMPLE 2.—What must be the volume of the cylinder of a two-horse-power engine, which makes 70 revolutions per minute, when e_1, or the coefficient of fill, and y, or the efficiency, are both 0.55?

From the formula $N = \frac{uFsl}{75}$, we have

$$Fs = \frac{75N}{ul}. \text{ In the present case, } l = 158.1, \text{ hence}$$

$$Fs = \frac{75 \times 2}{70 \times 158.1} = \frac{150}{11067} = 0.0136 \text{ cubic meters.}$$

Quantity of Cooling Water.—It remains now to calculate the amount of heat which must be abstracted from the cylinder for every cubic meter of gas used.

We assume that directly upon firing, so much heat is at once abstracted as is contained by $1 - \alpha$ cubic meters of gas. This heat is

$$(1 - \alpha) k \text{ heat units.}$$

If we put $\alpha = \frac{2}{3}$, we have

$$\tfrac{1}{3} \times 5580 = 1860 \text{ heat units.}$$

Heat is now abstracted from the gas mixture during expansion, since the law of pressure variation is given by

$$pv^2 = p_1 v_1^2.$$

In order to find this heat, we must first determine the temperature at the end of expansion or at the end of the stroke. This, as well as the final pressure, can be easily found.

Denote the last by p, then, according to our law,

$$p = p_1 \left(\frac{V_1}{V}\right)^2 = p_1 e_1^2.$$

If, for example, $e_1 = 0.50$, then

$$p = 5.19\,(0.50)^2 = 5.19 \times \tfrac{1}{4} = 1.297 \text{ atmospheres.}$$

The temperature T corresponding to this pressure is found from

$$\frac{T}{T_1} = \left(\frac{V_1}{V}\right)^{\frac{n-m}{m}} = e_1.$$

Since $T_1 = 1519$ (page 298), we have

$$T = 1519 \times 0.50 = 759.5^\circ \quad \text{or}$$

$$t = 759.5 - 273 = 486.5^\circ.$$

If we take $e_1 = 0.45$, we have

$$p = 5.19\,(0.45)^2 = 5.19 \times 0.2025 = 1.051 \text{ atmospheres,}$$

and the absolute temperature T at the end of stroke is

$$T = 1519 \times 0.45 = 683.6^\circ \quad \text{or}$$

$$t = 683.6 - 273 = 410.6^\circ.$$

We see, therefore, that even at the end of expansion, and in spite of the withdrawal of heat by the cooling water, the temperature of the gas mixture is still considerable, and hence that in course of time the engine may suffer injury.

The heat abstracted during expansion is found as follows: From a previous formula, the heat abstracted for 1 kilogram of air is

$$Q = \frac{mk - n}{m - n} c (T_1 - T).$$

In our present case, where $k = 1.386$, $m = 1$, $n = 2$, and $c = 0.1836$, we have

$$Q = \frac{1.386 - 2}{1 - 2} \times 0.1836 (T_1 - T) = 0.1127 (T_1 - T).$$

For $e_1 = 0.50$, we found $T = 759.5$, hence

$$Q = 0.1127 (1519 - 760) = 85.5 \text{ heat units.}$$

The gas mixture of $a + 1$ cubic meter weighs

$$1.225a + 0.535 \text{ kilograms,}$$

or since we have taken $a = 13$, 16.46 kilograms. Hence the heat abstracted during expansion from each cubic meter of mixture is

$$85.5 \times 16.46 = 1407 \text{ heat units.}$$

After the exhaust valve is opened, the pressure falls rapidly from p to that of the atmosphere p_0, and that portion of the heat disappears which went to increase the rectilinear motion of the gas molecules. The mixture then flows out under the constant pressure p_0.

If, disregarding this loss of heat, we allow the gas at the absolute temperature $T_2 = 273 + 150 = 423°$ to issue under constant pressure p_0, we must abstract

$$(1.225a + 0.535) c_1 (T - T_2) \text{ heat units.}$$

Since $c_1 = 0.2545$, and T in our example is 759.5 or 760, we have

$$16.46 \times 0.2545 \times 337 = 1414 \text{ heat units.}$$

Hence the entire heat abstracted is

$$1860 + 1407 + 1414 = 4681 \text{ heat units.}$$

Since a one-horse-power gas engine, with coefficient of fill of 0.50, requires per hour 2.13 cubic meters gas, the heat abstracted is

$$4681 \times 2.13 = 9970 \text{ heat units.}$$

If the cooling water is heated 50°, say from 10° to 60°, we have for the weight of water W which must circulate per hour around the cylinder,

$$50W = 9970 \quad \text{or} \quad W = 199.5 \text{ kilograms.}$$

This weight is, however, too great, as we disregard, as above remarked, that heat which disappears suddenly when the exhaust valve opens.

Hugon's Engine.—In this the heating of the gas mixture is less, by reason of the cold water injected into the cylinder, than in the Lenoir engine, and the piston packing suffers less. According to Grashof, we have here for l (page 300),

$$l = 344.4y\,[7.21\,(e_1 - e_1^{1.6}) - 1 + e_1] \quad . \quad . \quad \text{(LXXVIII.)}$$

where y can be taken at 0.55.

If $a = 13$, and the weight of water injected per cubic meter of gas used is 2 kilograms, we have, according to the same author, $\frac{p_1}{p_0} = 4.33$, and in the expression which gives the law of variation of pressure, we have $n = 1.6$.

For e =	0.4	0.45	0.5	0.55
l =	117.4	129.8	137.6	141.2
and G =	2.19	2.23	2.34	2.50 cubic meters.

The gas jets for firing require about 0.25 cubic meters of gas per hour additional.

From the values of l we can find the delivery by the same formula as before,

$$N = \frac{uFsl}{75} \text{ horse-power.}$$

Description of the Atmospheric Gas Engine.—This engine is, beyond doubt, one of the finest inventions of the century, and was justly awarded the gold medal by the jury at the Exposition in Paris, 1867. In fact, the inventor has overcome in a very ingenious manner the disadvantages connected with the engines of Lenoir and Hugon, and the essential principles of the machine are based upon the results of scientific investigation. The construction also, in the opinion of the author, leaves little to be desired.

The manner in which the inventor has happily overcome the defects of the older gas engines is best told in his own words (*Dingler's Polyt. Journal*, Bd. clxxxvi.).

"By the combustion of explosive mixtures in a confined space, the products of combustion are heated by the heat disengaged, and in consequence have a tendency to expand, which, when prevented, gives rise to a pressure upon the inclosing walls corresponding to the temperature. This pressure exists so long as the combustion products lose no heat.

"If they are cooled, they contract under the pressure of the atmosphere, which directly or indirectly surrounds them.

"If it is wished to utilize the pressure as a motive force, the question arises, what time elapses between the heating and cooling, between the expansion and contraction of the gases.

"This time is known to be very short, and we lose a portion of the heat generated by combustion, by conduction and radiation through the cylinder sides, when we endeavor to utilize the expansion of the heated gases, if we do not allow them to expand quickly after the combustion. This loss of heat corresponds, of course, to the loss of a portion of motive power.

"If, we conceive an engine as ordinarily constructed, that is, with piston connected by connecting-rod and crank with a fly-wheel, we shall have opposed to the explosion in the cylinder, back of the piston, not only the useful work, but also the mass of the entire system. Such an engine must work with enormous velocity, and the action must be an impulsive one, and since the moving masses can never take an acceleration corresponding to the intensity of the explosion, the heat not utilized must heat considerably the inclosing walls of the cylinder.

"Guided by such considerations, we have adopted in the

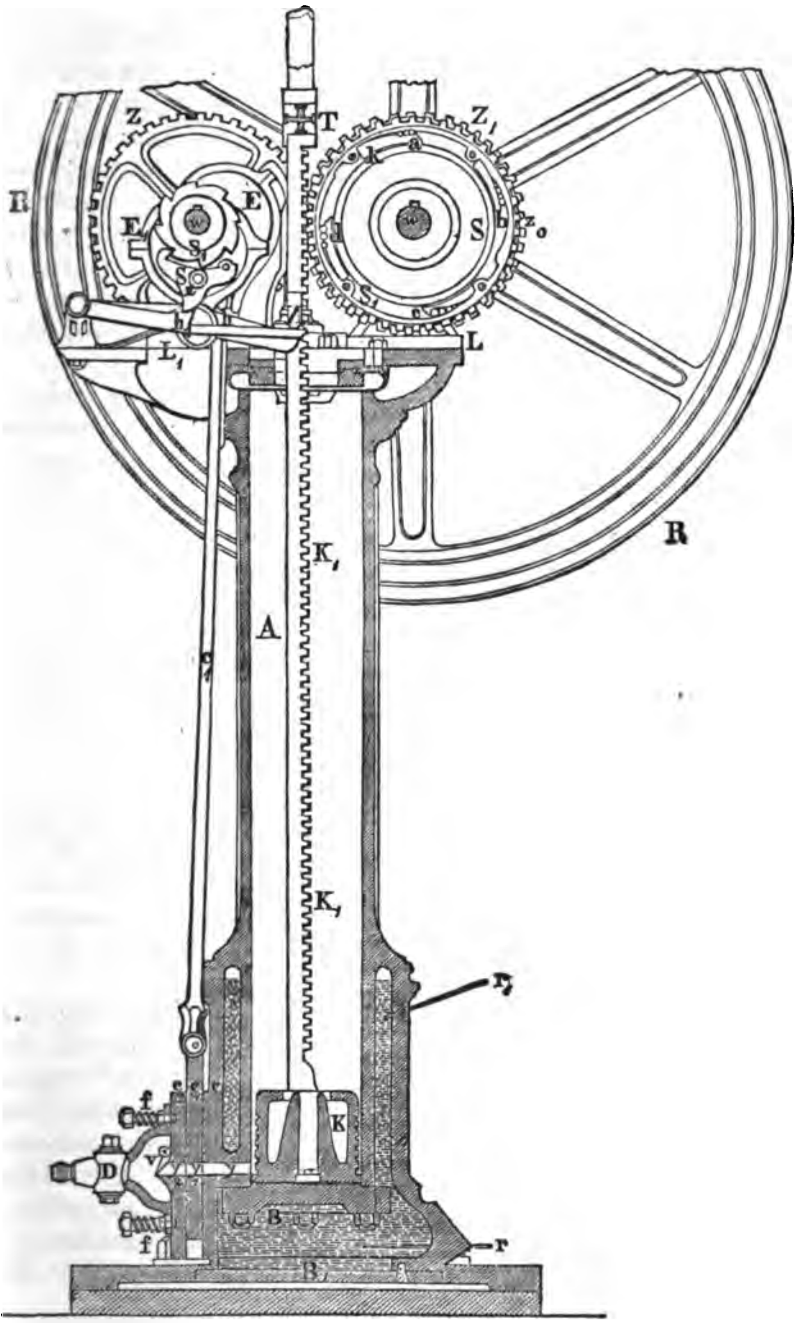


FIG. 45.

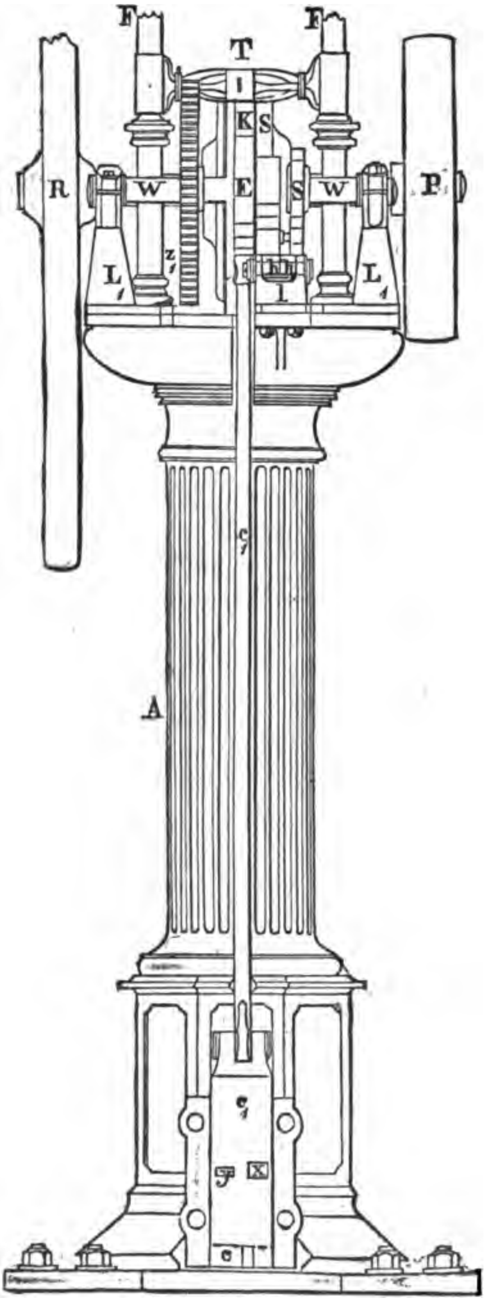


FIG. 46.

construction of our engine the principle that a direct utilization of the explosive force must be discarded.

"On the contrary, we utilize the heat set free by the explosion, by opposing to the expansion of the products of combustion but very slight resistance, and by employing, as the motive power, the contractile action. Thus the gases, as soon as they have lost their heat, and hence their tension, are compressed by the pressure of the atmosphere to that volume which, after cooling, corresponds to their temperature and chemical constitution.

"With these preliminary remarks we can now explain more in detail the construction and action of the engine.

"We shall make use for this purpose of Figs. 45 to 54. Fig. 45 is a vertical section, and Fig. 46 gives a vertical projection and other details of construction.

"*A* is a cast-iron cylinder, with two air-tight screw covers B and B_1. This cylinder is surrounded up to about ⅓d of its height with a double wall, the space between, as well as between B and B_1, being filled with cold water, which serves to cool the cylinder walls. This water is brought from a reservoir, not shown in our Figure, through the pipe r, and the warm water carried off through r_1. This simple circulation of the water is sufficient, in order to keep the cylinder jacket always at a low temperature, without any renewal of the cooling water being necessary.

"K is a metal piston, which can move air tight up and down in the cylinder. At every firing of the gas mixture this is shot up the cylinder. Thus the mixture expands suddenly and cools very quickly, so that its tension falls far below that of the atmosphere, and the piston is driven down by the outer air pressure. This last motion is that which is transferred to the engine and utilized. The transference is effected as follows:

"The piston K is fastened to a toothed rod K_1, which ends above in a cross-head T (Figs. 45 and 46), moving up and down vertically in guides F, F, only partly shown. Thus the vertical motion of the piston is insured.

"The cylinder plate carries two pair of standards, of which only one pair L_1 L_1 are visible in our Figure. These support the shaft W, which carries on the left the fly-wheel R, and on the right the belt pulley P. At the middle of the shaft the disc S is keyed, a cross-section of which is given in Fig. 47. Upon the prolonged bars of this disc are two loose discs S_1 S_1, between which a crown-wheel is fastened by means of four bolts which may be seen in Fig. 45. The teeth of the piston-rod K_1 engage with the teeth of this crown-wheel. When the piston and rod move upward, the crown-wheel and both discs S_1 and S_1 turn from left to right, and revolve loosely upon the boss of the middle disc S. When the piston and rod descend, the crown-wheel is made by a special arrangement to grip tightly the circumference of the disc S, and thus to impart motion to the shaft. The construction of this special arrangement is different according to the power of the engine.

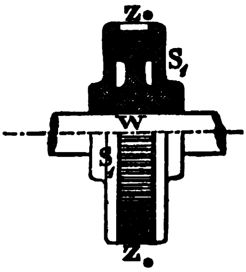


Fig. 47.

"In engines of small size (¼ horse-power) the inner surface of the crown-wheel carries a number of eccentric surfaces *ab*, *ab*, etc. (Fig. 48). Between these and the circumference of the disc there are the same number of rollers, *c*, *c*, etc. If the crown turns in the direction of the arrow, as it does when the piston descends, those surfaces which lie nearest the center *m*, press down the rollers upon the disc and compel it to take part in the motion. If the crown revolves in the reverse direction, there is no such pressure, and the small rollers *c*, *c*, roll upon the circumference of the disc.

"In larger engines the construction is similar. Here the rollers, however, do not lie directly upon the circumference of the disc, but upon the upper surface of four wedge-shaped bodies, *a*, *b*, *c*, *d* (Fig. 45). Here, also, the portions of the eccentric surfaces nearest the center, during motion from right to left, press the rollers against the wedges, and these upon the circumference of the disc, so that motion is transferred.

"If the wedges are not to slide upon the periphery of the disc *S*, the angle which the eccentric surfaces of the crown-wheel make with the circumference of the disc must be less than the angle of friction of the metals.

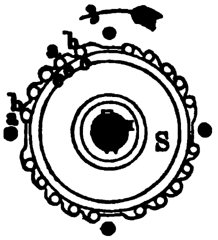


Fig. 48.

"Upon the shaft, *W* there is, besides the toothed wheel Z_0, also another one, Z_1. This engages with the wheel *Z* (Fig. 45), which is keyed to a second shaft *w*, parallel with *W*. Upon *w* is the ratchet wheel S_1, while the two eccentrics *E* and E_1 form one piece and revolve loosely upon the shaft. The catch S_2 couples or disconnects the eccentrics with the shaft *w*, according as it engages with the ratchet wheel S_1 or is shoved back by the bolt h_1.

"As the piston descends, the projection *N* upon the piston-rod strikes down the lever h_1, raises the left side of the catch S_2, which then engages with a tooth of the ratchet wheel S_1, and then the catch and both eccentrics *E* and E_1 partake of the motion of the shaft *w*. If now the number of strokes of the piston is equal to the revolutions of the shaft or fly-wheel, which, as we shall see, is by no means necessary, h_1 is raised during a revolution and the catch S_2 released.

"As soon as the catch is released, which must always occur when the piston is not far from the bottom of the cylinder, both eccentrics come to rest, and the slide C_1, which is moved by the eccentric *E*, has its central position. In this position the port y_1 (Fig. 49) corresponds with *y* in the cylinder jacket and y_2 in the cover C_2. This is pressed by springs *f*, *f*, upon the slide, so that air cannot get between the surfaces.

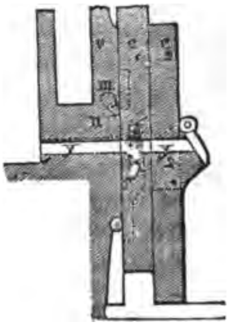


FIG. 49.

"While the slide C_1 remains thus a short time in its central position, the piston completes the lower portion of its stroke and presses the products of combustion out through the ports and valve *v* into the air. At the same time the lever h_1 is depressed, the catch S_2 engages the ratchet wheel S_1, and both eccentrics partake of the motion of the shaft. One of these, *E*, which moves the slide, moves downward, as is seen from Fig. 45, and the slide moves down. The other, E_1, moves up. As this moves the lever *h*, which in our figure is behind h_1, and the right end of which is below the tappet *N*, the lever and piston are raised. During this motion of the piston the descending slide has closed the canal *y*, and made another connection, which is represented in Fig. 50. Here the canal *x* in the cylinder side, which lies near *y* (Fig. 46), communicates with the channel *a* of the slide C_1, and thus by *m* and *n* with the outer air and a gas receiver. Thus as the piston is raised a mixture of gas and air enters the cylinder.

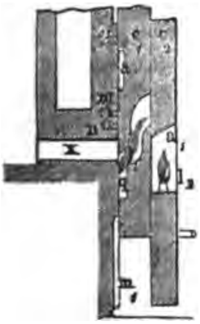


FIG. 50.

"As, however, the slide c_1 moves down, the canal *q* has been in communication with *m* and *n*, and is therefore also filled with a mixture of gas and air. When the piston has its lowest position, this canal communicates, as we see in Fig. 50, with the opening *a* in the cylinder, in which a lamp l_2 is burning, and thus the mixture kindled in *q*. Now the slide moves rapidly upward and takes the position shown in Fig. 51. The kindled gas in *q* is still burning, and so soon as *q* thus communicates with the canal *x*, the gas mixture under the piston in the cylin-

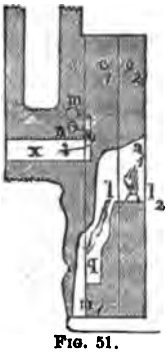


FIG. 51.

der is also kindled. By the expansive force developed by the explosion the piston is shot up with great velocity, and the eccentric E raises the slide somewhat more, and then returns it to its central position. The lever h_1 and detent S_2 then release the eccentric, and this as well as the slide remain in their positions until the above operation is repeated.

"Let us now point out how the action can be regulated, or the number of strokes made independent of the number of revolutions of the crank.

"In the gas conductor is a cock by which the relative proportion of gas and air can be so regulated that the piston is shot up by the explosion only a certain distance. We can thus alter the performance of the engine at pleasure. Since, however, the useful effect of the engine is greatest for a certain height of throw, it is desirable so to regulate the force exerted that the throw of the piston may be always the same, and hence independent of the delivery of the engine in a given time.

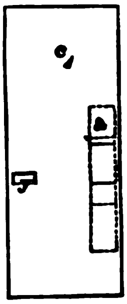


FIG. 52.

"The inventors accomplish this by making the number of piston strokes independent of the number of revolutions of the axis, regarded as constant. For this purpose they have a cock D (Fig. 45) in the end of the exhaust pipe, that is the pipe through which the products of combustion escape, which allows these products to escape more or less rapidly. By this arrangement we can cause the piston to descend more or less rapidly. Since the piston is pressed down by the difference of pressure of the atmosphere and inclosed gas mixture, it moves with the velocity of the periphery of the crown-wheel so long as the pressure in the cylinder is less than one atmosphere. As soon as this pressure is reached, the piston sinks by virtue of its own weight. If now D is but little opened, it sinks more slowly, if much opened, more rapidly, while the velocity of the periphery of the crown-wheel and disc is nearly constant. If D is fully opened, the engine works with its maximum power. If the cock is closed so much that the escape of the products of combustion is retarded, h_1 remains longer in its raised position, and the valve works more slowly. The position of the cock *D* depends therefore upon the delivery desired. Hence, under irregular resistance, the revolutions of the fly-wheel can be regulated by this cock.

"In Figs. 52 to 54, we have represented the slide and slide surfaces on the cylinder and cover.

"If we wish an automatic regulation, it is only necessary to fit the cock *D* with a governor run by the shaft in the usual manner."

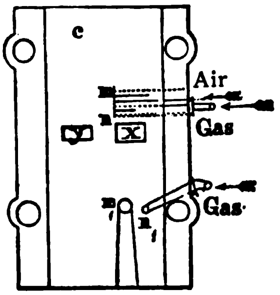


FIG. 53.

From this description it will not be disputed that the construction of the atmospheric gas engine is very complete, and that it deserves to be placed among the most ingenious inventions of the century, a century which includes also the engine of Ericsson.

The inventors add to their description in the *Journal* already cited the following:

"The atmospheric gas engine differs essentially from earlier gas engines in the following points:

"1. Regard is had to the physical principles noticed in the introductory remarks.

"2. The action of the piston is intermittent.

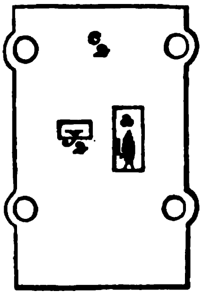


FIG. 54.

"3. A special construction transfers the downward motion of the piston to the fly-wheel shaft.

"4. The construction of the valve motion and the slide is essentially different from other gas engines.

"5. By changing the number of strokes of piston for a constant number of revolutions of the shaft, the performance can be regulated."

That which distinguishes in other respects this gas engine from ordinary hot-air engines is, that it is very easily set in action, and requires very little attendance. It is equally safe with all hot-air engines.

It has the disadvantage, as compared with hot-air engines, that it can only be set up in places where illuminating gas can be obtained or manufactured without great cost.

Theory of the Atmospheric Gas Engine.—Let T_0 be the absolute temperature of a gas mixture, consisting of a cubic meters of air and 1 cubic meter of gas, and T_1 the absolute temperature after firing, then the increase of temperature is, according to page 297, where it is assumed that of the heat which every cubic meter of gas furnishes by its combustion, $1-\alpha$ heat units are directly withdrawn,

$$T_1 - T_0 = \frac{1}{c} \; \frac{\alpha k}{1.225a + 0.535},$$

where $c = \dfrac{0.1684a + 0.286}{a + 0.48}$.

According to experiment, the ratio of air to gas is 8 to 1. If, then, we take $a = 8$, we have

$$c = \frac{0.1684 \times 8 + 0.286}{8.48} = 0.190.$$

If we put now $\alpha = \frac{2}{3}$, we have, since $k = 5580$,

$$T_1 - T_0 = \frac{1}{0.19} \; \frac{\frac{2}{3} \times 5580}{10.335} = \frac{3720}{1.964},$$

or

$$T_1 - T_0 = 1894^\circ.$$

For $T_0 = 273 + 15 = 288^\circ$, we have

$$T_1 = 1894 + 288 = 2182^\circ.$$

The increase of density D_1 is, from page 296,

$$D_1 = \frac{a^2 + 1.48a + 0.48}{a^2 + 1.2667a + 0.3625} = 1.024.$$

Hence the tension after the firing is

$$p_1 = \frac{1}{D_1} \frac{T_1}{T_0} p_0.$$

Inserting numerical values

$$p_1 = \frac{1}{1.024}\,\frac{2182}{288} = \frac{1}{1.024} \times 7.58 = 7.4 \text{ atmospheres (nearly).}$$

The law of variation during rise of piston may be represented by the equation

$$pv^n = p_1 v_1^n.$$

(In the Lenoir engine we had $n = 2$.)

The work which a gas volume performs by its expansion when the law of variation of pressure with volume is

$$p^m v^n = p_1^m v_1^n$$

is, from preceding principles,

$$L = \frac{m}{n-m} p_1 v_1 \left[1 - \left(\frac{V_1}{V}\right)^{\frac{n-m}{m}}\right].$$

Since we have assumed $m = 1$, in the present case

$$L = \frac{1}{n-1} p_1 V_1 \left[1 - \left(\frac{V_1}{V}\right)^{n-1}\right],$$

where V_1 is the least and V the greatest volume, or V_1 the volume before and V that after expansion.

If we denote now the cross-section of the cylinder by F square meters, the height to which piston is shot up measured from bottom of cylinder by s meters, the height which the gas mixture occupies before it is fired by $e_1 s$ meters, where e_1 is a proper fraction, then

$$\frac{V_1}{V} = \frac{Fe_1 s}{Fs} = e_1,$$

and we have again as before,

$$L = \frac{1}{n-1} Fse_1 p_1 (1 - e_1^{n-1}).$$

Inserting $n-1$, e_1 and p_1 in the parenthesis, and denoting the atmospheric pressure by p_0, we have

$$L = Fsp_0\left[\frac{p_1}{p_0}\frac{e_1}{n-1}(1-e_1^{n-1})\right].$$

This delivery goes in several directions.

A portion of it imparts living force to the piston, and shoots it up to a certain height. Another goes to overcome the resistance of the air, and a third the piston friction.

The overcoming of the resistance of the air requires the work

$$Fsp_0(1-e_1).$$

If we denote the piston friction by R, we have the work for overcoming it

$$Rs(1-e_1),$$

or if, with Grashof, we put $R = \rho Fp_0$,

$$\rho Fp_0 s(1-e_1).$$

If further, P is the weight of the piston and rod, the work required to raise it to the height $s - e_1 s = s(1-e_1)$ is

$$Ps(1-e_1).$$

If here we put $P = \varphi Fp_0$, we have

$$\varphi Fp_0 s(1-e_1).$$

These three works together must equal that of the gas during its expansion. Hence

$$Fsp_0\left[\frac{p_1}{p_0}\frac{e_1}{n-1}(1-e_1^{n-1})\right]$$

$$= Fsp_0[1-e_1+\rho(1-e_1)+\varphi(1-e_1)],$$

or

$$\frac{p_1}{p_0}\frac{e_1}{n-1}(1-e_1^{n-1}) = (1+\rho+\varphi)(1-e_1).$$

Putting for the sake of brevity $\frac{p_1}{p_0} = m$,

$$\frac{1 - e_1^{n-1}}{n-1} = \frac{(1 + \rho + \varphi)(1 - e_1)}{me_1}.$$

Since now ρ, φ, e, and m may be regarded as known, we can determine n from this equation.

According to experiments by Meidinger, in an engine of the kind in question of ½ horse-power, $F = 0.01767$ sq. meters, $s = 0.99$ meters (full rise of piston), $e_1 = 0.114$ meters, $P = 21.8$ kilograms, and $R = 7$ kilograms.

Hence

$$P = \varphi F p_0 = 21.8, \quad \text{or} \quad \varphi = \frac{21.8}{0.01767 \times 10334} = 0.119.$$

Further

$$R = \rho F p_0 = 7, \qquad \text{or} \quad \rho = \frac{7}{0.01767 \times 10334} = 0.038.$$

Substituting these values in our last equation,

$$\frac{(1 + 0.038 + 0.119)(1 - 0.114)}{7.4 \times 0.114} = \frac{1 - e_1^{n-1}}{n-1}$$

$$1.215 = \frac{1 - e_1^{n-1}}{n-1} \quad \text{or} \quad 1.215n - 2.215 = -e_1^{n-1},$$

$$e_1^{n-1} + 1.215n = 2.215.$$

This equation gives $n = 1.60$ very nearly, thus

$$(0.114)^{0.60} + 1.115 \times 1.60 = 2.2157,$$

or only $\frac{7}{10000}$ too great.

Hence the equation

$$pv^{1.60} = p_1 v_1^{1.60}$$

gives the law of variation of pressure with volume when the piston rises.

As soon as this law is known we are able to calculate the pressure and temperature at the end of the piston rise. If we denote this pressure by p, we have

$$p = p_1\left(\frac{v_1}{v}\right)^{1.60}.$$

Since $p_1 = 7.4$ and $\frac{v_1}{v} = 0.114$, we have

$$p = 7.4\,(0.114)^{1.60} = 7.4 \times 0.03098 = 0.2293 \text{ atmosphere.}$$

That is, *the final pressure is far below the atmospheric.*

For the sake of clearness we have represented graphically in Fig. 55 the law of change of the gas during the piston rise. *Oe* is the distance of the piston from the bottom of the cylinder at the moment the gas is fired. *Of* is about three, *Og* six times this distance, and *Oh* is the entire rise. At *f* the pressure is about 1.35 atmospheres. This is represented by the line *fb*. At *g* the pressure is *gc* = 0.45. Joining the points *abcd* we obtain a curve which gives the change of pressure with volume. We see that at a distance of hardly 0.33 meters above the cylinder bottom the pressure has already sunk to one atmosphere. The line *cm* = *hn* represents the pressure of the atmosphere increased by the weight of the piston. From *e* to *k*, the area *abim* denotes the excess of work of the gas above that required for overcoming the air pressure, the piston friction, and for raising the piston. This excess imparts to the piston its living force, by virtue of which it continues to rise from *k* to *h*, or from *i* to *n*, overcoming the resistance of the air and piston friction. During this the inclosed gas furnishes indeed a work represented by *icdhk*. If therefore we add this work to the living force of the piston, we have the work which goes to overcome the air resistance, the piston friction, and to raise the piston.

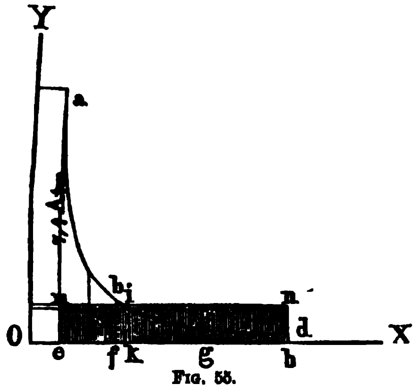


Fig. 55.

If now T is the absolute temperature at the highest position of the piston, we have

$$\frac{T}{T_1} = (0.114)^{\frac{1.60-1}{1}} = (0.114)^{0.60} = 0.2717.$$

Since $T_1 = 2182^\circ$,

$$T = 2182 \times 0.2717 = 593^\circ,$$

or

$$t = 593 - 273 = 320^\circ,$$

a temperature still pretty high.

The useful work which the engine furnishes is now performed during the descent of the piston, by its weight and by the air pressure. This work is not entirely applied to moving the engine however. A part serves to compress the inclosed air again to the pressure of the atmosphere, and then to drive it out of the exhaust valve. Since this compression takes place with the velocity of the periphery of the crown-wheel, and hence relatively rather slowly, a part of the heat set free has time to radiate from the cylinder walls, and another part is absorbed by the cold water which surrounds the cylinder up to about ⅓d of its height. In consequence of this the temperature sinks during the compression, and the departing products of combustion have, according to Meidinger, only about 200° temperature, or an absolute temperature of 200 + 273 = 473°. Another part of the work goes to overcome the piston friction.

We can now easily calculate at what height of piston above the cylinder bottom the exhaust valve is opened, or at what height the products of combustion are compressed back to one of atmosphere.

From known principles the weight G of a volume of air V, pressure p and absolute temperature T, is

$$G = \frac{pV}{RT}.$$

In our engine, where the air is not pure, but a mixture of air, carbonic acid, etc., R has evidently a different value. If we put for it R_1, the weight G in the moment after firing is

$$G = \frac{p_1 F e_1 s}{R_1 T_1}.$$

Assuming that there is no loss of air, this weight must remain the same at every position of the piston, and therefore when the mixture is compressed to the atmospheric pressure p_0. If, then, the absolute temperature is T_2 and the gas volume is Fe_2s, e_2s being the distance of the piston from the cylinder bottom, we have

$$G = \frac{p_0 F e_2 s}{R_1 T_2}.$$

From these two equations we obtain

$$\frac{p_1}{p_0} = \frac{e_2}{e_1}\frac{T_1}{T_2} \quad \text{or} \quad e_2 = \frac{p_1}{p_0}\frac{e_1 T_2}{T_1}.$$

Since now, p_1, e_1, T_2, and T_1 are known, we can calculate e_2. Inserting numerical values, we have

$$e_2 = 7.4\ \frac{0.114 \times 473}{2182} = 0.183.$$

As $s = 0.99^{\text{m.}}$, the distance e_2s is

0.183 × 0.99 = 0.181 meters, or 18.1 centimeters.

Let the law of change during the descent of the piston be

$$pv^{n_1} = p_0 v_0^{n_1},$$

p and v being specific pressure and volume at the highest position, and p_0v_0 at the height e_2s of piston.

Then

$$\frac{p}{p_0} = \left(\frac{V_0}{V}\right)^{n_1} = \left(\frac{Fe_2s}{Fs}\right)^{n_1} = e_2^{n_1}.$$

For the rise of the piston we had

$$pv^n = p_1v_1^n,$$

or

$$\frac{p}{p_1} = \left(\frac{V_1}{V}\right)^n = \left(\frac{Fe_1s}{Fs}\right)^n = e_1^n.$$

If we divide the two equations for $\frac{p}{p_0}$ and $\frac{p}{p_1}$ one by the other, we have

$$e_2^{n_1} = \frac{p_1}{p_0}e_1^n.$$

From this we can calculate n_1.
Inserting numerical values

$$(0.183)^{n_1} = 7.4 \times 0.114^{1.60},$$

or

$$n_1 \log 0.183 = \log 7.4 + 160 \log 0.114.$$

Hence

$$n_1 = \frac{\log 7.4 \times 1.60 \log 0.114}{\log 0.183} \quad \text{or} \quad n_1 = 0.866.$$

Therefore the law of relation between pressure and volume during the descent of the piston is

$$pv^{0.866} = p_0v_0^{0.866}.$$

Now we can calculate the mechanical work necessary for compressing the products of combustion from p to p_0. This is

$$L = \frac{1}{n_1 - 1}p_0Fe_2s\,(1 - e_2^{n_1-1})$$

$$= Fsp_0 \times \frac{e_2 - e_2^{n_1}}{0.861 - 1} = Fsp_0\frac{e_2 - e_2^{n_1}}{-0.134} = Fsp_0\frac{e_2^{n_1} - e_2}{0.134}.$$

Further, the air pressure and descending weight have to overcome the piston friction. This is

$$\rho F s p_0 (1 - e_2).$$

Hence the work required by both resistances is

$$F s p_0 \left[\rho (1 - e_2) + \frac{e_2^{n_1} - e_2}{0.134} \right].$$

The work furnished by the air pressure and descending weight is, however,

$$F s p_0 [1 - e_2 + \varphi (1 - e_2)] = F s p_0 (1 + \varphi)(1 - e_2).$$

If we subtract the preceding, we have the theoretical effective delivery per revolution,

$$F s p_0 \left[(1 + \varphi)(1 - e_2) - \rho (1 - e_2) - \frac{e_2^{n_1} - e_2}{0.134} \right]$$

$$= F s p_0 \left[(1 + \varphi - \rho)(1 - e_2) - \frac{e_2^{n_1} - e_2}{0.134} \right].$$

If there are u revolutions per minute, the delivery per second is

$$L_e = \frac{u}{60} F s p_0 \left[(1 + \varphi - \rho)(1 - e_2) - \frac{e_2^{n_1} - e_2}{0.134} \right] \text{ meter-kilograms.}$$

Or, since $p_0 = 10334$,

$$L_e = 172.23 F s u \left[(1 + \varphi - \rho)(1 - e_2) - \frac{e_2^{n_1} - e_2}{0.134} \right]. \quad \text{(LXXX.)}$$

In horse-power

$$N = 2.296 F s u \left[(1 + \varphi - \rho)(1 - e_2) - \frac{e_2^{n_1} - e_2}{0.134} \right]. \quad \text{(LXXXI.)}$$

EXAMPLE.

What is the theoretical delivery of the atmospheric gas engine experimented upon by Professor Meidinger, for which $F = 0.01767$ square meters, $s = 0.99^{m.}$, $e_1 = 0.114$, $P = 21.8$ kilograms, $R = 7$ kilograms, $u = 34$, and $T_2 = 273 + 200 = 473°$?

First, according to the preceding calculations, $1 + \varphi - \rho = 1 + 0.119 - 0.038 = 1.081$. Further, $1 - e_2 = 1 - 0.183 = 0.817$. Hence $(1 + \varphi - \rho)(1 - e_2) = 1.081 \times 0.817 = 0.8832$.

Then $e_2^{n_1} = 0.2297$ and $e_2^{n_1} - e_2 = 0.2297 - 0.183 = 0.0467$.

Therefore $\frac{e_2^{n_1} - e_2}{0.134} = \frac{0.0467}{0.134} = 0.349$. Hence

$$(1 + \varphi - \rho)(1 - e_2) + \frac{e_2^{n_1} - e_2}{0.134} = 0.8832 - 0.349 = 0.5342.$$

For Fsu we have

$$0.01767 \times 0.99 \times 34 = 0.5947, \quad \text{therefore}$$

$$Fsu\left[(1 + \varphi - \rho)(1 - e_2) - \frac{e_2^{n_1} - e_2}{0.134}\right] = 0.5947 \times 0.5342 = 0.3177.$$

If this result is finally multiplied by 172.23, we have for the theoretical delivery in meter-kilograms 54.75.

Experiment gave 40 meter-kilograms, and therefore the efficiency is

$$\frac{40}{54.75} = 0.73.$$

According to Grashof, the efficiency of the engine in question, when oiled carefully and at short intervals, is given by the equation

$$y = 0.838 - 0.054\frac{z}{u},$$

in which z is the number of revolutions of the fly-wheel or gear shaft. In Meidinger's experiment z was 75, hence

$$y = 0.838 - 0.54\frac{75}{34} = 0.72,$$

or agreeing almost exactly with our calculated result.

For ordinary practical working condition Grashof gives

$$y = 0.79 - 0.07\frac{z}{u}.$$

Hence the actual effect of an atmospheric gas engine is

$$N = 2.296Fsuy\left[(1 + \varphi - \rho)(1 - e_2) - \frac{e_2^{n_1} - e_2}{0.134}\right] \text{horse-power} \quad . \quad . \quad . \quad \text{(LXXXI.)}$$

From a circular of the inventors we take the following:

A convenient and advantageous motor for the minor industries is offered by the Otto Langen gas engine.

This engine can be set up in crowded areas, as its action is entirely without danger and it requires little space.

The consumption of illuminating gas per hour for every horse-power (actual) depends upon the size of the engine, and is on the average only about 1 cubic meter, or considerably less when the engine is not worked up to its limit.

The expense for gas is the only cost of working; wages for service do not increase with size.

The water for cooling requires no renewal. Its temperature does not exceed 50°.*

---

* According to Meidinger's experiments, the cooling water for the engine of ½ horse-power was 70 liters. The circulation was maintained independently, and during 10 hours of constant action it left the jacket with a temperature of 83° C. and returned with 67° C.

CHAPTER XII.

FORMULÆ FOR THE VELOCITY WITH WHICH AIR FLOWS OUT OF VESSELS.

IN Fig. 56 let the space between the two pistons *HI* and *G*, in the vessel *ABCD*, and the pipe *EF*, be filled with water or some other liquid. Let the piston *HI* be pressed toward the right with p kilograms per square meter, and G toward the left with p_1 kilograms, and let $p_1 < p$. If the area of the large piston is F, and that of the smaller f, the total pressure upon one is Fp, and upon the other fp_1. Let the distance passed over by the larger piston in one second be s, and that passed over by the other be s_1. Then the work of the force Fp is Fps, and that of the resistance fp_1 is fp_1s_1. If water fills the space between both pistons, then for every position $Fs = fs_1$, or $F : f = s_1 : s$.

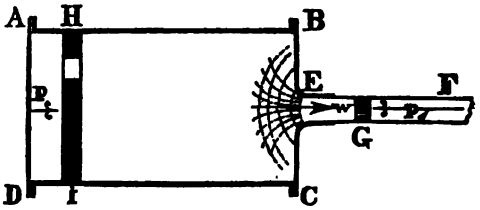


Fig. 56.

While now the water particles pass from the larger to the smaller vessel, they must take a greater velocity; the less velocity s passes into the greater s_1. If one cubic unit of water weighs γ kilograms, we have $Fs\gamma = fs_1\gamma$. The work Fps of the force Fp has not only, therefore, to overcome the work fp_1s_1 of the resistance fp_1, but also has to give to the weight $Fs\gamma = fs_1\gamma$ a greater living force. When, then, uniform velocity is attained in the vessel and pipe, we have

$$Fps = fp_1s_1 + \frac{s_1^2 - s^2}{2g} Fs\gamma,$$

or

$$Fps - fp_1s_1 = \frac{s_1^2 - s^2}{2g} Fs\gamma.$$

That is, the difference of the work of the force and of the resistance is equal to the increase of living force of the water, neglecting the loss of velocity due to friction, etc. Such an excess of work must always exist when we have a change of velocity.

If s is very small, with reference to s_1, we may neglect s^2 and have then

$$Fps - fp_1s_1 = \frac{s_1^2}{2g} Fs\gamma,$$

or, putting w in place of s_1,

$$Fps - fp_1s_1 = \frac{w^2}{2g} Fs\gamma \quad . \quad . \quad . \quad . \quad . \quad (1).$$

If, now, the space between the pistons were filled with an expansive fluid, as air, instead of liquid, the case would be somewhat different. Such a gas would expand when the pressure on one end was less than that on the other. This would be especially the case for those particles in the vicinity of E, as shown by the dotted lines. If we assume, as before, that the volume Fs passes per second into the pipe EF, then, if its weight is one kilogram, its specific volume is v. This volume increases in passing out to v_1, so that $v_1 = fs_1$. The works Fps and fp_1s_1 are then equal to pv and p_1v_1, and we have

$$pv - p_1v_1 = \frac{w^2}{2g} \cdot 1,$$

where 1 is the weight of one kilogram.

If we assume that the air in the pipe has the same temperature as in the vessel, then, by Mariotte's law,

$$pv = p_1v_1,$$

and the left side of our equation would be zero, hence $\frac{w^2}{2g}$ or w would be zero. We have to seek the cause of the change of velocity in the nature of the gas itself. If we examine more closely we shall recognize a force which causes this change. Thus, in order that the temperature may be constant in the pipe and vessel, heat must be imparted from without, and just so much heat as is equivalent to the work of expansion, or to the increase of living force of the molecules. This heat is

$$Q = 2.3026ART \log \frac{p}{p_1},$$

and the equivalent outer work is

$$L = \frac{Q}{A} = 2.3026RT \log \frac{p}{p_1}.$$

We have, then, this expression in the place of $pv - p_1v_1$ in Equation (2). Thus

$$w = \sqrt{4.6052RTg \log \frac{p}{p_1}}.$$

Putting in the place of R and g their values ($g = 9.81$ meters) for the velocity w in meters per second and Centigrade degrees,

$$w = 36.365\sqrt{T \log \frac{p}{p_1}}. \quad . \quad \text{(LXXXII.)}$$

For feet and Fahrenheit degrees,

$$w = 88.9\sqrt{T \log \frac{p}{p_1}}.$$

We can make use of this formula in every case when the expansion is very small; when, therefore, p is but little more than p_1. For, in such case, the heat required is but little, and we may assume that it is supplied by the outer air. According to Weisbach this is always the case when $p - p_1$ is less than $\frac{1}{10}p$.

If, however, no heat is imparted during the expansion from v to v_1, this must be supplied by the heat of the air itself, or, in other words, it must lose heat equivalent to the work done in expanding. This heat is

$$Q = (T - T_1)c$$

where T_1 is the absolute temperature of the air in the pipe. The equivalent work is

$$L = \frac{Q}{A} = \frac{c}{A}(T - T_1)$$

Now in the present case, by the combined law of Mariotte and Gay-Lussac,

$$\frac{pv}{T} = \frac{p_1v_1}{T_1},$$

and since $T_1 < T$, we must have $p_1v_1 < pv$. By the expansion, therefore, the specific volume increases less rapidly than the pressure diminishes. The expression $pv - p_1v_1$ in Equation (2) has then a positive value, or work must be performed by the piston HI, in order that the air may flow out with the velocity w. Hence the entire work necessary to impart the living force $\frac{w^2}{2g}$ to one kilogram of air, is

$$\frac{w^2}{2g} = pv - p_1v_1 + \frac{c}{A}(T - T_1).$$

A number of other cases may be conceived. Thus we may suppose heat abstracted during the expansion, according to the law $pv^{-2} = p_1v_1^{-2} =$ etc. Such cases have no practical interest.

If in the above equation we put RT in place of pv, and RT_1 for p_1v_1, we have

$$\frac{w^2}{2g} = RT - RT_1 + \frac{c}{A}(T - T_1) = \left(R + \frac{c}{A}\right)(T - T_1).$$

Since, however, $R = \frac{c\,(k-1)}{A}$,

$$\frac{w^2}{2g} = \left(\frac{ck}{A} - \frac{c}{A} + \frac{c}{A}\right)(T - T_1) = \frac{ck}{A}(T - T_1).$$

If we assume T as known, we can easily find T_1. Thus, since the expansion is adiabatic, we have

$$pv^k = p_1v_1^k.$$

Also,
$$\frac{T_1}{T} = \left(\frac{p_1}{p}\right)^{\frac{k-1}{k}}.$$

We have therefore for w in meters per second for Centigrade degrees,

$$w = \sqrt{2g\frac{ck}{A}T\left[1-\left(\frac{p_1}{p}\right)^{\frac{k-1}{k}}\right]}$$

$$= 44.449\sqrt{T\left[1-\left(\frac{p_1}{p}\right)^{0.2907}\right]}. \quad \text{(LXXXIII.)}$$

and for feet per second and Fahrenheit degrees,

$$w = 108.6\sqrt{T\left[1-\left(\frac{p_1}{p}\right)^{0.2907}\right]}.$$

These formulæ apply to the case where air flows out of a vessel into the atmosphere. In this case $p_1 = 1$. Instead of assuming the piston HI, by moving toward the right, to preserve a constant pressure, we may suppose fresh air constantly forced in.

Experiment shows that the above velocity is never attained completely. The particles are hindered by friction and mutual impact. These disturbances cause a loss of velocity which reappears as heat. Since, however, the velocity of efflux for moderate pressure is very great, the heat thus generated by loss of velocity is imparted almost entirely to the particles rather than to the walls of the vessel, and the temperature T_1 at the plane of the orifice, or where the pressure is constant, is somewhat greater than given by

$$T_1 = T\left(\frac{p_1}{p}\right)^{\frac{k-1}{k}}.$$

Experiment has also shown that the greatest velocity is not in the plane of the orifice, but some distance from it, within the pipe. The stream possesses, then, at this point, a somewhat smaller cross-section than the orifice. The phenomenon is almost exactly the same as for the efflux of water or similar liquids.

The number by which the area of the orifice must be multiplied in order to give the area of cross-section of the stream where the velocity is greatest, is called the coefficient of contraction. It depends not only upon the excess of pressure $p - p_1$ in the vessel, but also upon the form of the orifice, as well as upon its position.

According to Weisbach, for an effective pressure $p - p_1$ of 50 to $850^{\text{mm.}}$, for circular orifice in thin plate, of from 10 to $24^{\text{mm.}}$ diameter, the coefficient of contraction is $\alpha = 0.566$ to 0.811, increasing with $p - p_1$. If, for example, the area of orifice is 1 square centimeter, the cross-section of the stream at the place of greatest velocity is

0.566 to 0.811 square centimeters.

But in this cross-section the velocity is not w, as just found, but is somewhat less. We have therefore to multiply w by a proper fraction, in order to obtain the actual velocity, and this fraction we call the "*coefficient of velocity.*" If we denote it by φ, and the effective velocity by w_e, we have

$$w_e = \varphi w.$$

If, now, the cross-section of the orifice is F, that of the greatest velocity will be αF, and since the particles pass with the velocity $w_e = \varphi w$, the discharge per second is

$$V = w_e \alpha F = \varphi w \alpha F.$$

And putting for w its calculated value,

$$V = 44.449\, \varphi \alpha F \sqrt{T\left[1 - \left(\frac{p_1}{p}\right)^{0.2907}\right]}.$$

The product of the coefficients of contraction and velocity $(\alpha\varphi)$ by which we multiply the area F of the orifice and the theoretical velocity, in order to find the actual discharge, is called the "*coefficient of discharge,*" and is denoted by μ. Accordingly

$$V = 44.449 \mu F \sqrt{T\left[1 - \left(\frac{p_1}{p}\right)^{0.2907}\right]} \quad . \quad \text{(LXXXIV.)}$$

For English measures and Fahrenheit degrees, put 108.6 in place of 44.449.

If the mouthpiece through which the air flows is fitted to the shape of the stream, we have $\alpha = 1$, because the orifice has then the same cross-section as that where the velocity is greatest. For such orifices, shaped somewhat as shown in Fig. 56, Weisbach gives for an effective pressure $p - p_1$ of 180 to $850^{\text{mm.}}$, for area of orifice of about $10^{\text{mm.}}$

$$\varphi = \mu = 0.981.$$

(One atmosphere is $760^{\text{mm.}}$)

For circular orifices in a thin plate, from 10 to $24^{\text{mm.}}$ diameter and an effective pressure $p - p_1$ of from 50 to $850^{\text{mm.}}$, and taking $\varphi = 0.98$,

$$\mu = 0.556 \quad \text{to} \quad 0.795.$$

For short cylindrical pipes of the same diameter as the orifice, for the same limits of pressure,

$$\mu = \varphi = 0.737 \quad \text{to} \quad 0.839.$$

EXAMPLE 1.—With what velocity w_e will air flow out of a receiver into the air, when the pressure p is 1.033 atmospheres, the absolute temperature T 273 + 10 = 283°, and the coefficient of velocity $\varphi = 0.90$?

Since the pressure is but little in excess of the atmosphere, we may make use of the formula LXXXII., page 327. We have then

$$w_e = 0.9 \times 36.365 \ \sqrt{283 \log 1.033}$$

$$= 0.9 \times 36.365 \ \sqrt{283 \times 0.0141}$$

$$= 0.9 \times 36.365 \ \sqrt{3.9903}$$

$$w_e = 65.132 \text{ meters per second.}$$

If the temperature Fahrenheit is $T = 459.4 + 50 = 509.4$, we have

$$w_e = 0.9 \times 88.9 \sqrt{509.4 \log 1.033} = 214 \text{ feet per second.}$$

EXAMPLE 2.—In a receiver we have air under the constant pressure $p = 1.2$ atmospheres. The absolute temperature is $T = 273 + 10 = 283°$. With what velocity will the air issue when the discharge takes place through an orifice of the shape of the contracted stream ?

Here $p = 1.2$ and $p_1 = 1$, hence

$$\left(\frac{p_1}{p}\right)^{0.2907} = \left(\frac{1}{1.2}\right)^{0.2907} = 0.9485,$$

Hence

$$w = 44.449 \sqrt{283\,(1 - 0.9485)} = 44.449 \sqrt{14.5745},$$

or

$$w = 169.443 \text{ meters.}$$

Since $\varphi = 0.981$, the actual velocity is

$$w_e = \varphi w = 0.981 \times 169.443 = 166.224 \text{ meters.}$$

If, now, in the equation

$$\frac{w^2}{2g} = \frac{ck}{A}(T - T_1),$$

we put w_e in place of w, the absolute temperature T_1 in the plane of the orifice will be

$$T_1 = T - \frac{w_e^2}{2g} \cdot \frac{A}{ck} = 283 - \frac{(166.224)^2}{2 \times 9.81} \times 0.00993$$

$$= 283 - 13.98 = 269.02.$$

Hence the temperature, Centigrade, is $t_1 = 269.02 - 273 = -4.02°$.

We see, therefore, that even for a slight excess of pressure of only 0.2 atmospheres, there is a considerable reduction of temperature.

Without loss of velocity, the temperature T_1 would have been

$$T_1 = 283 - \frac{(169.443)^2}{2 \times 9.81} \times 0.00993 = 283 - 14.53 = 268.47°,$$

or $268.47 - 273 = 4.53°$ C. By the resistance, therefore, the air is heated $4.53 - 4.02 = 0.51°$.

EXAMPLE 3.—If the same receiver is required to furnish 100 cubic meters of air per minute, what must be the cross-section F of the orifice of discharge?

We have $V = w_e \alpha F$, or $F = \dfrac{V}{w_e \alpha}$, or

since $\alpha = 1$, $V = \dfrac{100}{60} = 1.666$, $w_e = 166.224$,

$$F = \frac{1.666}{166.224} = 0.01003 \text{ square meters} = 1.003 \text{ square dec.}$$

Finally, we remark that when air flows from a receiver into the atmosphere, the contracted stream expands again in a manner similar to that in which it contracted. It thus gradually loses its velocity and the particles come to rest. It is evident that the entire living force $\frac{w^2}{2g}$, inherent in one kilogram, is thus transformed into heat, and that, therefore, as soon as rest obtains, the temperature of the air is again the original temperature T which it had in the receiver before expanding.

CHAPTER XIII.

AIR COMPRESSORS AND COMPRESSED AIR ENGINES.

[The following pages comprise an abstract of a work entitled "L'Air Comprimé," by M. A. Pernolet, Paris, 1876. The abstract formed portion of a Graduation Thesis written by Mr. Bailey Willis, M.E., while a student in the School of Mines, Columbia College, and is, with his permission, given here with insignificant changes, precisely as prepared by him. Mr. Willis has in several places found occasion to differ from M. Pernolet, and such differences will be found noted in the Text. Mr. Willis has also converted all the formulæ and calculations into English measures.]

Work of Compression.—Suppose we have a given weight G of air, whose volume is V_1, pressure p_1, and absolute temperature T_1. By means of a piston let this air be compressed adiabatically to the volume V_2, pressure p_2, and temperature T_2. During compression, the pressure p_1 of outside air acts upon the piston to help compression. After compression, the volume V_2 is forced under the pressure p_2 into a reservoir. Then the work of compression is

$$L_c = \frac{Gc_v}{A}(T_2 - T_1) + p_2 V_2 - p_1 V_1.$$

But

$$p_2 V_2 = \frac{G}{A}(c_p - c_v) T_2$$

and

$$p_1 V_1 = \frac{G}{A}(c_p - c_v) T_1,$$

hence

$$L_c = \frac{G}{A}(c_v + c_p - c_v)(T_2 - T_1),$$

or

$$\left.\begin{aligned} L_c &= \frac{G}{A} c_p (T_2 - T_1) \\ &= 100.704G(T_2 - T_1) \text{ meter-kil.} \\ &= 183.86G(T_2 - T_1) \text{ foot-lbs.} \end{aligned}\right\} . \quad \text{(LXXXV.)}$$

where $\frac{1}{A} = 772$, and degrees are measured by Fahrenheit scale. For convenience of use we may write these equations in the following form :

$$\left.\begin{aligned} L_c &= 100.704 G T_1 \left(\frac{T_2}{T_1} - 1\right) \text{ meter-kil.} \\ \text{or} \quad L_c &= 183.36 G T_1 \left(\frac{T_2}{T_1} - 1\right) \text{ foot-lbs.} \end{aligned}\right\} \quad . \quad \text{(LXXXVI.)}$$

We can find from the table, page 178, $\frac{T_2}{T_1}$ for any given ratio of $\frac{p_2}{p_1}$, and then find the work, without first finding T_2.

We may also write the expression for the work of compression in the form

$$\left.\begin{aligned} L_c &= 100.704 G T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right] \text{ meter-kil.} \\ \text{or} \quad L_c &= 183.36 G T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right] \text{ foot-lbs.} \end{aligned}\right\} \text{(LXXXVII.)}$$

Again, if the volume V_1 of air to be compressed is given, instead of the weight G, then since

$$p_1 V_1 = \frac{G}{A}(c_p - c_v) T_1,$$

we have

$$G = \frac{A p_1 V_1}{(c_p - c_v) T_1},$$

and hence

$$\left.\begin{aligned} L_c &= p_1 V_1 \frac{c_p}{c_p - c_v} \left(\frac{T_2}{T_1} - 1\right) \\ &= 8.452 p_1 V_1 \left(\frac{T_2}{T_1} - 1\right) \end{aligned}\right\} \quad . \quad . \quad \text{(LXXXVIII.)}$$

Any of these formulæ may be used in finding the work of compression, as may be most convenient, and the table on page 178 will greatly facilitate computations.

Volume of the Compressing Cylinder.—Let the volume of the compressing cylinder be V, and the volume of air compressed per stroke be V_1. If the engine makes n revolutions per second, it will make $2n$ strokes per second, and the volume of air compressed at each stroke is $\frac{V_1}{2n}$.

The volume of the cylinder must be equal to this, or

$$V = \frac{V_1}{2n}.$$

But $\qquad p_1 V_1 = GRT_1, \quad$ hence

$$V = \frac{GRT_1}{2np_1}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(LXXXIX.)}$$

Final Temperature.—In the formulæ for the work of compression already deduced, the final temperature T_2 of the air, as it passes out of the compressing cylinder, occurs. In the following table the final temperatures for different values of $\frac{p_2}{p_1}$ are given, the initial temperature t_1 being assumed equal to 68° Fahr., or $T_1 = 459.4 + 68 = 527.4°$.

Final Temperature.—$T_1 = 459.4 + 68 = 527.4°$, or $t_1 = 68°$ Fahr.

$\frac{p_2}{p_1}$	Final absolute temperature T_2.	Final temperature in degrees Fahr. t_2.	$\frac{p_2}{p_1}$	Final absolute temperature T_2.	Final temperature in degrees Fahr. t_2.
2............	644.8	185.4	9............	997.42	538.02
3............	725.3	265.9	10............	1027.95	568.55
4............	788.35	328.95	11............	1058.2	598.8
5............	841.1	381.7	12............	1084.2	624.8
6............	886.7	427.3	13............	1109.6	650.2
7............	927.3	467.9	14............	1133.85	674.45
8............	963.9	504.5	15............	1156.6	697.2

These values are easily calculated from the following table of ratios, and the table can be readily extended if desired.

They can, if desired, be easily reduced to Centigrade degrees, or the Centigrade values can be calculated directly.

$\frac{p_2}{p_1}$	$\frac{t_1}{t_2}$	Diff.	$\frac{p_2}{p_1}$	$\frac{t_1}{t_2}$	Diff.	$\frac{p_2}{p_1}$	$\frac{t_1}{t_2}$	Diff.
1.2	0.773	78	3.2	0.460	12	6	0.355	21
1.4	0.695	53	3.4	0.448	10	7	0.334	17
1.6	0.642	39	3.6	0.438	10	8	0.317	14
1.8	0.603	32	3.8	0.428	9	9	0 303	13
2	0.571	25	4	0.419	9	10	0.291	10
2.2	0.546	22	4.2	0.410	8	11	0.281	9
2.4	0.524	19	4.4	0.402	7	12	0.272	9
2.6	0.505	17	4.6	0.395	7	13	0.263	7
2.8	0.488	15	4.8	0.388	6	14	0.256	6
3	0.473	13	5	0.382	27	15	0.250	

Compression in two Cylinders with Intermediate Reservoir.—We see from the expression for the work done in compression

$$L_c = \frac{G}{A} c_p (T_2 - T_1)$$

that in order to reduce the work of compression for any given ratio $\frac{p_2}{p_1}$, we must, if possible, reduce the final temperature T_2.

This may be in some measure accomplished by compressing the air in one cylinder to a pressure p_2' intermediate between p_1 and p_2, then cooling it under the constant pressure p_2' to the temperature T_1, and then further compressing it to the pressure p_2 in a second cylinder.

The work of compression in the first cylinder is then

$$L_1 = \frac{G}{A} c_p (T_2' - T_1) = \frac{G}{A} c_p T_1 \left[\left(\frac{p_2'}{p_1}\right)^{\frac{k-1}{k}} - 1 \right],$$

and in the second cylinder

$$L_2 = \frac{G}{A} c_p (T_2'' - T_1) = \frac{G}{A} c_p T_1 \left[\left(\frac{p_2}{p_2'}\right)^{\frac{k-1}{k}} - 1 \right].$$

The total work of compression is therefore

$$L_c' = \frac{G}{A} c_p T_1 \left[\left(\frac{p_2'}{p_1}\right)^{\frac{k-1}{k}} + \left(\frac{p_2}{p_2'}\right)^{\frac{k-1}{k}} - 2 \right]. \quad . \quad . \quad . \quad (\text{XC.})$$

Now L_c' is a minimum when $\left(\frac{p_2'}{p_1}\right)^{\frac{k-1}{k}} + \left(\frac{p_2}{p_2'}\right)^{\frac{k-1}{k}}$ is a minimum.

In any given case, p_1 and p_2 will be known, and hence the above expression is of the form

$$\frac{x}{a} + \frac{b}{x}.$$

Differentiating and placing the first differential coefficient equal to zero, we find

$$x = \sqrt{ab},$$

and this value substituted in the second differential coefficient gives a positive result.

Hence the work of compression will be a minimum when

$$p_2' = \sqrt{p_1 p_2},$$

or

$$\frac{p_2'}{p_1} = \frac{p_2}{p_2'} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{XCI.})$$

Therefore

$$\left.\begin{aligned} L_c'^{\text{min.}} &= \frac{2G}{A} c_p T_1 \left[\left(\frac{p_2'}{p_1}\right)^{\frac{k-1}{k}} - 1 \right] \\ \text{or} \qquad & \\ L_c'^{\text{min.}} &= \frac{2G}{A} c_p T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{2k}} - 1 \right] \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (\text{XCII.})$$

It follows also, from

$$\frac{p_2'}{p_1} = \frac{p_2}{p_2'},$$

that

$$\frac{T_2'}{T_1} = \frac{T_2''}{T_1} \quad \text{or} \quad T_2' = T_2'',$$

that is, the final temperatures in the two cylinders are equal.

Since

$$\frac{T_2'}{T_1} = \left(\frac{p_2'}{p_1}\right)^{\frac{k-1}{k}} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{2k}} \quad . \quad . \quad . \quad . \quad \text{(XCIII.)}$$

and since for a single cylinder and the same ratio $\frac{p_2}{p_1}$,

$$\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}},$$

we have

$$\frac{T_2'}{T_1} = \left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XCIV.)}$$

Comparing the work of compression in two cylinders with that in one, we have

$$\frac{L_c'}{L_c} = \frac{\frac{2G}{A} c_p T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{2k}} - 1\right]}{\frac{G}{A} c_p T_1 \left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right]} = \frac{2}{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{2k}} + 1} \quad . \quad . \quad \text{(XCV.)}$$

or

$$\frac{L_c'}{L_c} = \frac{2}{\left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} + 1} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XCVI.)}$$

Now $\frac{T_2}{T_1}$ is always greater than 1, therefore $\left(\frac{T_2}{T_1}\right)^{\frac{1}{2}} + 1$ is always greater than 2, and L_c, the work of compression in one cylinder for the same limits of p_1 and p_2, is always greater than the work of compression in two cylinders.

In the following table, the ratios $\frac{L_c'}{L_c}$ and the final temperatures in degrees Fahrenheit, for one and two cylinders, are given; it being assumed that $T_1 = 459.4 + 68 = 527.4$, or $t_1 = 68°$ Fahrenheit.

We find T_2' from XCIII.

$$\frac{T_2'}{T_1} = \left(\frac{p_2}{p_1}\right)^{0.145}.$$

T_2 may be found by aid of the table, page 173.

TABLE,

SHOWING FINAL TEMPERATURE FOR ONE AND TWO CYLINDERS AND RATIO $\frac{L_t'}{L_t}$,

$$T = 459.4 + t.$$

$\frac{p_2}{p_1}$.	Final temperature in degrees Fahr.		$\frac{L_t'}{L_t}$.	$\frac{p_2}{p_1}$.	Final temperature in degrees Fahr.		$\frac{L_t'}{L_t}$.
	One cylinder t_2.	Two cylinders t_2'.			One cylinder t_2.	Two cylinders t_2'.	
2........	185.4	123.8	0.95	9.......	538.0	265.9	0.84
3........	265.9	159.1	0.92	10.......	568.55	277.0	0.83
4........	328.95	185.4	0.90	11.......	598.8	287.3	0.83
5........	381.7	207.5	0.88	12.......	624.8	296.8	0.82
6........	427.3	224.5	0.87	13.......	650.2	305.6	0.82
7........	467.9	239.9	0.86	14... ...	674.45	313.9	0.81
8........	504.5	253.6	0.85	15.......	697.2	321.6	0.80

Friction of Air in Pipes.—The compressed air is conveyed by means of pipes from the reservoir to the point at which it is desired to utilise it, and therefore there is a certain loss of pressure due to friction.

This loss is independent of any changes of temperature of the air. It is directly proportional to the length of the pipe and the square of the velocity, and inversely as the diameter of the pipe.

Denoting the loss of pressure due to friction by F, we have from experiments at Mont Cenis

$$F = 0.00936 \frac{u^2 l}{d} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(XCVII.)}$$

where u is the velocity in feet per second, l the length of pipe in feet, and d the diameter of the pipe in inches.

In conveying the air through the pipe, there is also a loss of power due to the change of temperature. If we denote by T_3 the temperature at the point of application, we have for this loss

$$F' = \frac{G}{A} c_v (T_2 - T_3) \quad . \; . \; . \; . \; . \quad \text{(XCVIII.)}$$

hence T_2 should be as low and T_3 as high as possible.

The Compressed Air Engine.—The compressed air arrives at the end of the pipe with a pressure p_3 and at an absolute temperature T_3, while G pounds of it, the amount used per second, occupy a volume V_3.

It is then admitted into the cylinder of the compressed air engine and does work, its condition at the end of the stroke being p_4', T_4', and V_4'. The exhaust then opens and the air escapes into the atmosphere, its state changing to p_4, T_4, and V_4.

The air may perform work in the cylinder in three different ways.

1. It may act at full pressure during a portion of the stroke, and be then cut off at such a point that the pressure in the cylinder falls by expansion to the pressure of the atmosphere. In such case, p_4', T_4', and V_4' become equal to p_4, T_4, V_4.

2. The air may act at full pressure during the whole stroke. In this case p_4', T_4', and V_4' are equal to p_3, T_3, and V_3', and the work corresponding to the change of temperature $T_3 - T_4$ is lost.

3. The air may act at full pressure during a portion of the stroke, and be then cut off at such a point that the pressure within the cylinder is reduced indeed by expansion, but not to the pressure of the atmosphere. In this case p_3 is greater than p_4', and p_4' is greater than p_4.

We may call these three cases respectively :

1, *Complete expansion ;* 2, *Full pressure ; and* 3, *Incomplete expansion.*

1. COMPLETE EXPANSION.

We have to deal in this case with the initial conditions p_3, V_3, and T_3, and the final conditions p_4, V_4, and T_4 of G units of weight of air, the amount used per second.

Final Temperature.—The expansion takes place between the limits p_3 and p_4 according to the adiabatic law. Hence

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}} = \left(\frac{p_4}{p_3}\right)^{0.29}$$

or

$$\frac{T_3}{T_4} = \left(\frac{p_3}{p_4}\right)^{0.29}.$$

The ratio $\frac{T_3}{T_4}$ may be found in our table, page 178, for the given value of $\frac{p_3}{p_4}$.

Disposable Work.—The work of the air in the cylinder is made up of two parts, the work $p_3 V_3$ at full pressure plus the work of expansion. The disposable work is this amount less the work of the back pressure $p_4 V_4$.

Hence

$$L_d = p_3 V_3 + \frac{G}{A} c_v (T_3 - T_4) - p_4 V_4 .$$

This reduces, as we have already seen, to

$$L_d = \frac{G}{A} c_p (T_3 - T_4) = \frac{G}{A} c_p T_3 \left(1 - \frac{T_4}{T_3}\right).$$

Volume of the Cylinder.—We have found for the volume of the compressing cylinder

$$V = R \frac{G}{2n} \frac{T_1}{p_1}.$$

In an entirely analogous manner we have for the volume of the cylinder of the air engine

$$V' = R \frac{G}{2n} \frac{T_4}{p_4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XCIX.)}$$

where n is the number of revolutions per second.

Weight of Air per Second.—Let N denote the number of horse-power per second required, and η denote the efficiency of the engine. Then since

$$L_d = \frac{G}{A} c_p (T_3 - T_4),$$

we have for English measures

$$550N = \eta \frac{G}{A} c_p (T_3 - T_4);$$

hence

$$G = \frac{550A}{\eta c_p} \frac{N}{(T_3 - T_4)}$$

$$= \frac{3}{\eta} \cdot \frac{N}{T_3 - T_4} \textit{ pounds per second.} \quad . \quad . \quad . \quad \text{(C.)}$$

Efficiency of the Compressor and of the Engine.—Let ζ be the ratio between the work of the compressor L_c and the work of the air engine L_d. We have then

$$\zeta = \frac{L_d}{L_c} = \frac{\frac{G}{A} c_p (T_3 - T_4)}{\frac{G}{A} c_p (T_2 - T_1)} = \frac{T_3 \left(1 - \frac{T_4}{T_3}\right)}{T_2 \left(1 - \frac{T_1}{T_2}\right)}$$

or

$$\zeta = \frac{T_3 \left[1 - \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}\right]}{T_2 \left[1 - \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}\right]} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(CI.)}$$

In this expression the ratio of the terms in brackets is usually nearly equal to 1. It is therefore to the ratio $\frac{T_3}{T_2}$ that we must look for any increase in the efficiency of the combination of the compressor and the air engine.

T_3 is usually the temperature of the atmosphere at the air engine, and we see, therefore, that to increase the efficiency we must decrease T_2. This agrees with what has been said in discussing the compressor.

A rough approximation to the efficiency in any given case may be arrived at by placing

$$\zeta = \frac{T_3}{T_2}.$$

The values thus obtained will always be greater than the true values, and the latter will approach the nearer to it, as the loss by friction in the pipes is less.

For the sake of future comparison merely, we give in the following table the efficiency for complete expansion calculated from the formula

$$\zeta = \frac{T_3}{T_2},$$

T_3 being taken equal to 527.4° Fahr.

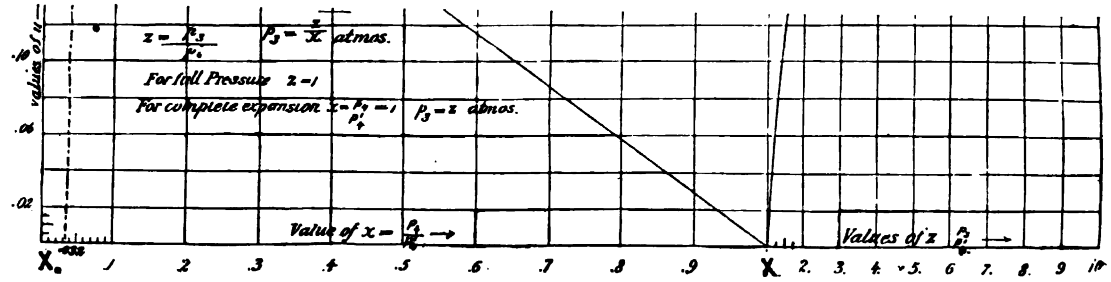
$\frac{p_2}{p_1}$.	ζ.	$\frac{p_2}{p_1}$.	ζ.
2..........	0.82	9..........	0.53
3..........	0.73	10..........	0.51
4..........	0.67	11..........	0.50
5..........	0.63	12..........	0.49
6..........	0.59	13..........	0.48
7..........	0.57	14..........	0.47
8..........	0.55	15..........	0.46

These values can be taken directly from the table, page 171, in the column for $\frac{T_1}{T_2}$.

In practice these efficiencies are reduced to less than half of these theoretical values. For if we denote the work of the engine that works the compressor by L, and the work performed by the air engine by L', and assume the efficiencies of the compressor and air engine at a fair average at 0.70, we have

$$L_e = 0.7L, \quad \text{and} \quad L' = 9.7L_d, \quad \text{hence}$$

$$\frac{L_1}{L} = \frac{0.7L_d}{\frac{L_e}{0.7}} = 0.49\,\frac{L_d}{L_e}.$$



$z = \frac{p_3}{p_4}$
$p_3 = \frac{z}{X}$ atmos.
For full Pressure $z = 1$
For complete expansion $x = \frac{p_4}{p_4'} = 1$ $p_3 = z$ atmos.
Value of $x = \frac{p_4}{p_4'}$ →
Values of z $\frac{p_3}{p_4'}$ →
values of u
.10
.06
.02
X_0
.032
.1
.2
.3
.4
.5
.6
.7
.8
.9
X
2.
3.
4.
5.
6
7.
8.
9
10

Moreover, it must be remembered that for long distances the term

$$\frac{1-\left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}}{1-\left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}}$$

has a very considerable influence, and reduces the value of ζ very materially.

Construction.—We may easily deduce formulæ for the cut-off which will produce complete expansion.

The volume of air at the pressure p_3 used per stroke is $\frac{V_3}{2n}$. When this expands to the pressure p_4, the volume must be that of the cylinder V', for which we have already deduced a value.

We have by the adiabatic law

$$2n\frac{V'}{V_3}=\left(\frac{p_3}{p_4}\right)^{\frac{1}{k}}$$

From our table, page 171, we can find the value of $2n\frac{V'}{V_3}$, for any given value of $\frac{p_3}{p_4}$, in the column for $\frac{v_1}{v_2}$.

Let now S be the stroke, and s the distance traversed by the piston during admission of air. Then

$$\frac{S}{s}=2n\frac{V_1}{V_3}=\left(\frac{p_3}{p_4}\right)^{\frac{1}{k}}$$

Let ω be the angular velocity of the crank, and t_1 the duration of admission. Then

$$s=\frac{S}{2}(1-\cos\omega t_1),$$

hence

$$1-\cos\omega t_1=2\frac{s}{S}=2\left(\frac{p_4}{p_3}\right)^{\frac{1}{k}},$$

or

$$\omega t_1=\text{arc.}\cos\left[1-2\left(\frac{p_4}{p_3}\right)^{\frac{1}{k}}\right] \quad . \; . \; . \; . \; . \quad \text{(CII.)}$$

If the time of entire stroke is t, we have

$$\omega t=\pi=3.14, \text{ and hence}$$

$$\frac{t_1}{t}=\frac{\omega t_1}{\omega t}=\frac{\omega t_1}{\pi} \quad . \; . \; . \; . \; . \; . \; . \; . \quad \text{(CIII.)}$$

In the last column of the table, page 178, we have given the volumes of $\frac{t_1}{t}$, calculated for the corresponding values of $\frac{p_3}{p_4}$, in the first column.

Air Engines with Two Cylinders and Intermediate Reservoir.—We have already given the formula for the final temperature T_4, for complete expansion (page 340), viz.:

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}.$$

Assuming $T_3 = 527.4°$, which corresponds to 60° Fahr., the values of T_4, given in the following table, may easily be calculated by the aid of table on page 178.

TABLE

FOR FINAL TEMPERATURE T_4.

$T_3 = 459.4 + 68 = 527.4.$

$\frac{p_3}{p_4}$.	Final absolute temperature T_4.	Final temperature Fahrenheit t_4.	$\frac{p_3}{p_4}$.	Final absolute temperature T_4.	Final temperature Fahrenheit t_4.
2..........	431.4	− 28	9..........	278.9	− 180.5
3..........	383.5	− 75.9	10..........	270.5	− 188.9
4..........	352.8	− 106.6	11..........	263.1	− 196.3
5..........	330.7	− 128.7	12..........	256.6	− 202.9
6..........	313.7	− 145.7	13..........	250.7	− 208.7
7..........	299.98	− 159.4	14..........	245.3	− 214.1
8..........	288.5	− 170.9	15..........	240.5	− 218.9

In discussing the air compressor, it has been shown that the final temperature T_2 could be reduced by compressing in two separate cylinders connected by a cooling reservoir (pages 336, 337). In a similar manner the very low temperature T_4, given in the preceding table, may be in a measure avoided.

The air expands in the first cylinder to a pressure p_4' which is greater than p_4. It then passes into a reservoir and is heated by jets of hot water, under the constant pressure p_4', to its original temperature T_3. It then passes into the second cylinder, and is there allowed to expand to the pressure p_4 of the atmosphere.

The work in the first cylinder is

$$L_1 = \frac{G}{A} c_p (T_3 - T_4') = \frac{G}{A} c_p T_3 \left[1 - \left(\frac{p_4'}{p_3}\right)^{\frac{k-1}{k}}\right].$$

The work in the second cylinder is

$$L_2 = \frac{G}{A} c_p (T_3 - T_4'') = \frac{G}{A} c_p T_3 \left[1 - \left(\frac{p_4}{p_4'}\right)^{\frac{k-1}{k}}\right].$$

Hence the total disposable work is

$$L_d' = \frac{G}{A} c_p T_3 \left[2 - \left(\frac{p_4'}{p_3}\right)^{\frac{k-1}{k}} - \left(\frac{p_4}{p_4'}\right)^{\frac{k-1}{k}}\right].$$

As already shown (page 337), the last two terms are a minimum when

$p_4' = \sqrt{p_3 p_4}$ and T_4' is then $= T_4''$.

Hence we have

$$L_d'{}_{\text{max.}} = 2 \frac{G}{A} c_p T_3 \left[1 - \left(\frac{p_4'}{p_3}\right)^{\frac{k-1}{k}}\right]$$

$$= 2 \frac{G}{A} c_p T_3 \left[1 - \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{2k}}\right]. \quad . \quad . \quad . \quad . \quad \text{(CIV.)}$$

Since

$$\frac{T_4'}{T_3} = \left(\frac{p_4'}{p_3}\right)^{\frac{k-1}{k}} = \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{2k}} \quad . \quad . \quad . \quad . \quad . \quad \text{(CV.)}$$

we have

$$\frac{T_4'}{T_3} = \left(\frac{T_4}{T_3}\right)^{\frac{1}{2}}$$

where T_4 denotes the final temperature for one cylinder. Comparing the work in one cylinder with that in two, we have

$$\frac{L_d'}{L_d} = \frac{2 \frac{G}{A} c_p T_3 \left(1 - \frac{T_4'}{T_3}\right)}{\frac{G}{A} c_p T_3 \left(1 - \frac{T_4}{T_3}\right)} = 2 \frac{1 - \sqrt{\frac{T_4}{T_3}}}{1 - \frac{T_4}{T_3}} = 2 \frac{1}{1 + \sqrt{\frac{T_4}{T_3}}} \quad . \quad . \quad \text{(CVI.)}$$

an expression entirely similar to XCVI., but in this case the denominator is less than 2, and L_d is therefore less than L_d'.

In the following table we give for comparison the final temperatures t_4 and t_4' in Fahrenheit degrees, and the ratio $\frac{L_d'}{L_d}$, for different values of $\frac{p_3}{p_4}$, T_3 being taken = 527.4°, or $t_3 = 68°$ Fahr. The values for one cylinder are taken from the table, page 344, and for two can be calculated from CVI.*

$$T_3 = 459.4 + 68 = 527.4°.$$

$\frac{p_3}{p_4}$.	Final temperature, Fahrenheit. One cylinder.	Final temperature, Fahrenheit. Two cylinders.	$\frac{L_a'}{L_a}$.
2............	− 28	+ 14.5	1.04
3............	− 75.9	− 10.2	1.08
4............	− 106.6	− 28.2	1.10
5............	− 128.7	− 41.8	1.12
6............	− 145.7	− 52.8	1.13
7............	− 159.4	− 61.7	1.14
8............	− 170.9	− 69.3	1.15
9............	− 180.5	− 75.8	1.16
10............	− 188.9	− 81.9	1.166
11............	− 196.3	− 86.9	1.172
12............	− 202.9	− 91.6	1.177
13............	− 208.7	− 95.7	1.183
14............	− 214.1	− 99.7	1.19
15............	− 218.9	− 103.3	1.194

2. FULL PRESSURE.

The action of the compressed air at full pressure is as follows: The air enters the cylinder and follows the piston throughout the stroke at full pressure p_3. The pressure and volume remain the same, and there is therefore no change of temperature. At the end of the stroke, the exhaust opens and the pressure of the air suddenly falls from p_3 to p_4, with a corresponding lowering of temperature from T_3 to T_4.

Thus the work is performed within the cylinder without any change of the internal energy of the air, and the work corresponding to the change of temperature from T_3 to T_4 is lost in the exhaust.

---

* [Mr. Willis omits at this point a discussion by Mons. Pernolet, from which he deduces an expression for the ratio $\frac{\zeta'}{\zeta}$ of the efficiencies of a single and double cylinder air engine with the compressor. The result obtained is $\frac{\zeta'}{\zeta} = \frac{2}{\frac{T_4}{T_3} + 1}$, which, Mr. Willis observes, is correct as far as it goes, but is based upon the supposition that $\zeta = \frac{T_3}{T_2}$. From Eq. CI. we see that $\zeta = \frac{T_3}{T_2} \frac{\left[1 - \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}\right]}{\left[1 - \left(\frac{p_1}{p_2}\right)^{\frac{k-1}{k}}\right]}$, and hence the value deduced for $\frac{\zeta'}{\zeta}$ by M. Pernolet is approximately true for short distances only.]

Disposable Work.—The available or disposable work for one second is equal to the pressure p_3 less the back pressure p_4, into the volume V_3 ; V_3 representing the volume of the weight G of air used per second at the pressure p_3. Hence

$$L_d = V_3 (p_3 - p_4).$$

But

$$V_3 = \frac{G}{A} (c_p - c_v) \frac{T_3}{p_3},$$

therefore

$$L_d = \frac{G}{A} (c_p - c_v) T_3 \left(1 - \frac{p_4}{p_3}\right)$$

$$= 53.268 T_3 G \left(1 - \frac{p_4}{p_3}\right). \quad . \quad . \quad . \quad . \quad . \qquad \text{(CVII.)}$$

in English measures.

This equation gives a simple value for the disposable work in terms of the initial temperature and of the pressure ratio $\frac{p_4}{p_3}$.

But for the purpose of further discussion, it is desirable to obtain an expression of the form $\frac{G}{A} c_p (T_3 - T_x)$, as is the case for complete expansion.

This may be readily done by placing

$$L_d = \frac{G}{A} (c_p - c_v) T_3 \left(1 - \frac{p_4}{p_3}\right) = \frac{G}{A} c_p (T_3 - T_x),$$

where T_x is an unknown quantity, and $(T_3 - T_x)$ denotes the degree of expansion which would give, with complete expansion, the same disposable work as that obtained at full pressure.

Solving the above equation we obtain

$$\frac{T_x}{T_3} = \frac{1}{k} + \frac{k-1}{k} \frac{p_4}{p_3} = 0.71 + 0.29 \frac{p_4}{p_3} \quad . \quad . \quad . \qquad \text{(CVIII.)}$$

From this formula the values in the following table have been calculated :

$\frac{p_3}{p_4}$.	$\frac{T_x}{T_3}$.	$\frac{p_3}{p_4}$.	$\frac{T_x}{T_3}$.
2..........	0.855	9..........	0.742
3..........	0.807	10..........	0.739
4..........	0.783	11..........	0.736
5..........	0.768	12..........	0.734
6..........	0.759	13..........	0.732
7..........	0.752	14..........	0.731
8..........	0.746	15..........	0.730

It is important to distinguish that T_x and T_4, the temperature of exhaust for full pressure, have no direct relation to each other.

Thus

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}, \quad \text{while} \quad \frac{T_x}{T_3} = 0.71 + 0.29\,\frac{p_4}{p_3}.$$

T_x is a theoretical temperature, which has no actual existence in the action of the air at full pressure. It simply means, that if an engine worked with complete expansion between T_3 and T_x, this engine would do the same work as the full-pressure engine does between T_3 and T_4.*

Final Temperature.—The final temperature T_4 is found from

$$\frac{T_4}{T_3} = \left(\frac{p_4}{p_3}\right)^{\frac{k-1}{k}}$$

for the expansion, on opening the exhaust takes place rapidly, and according to the adiabatic law.

This relation is the same as that already given for complete expansion, and the values of T_4, in table on page 344, apply also for full pressure for the same ratios of $\frac{p_3}{p}$.

Comparison of Efficiencies for Full Pressure and Complete Expansion.—We have already found an expression for the efficiency of an air engine working with complete expansion, and its compressor, and have given the values of ζ, calculated from the approximate formula

$$\zeta = \frac{T_3}{T_2}$$

in the table, page 342.

If we denote by ζ' the efficiency of an air engine acting at full pressure, with its compressor, we have

$$\zeta' = \frac{\frac{G}{A}(c_p - c_v)\,T_3\left(1 - \frac{p_4}{p_3}\right)}{\frac{G}{A}\,c_p T_1\left[\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1\right]}.$$

---

* [Mr. Willis calls attention to this point, because Mons. Pernolet distinctly states that T_x is T_4, and then deduces the quantities given in his table viii., p. 62 of "*L'air comprimé*," under this assumption. The results, therefore, of the last three columns of that table are incorrect.]

Hence

$$\zeta = \frac{c_p - c_v}{c_p} \frac{1 - \frac{p_4}{p_3}}{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1} = 0.29 \frac{1 - \frac{p_4}{p_3}}{\left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}} - 1} \quad . \quad \text{(CIX.)}$$

It being assumed that, as before,

$$T_1 = T_3, \quad \frac{p_2}{p_1} = \frac{p_3}{p_4}, \quad \text{and} \quad T_3 = 527.4°.$$

$\frac{p_3}{p_4}$.	Efficiency for complete expansion ζ.	Efficiency for full pressure ζ'.	$\frac{\zeta'}{\zeta}$.	$\frac{p_3}{p_4}$.	Efficiency for complete expansion ζ.	Efficiency for full pressure ζ'.	$\frac{\zeta'}{\zeta}$.
2........	0.82	0.65	0.79	9.......	0.53	0.29	0.55
3........	0.73	0.51	0.70	10.......	0.51	0.275	0.54
4........	0.67	0.44	0.66	11.......	0.50	0.26	0.52
5........	0.63	0.39	0.62	12.......	0.49	0.25	0.51
6........	0.59	0.35	0.59	13.......	0.48	0.24	0.50
7........	0.57	0.33	0.58	14.......	0.47	0.23	0.49
8........	0.56	0.31	0.56	15.......	0.46	0.227	0.49

Weight of Air per Second.—If N is the number of horse-power required, and η the efficiency of the air engine, we have

$$\eta L_d = 550N = \eta 53.268 G T_3 \left(1 - \frac{p_4}{p_3}\right).$$

Hence

$$G = \frac{550}{53.268} \frac{N}{\eta T_3 \left(1 - \frac{p_4}{p_3}\right)} = 10.825 \frac{N}{\eta T_3 \left(1 - \frac{p_4}{p_3}\right)} \quad . \quad . \quad \text{(CX.)}$$

in English measures.

Or we may place

$$\eta L_d = \eta \frac{G}{A} c_p (T_3 - T_e) = 550N,$$

whence

$$G = \frac{550A}{c_p} \frac{N}{\eta T_3 \left(1 - \frac{T_e}{T_3}\right)} = 8 \frac{N}{\eta T_3 \left(1 - \frac{T_e}{T_3}\right)} . \quad . \quad . \quad \text{(CXI.)}$$

in English measures.

This last expression is perfectly similar to Equation C., page 341, for complete expansion.

Volume of the Cylinder.—The volume of the cylinder is $\frac{V_3}{2n}$, n denoting the number of revolutions per second. From the formula

$$\eta L_d = \eta V_3 (p_4 - p_3) = 550N$$

we obtain

$$V' = \frac{V_3}{2n} = \frac{550}{2} \frac{N}{n\eta (p_3 - p_4)} = 275 \frac{N}{n\eta (p_3 - p_4)}.$$

Or we may place

$$V' = \frac{V_3}{2n} = \frac{G}{2n} R \frac{T_3}{p_3}.$$

3. INCOMPLETE EXPANSION.

When the air is cut off at such a point that the pressure at the end of the stroke is still greater than that of the atmosphere, we have to deal with three sets of conditions, viz.,

1. The initial conditions, p_3, V_3, and T_3.
2. The conditions at the end of stroke, p_4', V_4', T_4'.
3. The conditions at exhaust, p_4, T_4, V_4.

Disposable Work.—The disposable work is the work at full pressure, $p_3 V_3$, plus the work of expansion $\frac{G}{A} c_v (T_3 - T_4')$, less the work of the back pressure $p_4 V_4'$. Hence

$$L_d = p_3 V_3 + \frac{G}{A} c_v (T_3 - T_4') - p_4 V_4'.$$

This expression is not convenient for use, hence we seek a theoretical quantity, T_x, which, inserted in the expression for disposable work for complete expansion, will give the same value for L_d as would be obtained by the expression above. This is precisely similar to what we have done for full pressure. We obtain T_x from the expression

$$p_3 V_3 + \frac{G}{A} c_v (T_3 - T_4') - p_4 V_4' = \frac{G}{A} c_p (T_3 - T_x).$$

Substituting for V_3 and V_4' their values, $\frac{G}{A} (c_p - c_v) \frac{T_3}{p_3}$ and $\frac{G}{A} (c_p - c_v) \frac{T_4'}{p_4'}$, and divided through by $\frac{G}{A}$, we obtain

$$(c_p - c_v) T_3 + c_v (T_3 - T_4') - (c_p - c_v) T_4' \frac{p_4}{p_4'} = c_p (T_3 - T_s),$$

or by reduction

$$c_v T_4' + (c_p - c_v) T_4' \frac{p_4}{p_4'} c_p T_s,$$

whence

$$\frac{T_s}{T_4'} = \frac{c_v}{c_p} + \frac{c_p - c_v}{c_p} \frac{p_4}{p_4'} = \frac{1}{k} + \frac{k-1}{k} \frac{p_4}{p_4'} \quad . \quad . \quad . \quad \text{(CXII.)}$$

This expression is the same as CVIII., found on page 347, for full pressure. Therefore the values given in the table on page 347, for $\frac{T_s}{T_3}$ hold good here also for $\frac{T_s}{T_4'}$.

We have then

$$L_d = \frac{G}{A} c_p (T_3 - T_s) = \frac{G}{A} c_p T_3 \left(1 - \frac{T_s}{T_3}\right)$$

and

$$L_d = \frac{G}{A} c_p T_3 \left(1 - \frac{T_s}{T_4'} \frac{T_4'}{T_3}\right) \quad . \quad . \quad . \quad . \quad . \quad \text{(CXIII.)}$$

The values of $\frac{T_4'}{T_3}$ and $\frac{T_s}{T_4'}$ are easily found from the tables on pages 346 and 347, when the ratio $\frac{p_4'}{p_3}$ and $\frac{p_4}{p_4'}$ are given.

Weight of Air per Second and Volume of Cylinder.—From the formula

$$L_d = \frac{G}{A} c_p (T_3 - T_s) = \frac{550 N}{\eta} \text{ we obtain}$$

$$G = \frac{550 A N}{\eta c_p (T_3 - T_s)} = \frac{3}{\eta} \frac{N}{(T_3 - T_s)} \quad . \quad . \quad . \quad . \quad \text{(CXIV.)}$$

in English measures.

This expression agrees exactly with that given on page 341 for complete expansion.

For the volume of the cylinder we have

$$V' = R \frac{G}{2n} \frac{T_4'}{p_4'}.$$

If in this expression we substitute for G the values just deduced, we get

$$V' = 275\frac{c_p - c_v}{c_p}\,\frac{N}{\eta n}\,\frac{T_4'}{p_4'(T_3 - T_x)},$$

since for R we can substitute $\frac{c_p - c_v}{A}$. This reduces to

$$V' = 275\,\frac{k-1}{k}\,\frac{N}{\eta n}\,\frac{1}{p_4'\,\frac{T_3}{T_4'}\left(1 - \frac{T_x}{T_3}\right)},$$

or

$$V' = 79.75\,\frac{N}{\eta n}\,\frac{1}{p_4'\,\frac{T_3}{T_4'}\left(1 - \frac{T_x}{T_3}\right)} \quad . \; . \; . \; . \; . \quad \text{(CXV.)}^*$$

Graphic Representation of the Theory of Compressed Air Engines.—The general expression for the available work of an air engine, which includes all three cases of complete expansion, full pressure, and incomplete expansion, is then the Eq. CXIII., found on page 351, viz.,

$$L_d = \frac{G}{A}\,c_p T_3\left(1 - \frac{T_x}{T_4'}\,\frac{T_4'}{T_3}\right),$$

or in English measures

$$L_d = 183.35 G T_3\left(1 - \frac{T_x}{T_4'}\,\frac{T_4'}{T_3}\right) \quad . \; . \; . \; . \quad \text{(CXVI.)}$$

We have shown that

$$\frac{T_x}{T_4'} = 0.71 + 0.29\,\frac{p_4}{p_4'},$$

and

$$\frac{T_4'}{T_3} = \left(\frac{p_4'}{p_3}\right)^{\frac{k-1}{k}}$$

Substituting these values we obtain

$$L_d = 183.35 G T_3\left[1 - \left(0.71 + 0.29\frac{p_4}{p_4'}\right)\left(\frac{p_4'}{p_3}\right)^{\frac{k-1}{k}}\right],$$

whence

$$\frac{L_d}{183.35 G T_3} = 1 - \left(0.71 + 0.29\,\frac{p_4}{p'_4}\right)\left(\frac{p'_4}{p_3}\right)^{\frac{k-1}{k}} \quad . \; . \quad \text{(CXVII.)}$$

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* [This expression, Mr. Willis observes, corresponds to that given by Mons. Pernolet, p. 64 of "*L'air comprimé*, but the latter is defective in that it contains no factor representing the efficiency of the air engine.]

If now we let $\frac{L_d}{188.85GT_3} = y$, $\frac{p_4}{p_4'} = x$, and $\frac{p_4'}{p_3} = \frac{1}{z}$, and $\frac{k-1}{k} = m$, we have

$$y = 1 - (0.71 + 0.29x)\frac{1}{z^m} \quad . \quad . \quad . \quad . \quad . \quad \text{(CXVIII.)}$$

which is the equation of a surface whose co-ordinates are x, y, and z.

If in this equation we suppose z known, and therefore $\frac{1}{z^m}$ = a constant $= C$, we have

$$y = 1 - (0.71 + 0.29x)\,C,$$

which is the equation of a straight line.

Hence if we give to z various values from 1 up to any desirable limit, we shall get a corresponding number of equations of straight lines, in which

$$y = \frac{L_d}{188.85GT}$$

depends upon the disposable work and

$$x = \frac{p_4}{p_4'}.$$

Since $z = \frac{p_3}{p_4'}$, and $x = \frac{p_4}{p_4'}$, we have

$$\frac{z}{x} = \frac{p_3}{p_4}, \text{ or}$$

$$p_3 = \frac{z}{x}p_4 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(CXIX.)}$$

Now p_4 is the pressure of the exterior air = 1 atmosphere. Hence the initial pressure

$$p_3 = \frac{z}{x} \text{ atmospheres.}$$

Let us assume that we have given the number of disposable foot-lbs. of work required per second, the weight G of air to be used per second, and the initial temperature T_3. From these data we can calculate y.

If now we have a diagram upon which are drawn the lines whose equations are of the form

$$y = 1 - (0.71 + 0.29x)\,C,$$

we can read off from it the values of x and z that will give the desired value of y. In other words, we can find at once the initial pressure $\frac{z}{x}$ in atmospheres, and the degree of expansion $z = \frac{p_3}{p_4'}$ that will give the required power with the given weight of air per second.

Such a diagram has been constructed (see diagram at end of this chapter), in which the values of z or $\frac{p_3}{p_4'}$ range from 1 up to 10.

Construction of the Diagram.—Let the plane of the paper upon which the diagram is drawn be a horizontal plane, whose vertical ordinate, referred to the same origin as the surface, is $z = 1$. The equation of the surface, when z is constant, that is, when $\frac{p_3}{p_4'}$ is given, is the equation of a straight line,

$$y = 1 - (0.71 + 0.29x)\,C.$$

If now, in this equation, we fix the value of x, and calculate the corresponding value of y, we can construct a point of this line. By finding two such points we can draw the line.

The most convenient values for x are $x = 0$ and $x = 1$. These give us

$$\text{for } x = 0, \qquad y = 1 - \frac{0.71}{z^m} = 1 - \frac{0.171}{\left(\frac{p_3}{p_4'}\right)^{\frac{k-1}{k}}} = 1 - 0.71\frac{T_4'}{T_3},$$

and

$$\text{for } x = 1, \qquad y = 1 - \frac{1}{z^m} = 1 - \frac{T_4'}{T_3}.$$

Now for any given values of z or $\frac{p_3}{p_4'}$ we can find $\frac{T_4'}{T_3}$ from the table on page 173, under the head of $\frac{T_1}{T_2}$, and thus can easily calculate y.

The second result is given by the intersection of the horizontal line 0.28 with the line $z = 2$. The value of x is 0.61. Hence

$$p_3 = \frac{z}{x} = \frac{2}{0.61} = 3.27 \text{ atmospheres.}$$

That is, if the cut-off is such that the pressure is reduced by expansion to ½, the engine will give 50 H. P. with one pound of air at an initial pressure of 3.27 atmospheres.

The third intersection gives us $z = 3$ and $x = 0.95$, hence

$$p_3 = \frac{z}{x} = 3.15 \text{ atmospheres.}$$

There are, of course, many other degrees of expansion, with their corresponding initial pressures, intermediate between those just found, that will give the required result, but which do not appear on the diagram because the diagonal lines corresponding to the ratio $\frac{p_3}{p_4}$ are not drawn.

In order that the initial pressure may be found for any, degree of expansion, the curves zBC and $x_1B'C''$ are introduced.

These are obtained as follows :

If we intersect the surface whose equation is

$$y = 1 - (0.71 + 0.29x)\frac{1}{z^m}$$

by any plane parallel to the plane YOZ, x is for that intersection a constant, and the equation of the curve of intersection is

$$y = 1 - (0.71 + 0.29a)\frac{1}{z^m}.$$

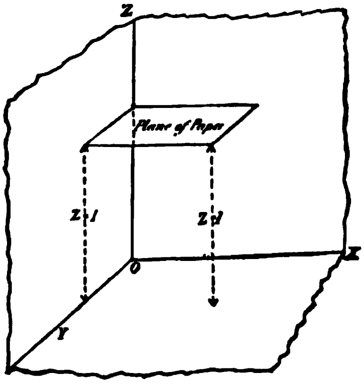

Fig. 57.

The values of y can then be laid off upon the vertical lines X_0Y and X_1Y' in the diagram, and the straight lines EE' drawn through each two points which have a common value for Z.

For each of these lines zz' there is a certain given value for z or $\frac{p_3}{p_4}$, hence each line corresponds to a given degree of expansion, which is denoted by the figures 1, 2, 3, etc., written at the intersections of the lines with x_1y'.

To illustrate the use of this portion of the diagram, let us assume a case.

Given an engine of 50 H. P. which is to use 1 pound of air per second. Required the amount of expansion and initial pressure which will give this result.

We have $L_d = 550 \times 50 = 27500$ ft.-lbs., and we may assume $T_3 = 527.4°$ Fahr. We have then

$$y = \frac{L_d}{183.35GT_3} = \frac{27500}{183.35 \times 1 \times 527.4} = 0.28.$$

Find 0.28 on the line X_0Y, and following it horizontally across the diagram, we can determine three methods of arriving at the desired result.

The horizontal line first intersects the diagonal line for $z = 1$, or the line of full pressure, at a point whose abscissa is $x = 0.032$. Hence

$$p_3 = \frac{z}{x} = \frac{1}{0.032} = 31.25 \text{ atmospheres.}$$

That is, if the engine works at full pressure and uses 1 pound of air per second, the air must have a pressure $p_3 = 31.25$ atmospheres to give 50 H. P.

If now we make $x = a = 0$, and $x = a' = 1$, we shall get the equations of two intersections of the surface, viz.,

$$\text{for } x = 0, \qquad y = 1 - \frac{0.71}{z^n},$$

and

$$\text{for } x = 1, \qquad y = 1 - \frac{1}{z^n}.$$

If we revolve the intersecting plane about its line of intersection with the drawing paper, the curve of intersection with the surface takes the positions zBC and $x_1B'C'$.

In this position we can construct them, remembering that since the plane of the drawing paper is assumed to have a vertical ordinate $z = 1$, we must lay off $z - 1$ instead of z in the direction x_0x_1 from x_0 and x_1.

If now we have any value for z, whose diagonal line is not given in the diagram, we seek the points on the curves EBC and $x_1B'C'$ which correspond to this desired z, and project these points upon the lines x_0y and x_1y'. Connecting these two projections, we have the desired diagonal line, and may then proceed to find the initial pressure that will give any desired power with this amount of expansion.

It may be required to find the initial pressure which will give some desired power at complete expansion.

In this case $p_4' = p_4$, and therefore

$$x = \frac{p_4}{p_4'} = 1:$$

hence

$$p_3 = \frac{z}{x} = z.$$

Having found, therefore, from given data, the value of y, for instance, in the example assumed, $y = 0.28$, we follow the corresponding horizontal line across to its intersection with the curve $x_1B'C'$, for in the equation of this curve $x = 1$. The value of z for the point of intersection is read off on the bottom line of the diagram, and we have $p_3 = z$ atmospheres, or, in the assumed example, $p_3 = 3.12$ atmospheres.

The case may arise in which we have given the desired power, the weight of air per second, and the initial pressure, required the amount of expansion.

$$\text{We have } p_3 = \frac{z}{x}, \text{ hence } z = p_3x.$$

If therefore we can draw upon the diagram curves of constant initial pressure, from which for a given y we can readily find x, z is easy to calculate.

Such curves are drawn upon the diagram. To construct them, we take the equation $x = \frac{z}{p_3}$. For instance, assume $p_3 = 10$ atmospheres, and give to z various values from 1 up to 10.

For each value of z we have a value of x, which we can fix on the diagonal corresponding to that z. Through the points thus obtained we draw the curve. If now we desire to know the degree of expansion that will give 50 H. P. from 1 pound of air per second at an initial pressure of 10 atmospheres, we have as before $y = 0.28$. Following this horizontal line to its intersection with the curve of constant initial pressure $p_3 = 10$ atmospheres, we get $x = 0.118$, hence

$$z = \frac{p_3}{p_4'} = p_3 x = 1.18.$$

There are one or two points connected with the diagram to which it is well to call attention.

A small variation in the value of x makes considerable difference in the degree of expansion. That portion of the diagram upon which x is read off, is therefore drawn on a scale twice that assumed for z. For the same reason the scale upon which y is measured is very much greater than either of the others. The scale for measurement on the curve EBC is the same as that for $x_1B'C'$, and is found at the top of the diagram.

Variations of Work with Different Degrees of Expansion.—Referring to the diagram, we see that the curves of constant initial pressure have a horizontal tangent at their intersections with the line of complete expansion x_1y', where $x = 1$, and that their curvature is slight for some distance from this point. Since y, the representative of the disposable work, is measured vertically, we see that the loss of work is not very great when the expansion is not complete within certain limits. Assume, for instance, that we have an engine working with complete expansion and initial pressure = 10 atmospheres. We find $y = 0.487$. Assume, now, that with the same initial pressure we cut off at such a point that $z = \frac{p_3}{p_4'} = 5$. Then $x = \frac{z}{p_3} = 0.5$. Seeking on the diagram for the curve of constant initial pressure $p_3 = 10$, the value of y for the abscissa $x = 0.5$, we find $y = 0.463$.

The loss of work is then $0.487 - 0.418 = 0.05$ per cent. The point of cut-off is readily found from the equation

$$\frac{p_3}{p_4'} = \left(\frac{v_4'}{v_3}\right)^k$$

Referring to table on page 173, we find for $\frac{p_3}{p_4'} = 5$, $\frac{v_4}{v_3} = 0.319$.

That is, cutting off at about ⅓ the stroke, we lose but 0.05 per cent. of the work.

Influence of the Vapor of Water Contained in the Air.—The preceding discussion has been based on the assumption that the air acted upon, or acting, was dry, and followed the laws of perfect gases.

In practice water is always injected into the compressing cylinder, to keep down the final temperature, and the working medium is therefore a mixture of water with air saturated with vapor of water. M. Pernolet has discussed at length the influence of water and its vapor upon the work of compression, the disposable work of the compressed air, the final temperature in the compressor and the air engine, etc. He concludes from his discussion,

1. That the influence of the vapor of water upon the work of compression, as well as upon the disposable work of the compressed air is relatively slight, and can be neglected in all approximate calculations for which the formulæ deduced for dry air are sufficiently correct.

2. Vapor of water in the compressor and in the air engine reduces materially the final temperature of compression, and raises the final temperature of expansion.

The assumption that compression and expansion take place according to the adiabatic law is also not strictly true, because the walls of the cylinder are always either receiving heat from or imparting heat to the air.

An analysis of this question by M. Mallard shows that the quantity of heat thus transmitted is small, and may be neglected.

Thus, for all practical purposes, the formulæ deduced for dry air acting adiabatically are sufficiently accurate.

EXAMPLES FOR PRACTICE.

1. If the specific heat of mercury under constant pressure is 0.033, how many pounds at the temperature of 240° will be necessary to raise 12 pounds of water from 50° to 58° ? *Ans.* 16 lbs.

2. Let w be the weight of one body, $t°$ its temperature, and c its specific heat.

Let w' be the weight of a second body, $t'°$ its temperature, and c' its specific heat.

What is the temperature x of the mixture ? *Ans.* $x = \frac{wct + w'c't'}{wc + w'c'}$.

3. Reduce − 40° Fahrenheit to Centigrade. *Ans.* − 40°.

Reduce − 273° C. to Fahrenheit. *Ans.* − 459.4.

4. How do you reduce generally Fahrenheit to Centigrade degrees ?
Ans. $\frac{9}{5}$ C. = F. − 32.

5. What outer work is performed when 1 kilogram of air is heated under the pressure of the atmosphere from 0° to 1° C ? *Ans.* 29.272 meter-kil.

6. What outer work is performed when 1 pound of air is heated under the pressure of the atmosphere from 0° to 1° F. ? *Ans.* 53.299 foot-lbs.

7. If the specific heat for air under constant volume is 0.16847, and under constant pressure 0.23751, what is the mechanical equivalent of heat in French measures ? *Ans.* 424 meter-kil.

8. If 10 cubic feet of air are heated from 0° to 10° C., what is the new volume ? *Ans.* 10.367 cubic feet.

9. What is the density ? *Ans.* 0.9646.

10. If two kilograms of water are heated under atmospheric pressure from 0° to 100° C., what expenditure of work is equivalent to the heat imparted ?
Ans. 84800 meter-kil.

11. What is the weight of 3 cubic meters of air at atmospheric pressure and 20° C. temperature ? *Ans.* 3.615 kilograms.

12. What is the volume of 2 kilograms of air at the temperature of 100° C., and pressure of 2 atmospheres ? *Ans.* 1.0565 cubic meters.

13. What is the pressure of 4 kilograms of air, when the volume is 2 cubic meters and temperature 200° C. ? *Ans.* 27691.3 kil.

14. What is the temperature of 8 kilograms of air, when the pressure is 6 atmospheres and volume 2 cubic meters ? *Ans.* 256.55°.

15. What work is performed by 10 kilograms of air at 2 cubic meters volume, and 5 atmospheres pressure, when it expands to 6 cubic meters, overcoming an

outer pressure equal at any moment to the tension, the temperature being kept constant ? *Ans.* 113581 meter-kil.

What is the constant temperature during expansion ? *Ans.* 80° C.

What is the final pressure ? *Ans.* 1.66 atmos.

How much heat must have been imparted during expansion in order to keep the temperature constant ? *Ans.* 267.76 heat units.

16. If 2 kilograms of air, having the volume of 3 cubic meters, expands as above, performing work, from the pressure of 4 atmospheres down to one atmosphere, and the temperature, during expansion, remains constant, what is the work done ? *Ans.* 171895 meter-kil.

What is the constant temperature during expansion ? *Ans.* 1845° C.

What is the final volume ? *Ans.* 12 cubic meters.

How much heat must have been imparted during expansion ? *Ans.* 405.4 heat units.

17. If a blowing engine changes per second 10 cubic feet of air, at a pressure of 28 inches, into a blast at a pressure of 30 inches, what is the work per second ? *Ans.* 1366.4 foot-lbs., or 189 meter-kil. per sec., or about 2.5 horse-power.

What is the volume after compression of the 10 cubic feet ? *Ans.* 9.33 cubic feet.

If the temperature is 60° F., what is the weight of 10 cubic feet ? *Ans.* 0.71404 lbs.

How much heat must have been abstracted during compression? *Ans.* 1.77 heat units per second.

18. If one kilogram of air is heated under the pressure of the atmosphere from 0° to 1° C., how much work does it perform during expansion? *Ans.* 29.272 meter-kil.

19. If one pound of air is heated under the pressure of the atmosphere from 32° to 33° Fahrenheit, how much work does it perform during expansion? *Ans.* 53.268 foot-lbs.

20. What is the weight of one cubic foot of air at atmospheric pressure and 32° F. temperature ? *Ans.* 0.08073 lbs.

21. What is the weight of one cubic meter of air under the same pressure and 0° C. temperature ? *Ans.* 1.29318 kil.

22. If under the piston of a steam engine, whose diameter is 16 inches, there is a quantity of steam 15 inches high and at a tension of 3 atmospheres, and if this steam in expanding moves the piston 25 inches, what is the work done, if we assume Mariotte's law to be true for the expansion of steam? and what is the mean force upon the piston when we neglect friction and the opposing pressure? *Ans.* 10866 foot-lbs. 5217 pounds.

23. If a given weight of air, say 2 kilograms, at a temperature of 30° C. expands adiabatically, performing work, till its volume is doubled, what is the final temperature ? *Ans.* – 44.9°.

What is the original volume? *Ans.* 1.72 cubic meters.

What is its final pressure if the initial pressure is 1 atmosphere? *Ans.* 0.38 atmosphere.

What work does it perform? *Ans.* 10714.65 meter-kilograms.

24. What is the C. equivalent of 15° F. ? *Ans.* – 9.444.

25. Zinc boils at 1204° F., mercury at 608° F. Change these readings to C. *Ans.* 650° C. and 320° C.

26. Change the following readings: Polished steel is of a deep blue color at 580° F.; polished steel is of a pale straw color at 460° F.; sea-water freezes at 28° F. *Ans.* 304.5° C.; 237.75° C.; – 2.2° C.

27. At 0° C. a cast-iron beam is 12 feet long. What is its length at 1000° C., supposing the law of similar increments to hold for that temperature? *Ans.* 12.13524 feet.

28. At 25° C. a bar of wrought iron was 16 feet long. What is its length at 96° C.? *Ans.* 16.01403 feet.

29. By how much would the length of a submarine copper cable at 0° shorten, if the temperature became – 20° C.? *Ans.* 0.0003434 of the length at 0°.

30. A wheel of wrought iron has an inside diameter of 5 feet when at the temperature of 970° C. What is its diameter at 0°? *Ans.* 4.9 feet.

31. A cylindric plug of copper just fits into a hole 4 inches in diameter in a piece of cast iron. After heating the mass to the temperature of 1240°, by how much is the diameter of the hole too small for the plug? *Ans.* 0.0293 inches.

32. Two rods, one of copper the other of iron, measure 9.8 decimeters each in length at 0°. What is their difference in length at 57°? *Ans.* 0.0027 decimeters.

33. The wooden pattern of a cast-iron beam must be longer than the casting at 0°. For a beam 12 meters long at 0°, what is the length of pattern? Cast iron melts at 1530° C. *Ans.* 12.207 meters.

34. What amount of work is involved in lifting 70 lbs. 6 feet high? *Ans.* 420 foot-pounds.

35. What work is involved in lifting 9000 cubic feet of water 46 feet? (A cubic foot of water weighs 62.4 lbs.) *Ans.* 25833600 foot-pounds.
How many units of heat (English) does this correspond to? *Ans.* 33463.

36. What work is involved in a piston moving 6 feet under an effective pressure of 17 lbs. per sq. inch, its area being 1670 sq. inches? *Ans.* 170340.

37. The piston of a steam engine is 21 inches in diameter, stroke 6 feet, mean pressure 16 lbs. per sq. inch; the engine makes 40 revolutions per minute. What is the horse-power (English)? *Ans.* 80.6144.

38. What time will be taken by a steam engine of 64 H. P. to lift 5360 tons of water 20 feet? *Ans.* 114 minutes.

39. How many heat units (English) per hour are involved in the idea of 63 H. P.? *Ans.* 88912.82.

40. A cubic mile of water is to be lifted from a depth of 2 feet in 800 hours. How many H. P. of a steam engine is necessary? A cubic foot of water weighs 62.4 lbs. *Ans.* 11597.6 H. P.

41. The resistance of friction, etc., to a train is a force equivalent to the weight of 600 lbs. How many H. P. of the locomotive will draw the train at the rate of 35 miles an hour? *Ans.* 56 H. P.

42. How many cubic feet of water will an engine of 10 H. P. raise from a depth of 150 feet in 24 hours? *Ans.* 50770 cubic feet.

43. What work is performed on a train weighing 500 tons in 8 miles of a level road, the resistance to traction being [illegible]th of the load? If this work were done in 6 minutes, what would be the H. P. of the engine?
Ans. 65736000 foot-pounds. 332 H. P.

44. Suppose the resistance to the progress of a vessel weighing 1260 tons to be 18 lbs. a ton when the speed is 6 knots, and that the resistance varies as the square of the speed; what work will be done by a vessel in 5 nautical miles, and what will be the H. P. of the engine when the speed is 12 knots? There are 6080 feet in a nautical mile, and a *knot* is a velocity of one nautical mile per hour.
Ans. About 2757894000; 3342.88 H. P.

45. What amount of work will be spent in the friction of a weight of 6 tons dragged along a level table for a length of 7 feet, when the coefficient of friction is 0.235? What will be the H. P. of an engine which would do this work in one second? *Ans.* 22108.8. 40.19 H. P.

46. In a cylinder we have 2 kilograms of air at a tension of 1½ atmospheres and a temperature of 30° C. What is the volume of this air?
Ans. 1.14436 cubic meters.

If this air expands adiabatically till the tension is 1 atmosphere, what will be the final temperature? *Ans.* − 3.77°.

What will be the final volume? *Ans.* 1.5258 cub. meters.

What work does it perform? *Ans.* 4824 meter-kil.

How many units of heat disappear? *Ans.* 11.38 heat units.

47. If 3 cubic meters of air at 150° and a pressure of 4 atmospheres expands adiabatically to double its former volume, what is the final temperature?
Ans. 45.85°.

What is the weight of this air? *Ans.* 10 kil.

What is the work performed? *Ans.* 74856 meter-kil.

48. If 10 kilograms of air occupy a space of 2 cubic meters, under a pressure of 6 atmospheres, what must be the temperature? *Ans.* 150.64°.

If it expands adiabatically till the final temperature is 48°, what is the final volume? *Ans.* 3.94 cub. meters.

What is the final pressure? *Ans.* 2.3 atmos.

What is the work done? *Ans.* 73316.8 meter-kil.

49. Two kilograms of air at 10° is heated under constant atmospheric pressure till the temperature is 80°. What is the initial volume?
Ans. 1.603 cub. meters.

What is the final volume? *Ans.* 2 cub. meters.

What is the work done? *Ans.* 4098 meter-kil.

What is the heat imparted? *Ans.* 33.25 heat units.

Of this heat how much disappears as outer work? *Ans.* 9.66 heat units.

50. Four kilograms of air at 20° C. and under atmospheric pressure is heated and expands isopiestically. After expanding till its volume is doubled, what is its temperature? What was its initial volume? and final volume?
Ans. 313°. 3.32 and 6.64 cub. meters.

What is the work done? *Ans.* 8576 meter-kil.

How many units of heat are imparted? *Ans.* 278.4 heat units.

How many disappear as outer work? *Ans.* 81 heat units.

51. If the air had not been allowed to expand, and still the same amount of heat had been imparted, what would have been the temperature? what the pressure? *Ans.* 431.34°; 2.4 atmospheres.

52. If one kilogram of air has the temperature 30° and the pressure of 1.5 atmospheres, and is cooled, the volume remaining constant, till the pressure is one atmosphere, what is the final temperature? *Ans.* − 71°.

What is the amount of heat abstracted? *Ans.* 17.015 heat units.

53. If one kilogram of air has the temperature 30°, and is heated under constant pressure till the final volume is $\frac{4}{3}$ of the initial, what is the final temperature? *Ans.* 131°.

What is the amount of heat imparted? *Ans.* 23.9885 heat units.

What is the outer work? *Ans.* 2956.5 meter-kil.

54. Suppose one kilogram of air of the temperature 30° expands, according to the law $p_1v_1^2 = p_2v_2^2$. until its final volume is $\frac{4}{3}$ of its initial. What is the final temperature? *Ans.* – 45.75°.

What is the outer work? *Ans.* 2217.35 meter-kil.

What is the specific heat in this case? *Ans.* 0.09940.

Must heat be added or subtracted during this expansion? *Ans.* Subtracted.

How much? *Ans.* 7.530 heat units.

55. One kilogram of air of one atmosphere pressure, 30° temperature, expands adiabatically, without overcoming any outer pressure till its volume is doubled. What is the final pressure? *Ans.* 0.5 atmosphere.

56. Suppose 10 units of heat are added during expansion. What is the final temperature? *Ans.* 79.85°.

What is the final pressure? *Ans.* 0.581 atmos.

57. At midday a person observed that his Fahrenheit thermometer marked 77 degrees, and at sunset that his Centigrade thermometer marked 10 degrees. How much has each thermometer fallen in the interval?

Ans. { F. has fallen 27°.
C. has fallen 15°.

58. What temperature is denoted by the same number in the Centigrade and Fahrenheit scales? *Ans.* – 40°.

59. What is the temperature when the sum of the readings of the same thermometer graduated according to Fahrenheit and Centigrade scales is 130? *Ans.* 95° F.

60. What is the temperature when the difference of the readings of the same thermometer graduated according to Fahrenheit and Centigrade scales is 60? *Ans.* 95° F. or – 175° F.

61. A thermometer is graduated both on Fahrenheit and Centigrade scales, and the reading on the former exceeds that on the latter by 24. What is the reading of each? *Ans.* 14° F. or – 10° C.

62. On the summit of a mountain water boils at 152° F. If a thermometer be graduated there without taking into account the atmospheric pressure, what would be the real temperature when this thermometer registers 92° F.? *Ans.* F. = 72°.

63. A thermometer is constructed which registers – 10° at freezing point and 110° at boiling point. What will be the temperature by Fahrenheit scale, when the reading on this thermometer is 60°? *Ans.* F. = 187°.

64. If a line of railway be laid with iron rails 5 yards long at 0° C., what should be the distance between two consecutive rails, to allow for the expansion due to an increase of temperature of 55° C.? (Coefficient of linear expansion for one degree Centigrade = 0.00001235.) *Ans.* 0.122 inch.

65. If a line of railway be laid with steel rails 25 feet long at 60° Fah., what should be the distance between two consecutive rails, to allow for the expansion due to a range of temperature from – 20° F. to 100° F.? (Coefficient of linear expansion for one degree *Centigrade*, counting from zero C. = 0.00001079). *Ans.* 0.215 inch.

66. A mass of mercury occupies 8.144 cubic inches at 100° C. What will be the increase in volume when its temperature is raised to 150° C.? (Coefficient of *linear* expansion of mercury = 0.00006 for one degree Cent.) What will be its volume at 0° C.? *Ans.* 0.072 cub. inch. 8 cubic inches.

67. A straight vertical tube, closed at its lower end, contains a small quantity of mercury which is supported at the height of 28 inches from the bottom of the tube by the confined air. The temperature of the air is 7° C. If the temperature is increased to 27° C., what position will the mercury occupy? *Ans.* 30 inches from bottom of tube.

68. If the temperature in the last example is 100.6° F., and is raised to 140.6° F., what is the position of the mercury? *Ans.* 30 inches from bottom of tube.

69. A quantity of gas occupies 150 cubic inches at a temperature of 2° C. when the barometer stands at 29.7 inches. What space will it occupy if the temperature rise to 16° C. and the barometer to 30.6 inches? *Ans.* 153 cubic inches.

70. If the temperature in the preceding example is 90.6° F., and rises to 118.6° F., what space does the gas occupy? *Ans.* 153 cubic inches.

71. If the compressed air in a flooded coal pit occupies 2500 cubic feet at a temperature of 50° F., and under a pressure of 70 inches of mercury, how much space would it fill at a temperature of 60° F. and under a pressure of 29.5 inches of mercury? *Ans.* $6048\frac{187792}{400846}$ cub. feet.

72. Two condensers contain equal quantities of air. One of them, at temperature 52° C., is 20 in. long, 15 in. broad, and 13 in. high, and the other at 57° C., is 2 feet long. 15 in. broad, and 11 in. high. Show that the air pressure is the same in both.

73. When the roof of a cylindrical diving bell is 27 feet below the surface of the water the temperature of the air within it is 2° C., and the level of the water in the bell is 5 feet from the roof. If now the temperature of the air in the bell is raised to 7° C., and the bell is moved until the level of the water is 1 ft. lower than at first, has the bell been raised or lowered, and how much? (Pressure of the atmosphere is supposed to be equal to a column of water 34 feet high.) *Ans.* Raised 11 feet.

74. A certain volume of gas is at a temperature of − 3° C. and under a pressure of 800 mm. If the pressure is diminished to 540 mm., what temperature must the gas have in order that its volume may be doubled? *Ans.* $t = 91.5°$ C.

75. A straight vertical tube whose section is one square inch, is closed at its lower end, and contains a quantity of air which supports an air-tight piston whose weight is one pound. The position of the piston is observed when the temperature of the air is 31° C., and the weight of the piston is then increased by 1 pound. Find what increase of temperature will be required to bring back the piston to its former position, the atmospheric pressure being 15 lbs. per sq. inch. *Ans.* New temp. = 50°, increase = 19°.

76. A volume of air at 0° C. and at a given pressure weighs 8 oz.; what weight of air would occupy the same volume at 30° C., the pressure being 4 times as great? *Ans.* $28\frac{84}{101}$ oz.

77. If the air in a fire balloon, the volume of which is 100 cub. ft., is at a temperature of 127° C., when the temperature of the surrounding air is 27° C., what weight, including that of the balloon itself, will just prevent it from ascending? The weight of a cubic ft. of air at 0° C. and atmospheric pressure being 1.2 oz. *Ans.* 27.3 oz.

78. If 1 cub. ft. of air at 0° and under a pressure of 760 mm. weighs 1.2 oz., what will a cub. ft. at 27° and under a pressure of 600 mm. weigh? *Ans.* 0.862 oz., nearly.

79. A body whose volume is 10 cub. ft. weighs 251 lbs. 5 oz. when weighed in air at a temperature of $-8°$ C. and under a pressure of 800 mm. What would be its weight in vacuo? (One cubic foot of air at 0° and pressure of 760 mm. weighs 1.2 oz.) *Ans.* 252 lbs. 1.66 oz., nearly.

80. A certain volume of air at 0° C. and at a given pressure, weighs 10 oz. What weight of air would occupy the same volume at 20° C., the pressure being doubled? *Ans.* $18\frac{186}{293}$ oz.

81. A room contains 4480 cubic feet of air ($c_p = 0.2375$), weighing 1.25 oz. per cubic foot. How many units of heat will be required to raise the temperature of the air 20°? If the initial temperature of the air is 16°, how many pounds of water at 74° will be required to raise the temperature of the air by the required amount? *Ans.* 1662.5 heat units, $43\frac{3}{4}$ lbs.

82. If the heat obtained from the combustion of 1 lb. of coal raise the temperature of 1000 lbs. of iron from 0° to 50° C., find the number of units of heat given out; the sp. heat of iron being 0.12 (a heat unit being taken as 1 lb. of water 1° C.). *Ans.* 6000.

83. A bar of iron weighing 25 lbs., whose temperature is 16° C., is plunged into 15 lbs. of water at 10° C., when the common temperature becomes 11° C. Find the specific heat of iron. *Ans.* 0.12.

84. A piece of tin weighing 125 oz. is immersed in boiling water, until it is of the same temperature; it is then taken out and dropped into a vessel containing 68 oz. of water at freezing, and the temperature of the water rises to 10° C. Find the specific heat of tin. *Ans.* 0.056.

85. A pound of platinum is placed in a furnace, and having acquired the temperature of the furnace, is plunged into a vessel containing 10 lbs. of water at 10° C. The temperature of the water rises to 14.3° C. What is the temperature of the furnace? (The specific heat of platinum is 0.032).
Ans. 1858.05° C.

86. If some water at 20° C. be mixed with a fluid at 150° C. weighing half as much as the water, and the temperature of mixture be 19° C., find the specific heat of the fluid. *Ans.* 0.5.

87. A mass of 10 lbs. of iron at the temperature 175° C., is immersed in 1 lb. of ice and water at 0° C., the masses of the ice and water being in the ratio 1:9; and the temperature of the whole mass becomes 20° C. Find the specific heat of iron, the latent heat of ice being 79. *Ans.* 0.018.

88. Two chains, *A* and *B*, are contained in a jar which is maintained at a temperature 114° C. *A* consists of 9 links of iron and 11 links of copper, and *B* consists of 9 links of copper and 11 links of iron, and each link weighs one ounce. The chain *A* is taken out of the jar and placed in a vessel containing 198 oz. of water at 18° C., and causes the temperature to rise by 1° C. Had both chains been placed in the vessel, the temperature would have risen by 2° C. What are the specific heats of iron and copper? *Ans.* Iron 0.11, copper 0.09.

89. Weights w. w' of two substances, whose specific heats are c, c', and temperatures t, t' are mixed. If no heat is lost, what is the temperature of the mixture?

$$Ans. \ \frac{wct + w'c't'}{wc + w'c'}.$$

90. One kilogram of mercury, specific heat $\frac{1}{30}$, at 130° C. is mixed with 4 kilog. of water at 10° C. What is the temperature of the mixture, if no heat is lost? *Ans.* 11°.

91. A mass of iron weighing 7 lbs. at the temperature 109° C., is placed upon a mass of ice at temperature 0° C. If the specific heat of iron is 0.118, and the latent heat of water 79, calculate the weight of water produced. *Ans.* $1\frac{1}{790}$ lbs.

92. Two lbs. of melting silver, at a temperature of 1000° C., are placed in a Laplace calorimeter and 1.4 lbs. of water drawn off. What is the specific heat of silver, the latent heat of water being 79? *Ans.* 0.0553.

REDUCTION TABLES.

TABLES FOR THE CONVERSION OF ENGLISH AND METRIC UNITS.

1 Meter = 3.2807 feet.
1 Foot = 0.3048 meter.
1 Liter (vol. of 1 kilog. water) = 0.2202 gal.
1 Gallon (vol. of 10 lbs. water) = 4.541 liters.
1 Kilogram = 2.2046 lbs. av.
1 Kilog. per sq. meter = 0.2040 lbs. per sq. ft.
1 Kilog. per sq. mm. = 1422.28 lbs. per sq. inch.
1 Lb. per sq. in. = 703.0958 kilog. per. sq. meter.
1 Gram = 15.4323 gr.
1 Grain = 0.0648 gram.
1 Meter-kilogram = 7.2331 foot-lbs.

1 Foot-pound = 0.1383 meter-kilog.
1 Atmosphere = 14.7 lbs. per sq. in. = 10334 kilog. per sq. meter = 29.922 inches, or 760 mm. of mercury = 33.9 ft., or 10⅓ meters of water.
1 Pound av. = 0.4536 kilog.
Deg. Cent. = $\frac{5}{9}$ (F.° − 32).
Deg. Fahr. = $\frac{9}{5}$ C.° + 32.
1 Calorie (kilog. water raised 1° C.) = 424 meter-kilog. = 3.9683 Eng. heat units.
1 Eng. heat unit (lb. water raised 1° F.) = 722 ft.-lbs. = 0.252 calorie.

TABLE I.

FOR CONVERTING METERS INTO INCHES.

Meters.	0	1	2	3	4	5	6	7	8	9
0	0	39.4	78.7	118.1	157.5	196.8	236.2	275.6	315.0	354.3
10	393.7	433.1	472.4	511.8	551.2	590.5	629.9	669.3	708.7	748.0
20	787.4	826.8	866.1	905.5	944.9	984.2	1023.6	1063.0	1102.4	1141.7
30	1181 1	1220.5	1259.8	1299.2	1338.6	1377.9	1417.3	1456.7	1496.1	1535.4
40	1574.8	1614.2	1653.5	1692.9	1732.3	1771.6	1811.0	1850.4	1889.8	1929.1
50	1968.5	2007.9	2047.2	2086.6	2126.0	2165.3	2204.7	2244.1	2283.5	2322.8
60	2362.2	2401.6	2440.9	2480.3	2519 7	2559.0	2598.4	2637.8	2677.2	2716.5
70	2755.9	2795.3	2834.6	2874.0	2913.4	2952.7	2992.1	3031.5	3070.9	3110.2
80	3149.6	3189.0	3228.3	3267.7	3307.1	3346.4	3385.1	3425.2	3464.6	3503.9
90	3543.3	3582.7	3622.0	3661.4	3700.8	3740.1	3779.5	3818.9	3858.3	3897.6

TABLE II.

FOR CONVERTING INCHES INTO CENTIMETERS.

Inches.	0	1	2	3	4	5	6	7	8	9
0	0	0.254	0.508	0.762	4.016	1.2699	1.5239	1.7779	2.0319	2.2859
1	2.5400	2.7940	3.0480	3.3020	3.5560	3.8090	4.0639	4.3179	4.5719	4.8259
2	5.0799	5.3339	5.5879	5.8419	6.0959	6 3498	6.6038	6.8578	7.1118	7.3658
3	7.6199	7.8739	8.1279	8 3819	8 6359	8.8898	9.1438	9.3978	9.6518	9.9058
4	10.1599	10.4139	10.6679	10.9219	11.1759	11 4298	11.6838	11.9378	12.1918	12 4458
5	12.6998	12.9538	13.2078	13 4618	13.7158	13 9697	14.2237	14.4777	14 7317	14.9857
6	15.2398	15.4938	15.7478	16 0018	16.2558	16.5097	16.7637	17.0177	17.2717	17.5257
7	17.7798	18.0338	18.2878	18.5418	18.7958	19.0497	19.3037	19.5577	19 8117	20.0657
8	20.3197	20.5737	20.8277	21.1817	21 3367	21.5896	21.8436	22.1976	22 3516	22.6056
9	22.8597	23.1137	23.3677	23.6217	23.8757	24.1296	24.3836	24.6376	24.8916	25.1456
10	25.3997	25.6537	25.9077	26.1617	26 4157	26.6696	26.9236	27.1776	27.4316	27.6856
11	27.9396	28.1936	28.4476	28.7016	28.9556	29.2095	29.4635	29.7175	29.9715	30.2255

TABLE III.

FOR CONVERTING FRENCH MEASURES INTO ENGLISH.

Meter, sq. m., cubic m.	Feet.	Inches.	Sq. ft.	Sq. in.	Cub. ft.	Cub. in.
1	3.2809	39.3706	10.7642	1550.05	35.3161	61026.2
2	6.5618	78.7412	21.5284	3100.09	70.6322	122052.4
3	9.8427	118.1118	32.2926	4650.13	105.9483	183078.6
4	13.1235	157.4824	43.0568	6200.18	141.2644	244104.9
5	16.4044	196.8530	53.8201	7750.23	176.5805	305131.1
6	19.6853	236.2237	64.5853	9300 27	211.8966	366157.3
7	22.9662	275.2943	75.3494	10850.31	247.2126	427183.5
8	26.2471	314.9649	86.1136	12400.36	282.5287	488209.7
9	29.5280	354.3355	96.8778	13950.40	317.8448	549235.9

TABLE IV.

FOR CONVERTING ENGLISH MEASURES INTO FRENCH.

Foot, sq. ft., cub. ft.	Meter.	Sq. m.	Cub. m.	Inch, sq. in., cub. in.	Centimeters.	Sq. cent.	Cub. centimeters.
1	0.304796	0.092901	0.028316	1	2.5400	6.4514	16.386
2	0.609592	0.185801	0.056631	2	5.0799	12.9029	32.773
3	0.914388	0.278702	0.084947	3	7.6199	19.3543	49.159
4	1.219184	0.371602	0.113263	4	10.1599	25.8057	65.546
5	1.523979	0.464503	0.141579	5	12.6998	32.2571	81.932
6	1.828775	0.557403	0.169894	6	15.2398	38.7086	98.318
7	2.133571	0.650304	0.198210	7	17.7799	45.1600	114.705
8	2.438367	0.743204	0.226526	8	20.3197	51.6114	131.091
9	2.743163	0.836105	0.254841	9	22.8597	58.0628	147.478
10	3.047959	0.929005	0.283157	10	25.3997	64.5143	163.864
11	3.352755	1.021906	0.311473	11	27.9396	70.9657	180.250

TABLE V.

FOR CONVERTING KILOGRAMS INTO POUNDS AVOIRDUPOIS, OR CALORIES INTO ENGLISH (CENTIGRADE) HEAT UNITS.

Kilograms.	0	1	2	3	4	5	6	7	8	9
0	0	2.205	4.409	6.614	8.818	11.023	13.228	15.432	17.637	19.841
10	22.046	24.251	26.455	28.660	30.864	33.069	35.274	37.478	39.683	41.887
20	44.092	46.297	48.501	50.706	52.910	55.115	57.320	59.524	61.729	63.933
30	66.138	68.343	70.547	72.752	74.956	77.161	79.366	81.570	83.775	85.979
40	88.184	90.389	92.593	94.798	97.002	99.207	101.412	103.616	105.821	108.025
50	110.230	112.435	114.639	116.844	119.048	121.253	123.458	125.662	127.867	130.071
60	132.276	134.481	136.685	138.890	141.094	143.299	145.504	147.708	149.913	152.117
70	154.322	156.527	158.731	160.936	163.140	165.345	167.550	169.754	171.959	174.163
80	176.368	178.573	180.777	182.982	185.186	187.391	189.596	191.800	194.005	196.209
90	198.414	200.619	202.823	205.028	207.232	209.437	211.642	213.846	216.051	218.255

TABLE VI.

FOR CONVERTING AVOIRDUPOIS POUNDS INTO KILOGRAMS, OR ENGLISH (CENTIGRADE) HEAT UNITS INTO CALORIES.

Pounds.	0	1	2	3	4	5	6	7	8	9
0	0	0.4536	0.9072	1.3608	1.8144	2.2680	2.7216	3.1752	3.6288	4.0824
10	4.536	4.9896	5.4432	5.8968	6.3504	6.8040	7.2576	7.7112	8.1648	8.6184
20	9.072	9.5256	9.9792	10 4328	10.8864	11.3400	11.7936	12.2472	12.7008	13.1544
30	13.608	14.0616	14.5152	14.9688	15.4224	15.8760	16.3296	16.7832	17.2368	17.6904
40	18.144	18.5976	19.0512	19.5048	19.9584	20.4120	20.8656	21.3192	21.7728	22.2264
50	22.680	23.1336	23.5872	24.0408	24.4944	24.9480	25.4016	25.8552	26.3088	26.7624
60	27.216	27.6696	28.1232	28.5768	29.0304	29.4840	29.9376	30.3912	30.8448	31.2984
70	31.752	32.2056	32.6592	33.1128	33.5664	34.0200	34.4736	34.9272	35.3808	35.8344
80	36.288	36.7416	37.1952	37.6488	38.1024	38.5560	39.0096	39.4632	39.9168	40.3704
90	40.824	41.2776	41.7312	42.1848	42.6384	43.0920	43.5456	43.9992	44.4528	44.9064

TABLE VII.

FOR CONVERTING METER-KILOGRAMS INTO FOOT-POUNDS.

Meter kilog.	0	1	2	3	4	5	6	7	8	9
0	0	0.72331	1.44662	2.16993	2.89324	3.61655	4.33986	5.06317	5.78648	6.50979
1	7.2331	7.9564	8.6797	9.4030	10.1263	10.8496	11.5729	12.2962	13.0195	13.7428
2	14.4662	15.1895	15.9128	16.6361	17.3594	18.0827	18.8060	19.5293	20.2526	20.9759
3	21.6993	22.4226	23.1459	23.8692	24.5925	25.3158	26.0391	26.7624	27.4857	28.2090
4	28.9324	29.6557	30.3790	31.1023	31.8250	32.5489	33.2722	33.9955	34.7188	35.4421
5	36.1655	36.8888	37.6121	38.3354	39.0567	39.7820	40.5053	41.2286	41.9519	42.6752
6	43.3986	44.1219	44.8452	45.5685	46.2918	47.0151	47.7384	48.4617	49.1850	49.9083
7	50.6317	51.3550	52.0783	52.8016	53.5249	54.2482	54.9715	55.6948	56.4181	57.1414
8	57.8648	58.5881	59.3114	60.0347	60.7580	61.4813	62.2046	62.9279	63.6512	64.3745
9	65.0979	65.8212	66.5445	67.2678	67.9911	68.7144	69.4377	70.1610	70.8843	71.6076

TABLE VIII.

FOR CONVERTING FOOT-POUNDS INTO METER-KILOGRAMS.

Foot lbs.	0	1	2	3	4	5	6	7	8	9
0	0	0.01383	0.02766	0.04149	.05532	.06915	.08291	.09681	.11064	.12447
1	0.1383	0.1521	0.1659	0.1797	0.1935	0.2073	0.2211	0.2349	0.2487	0.2625
2	0.2766	0.2904	0.3042	0.3180	0.3318	0.3456	0.3594	0.3732	0.3870	0.4008
3	0.4149	0.4287	0.4425	0.4563	0.4901	0.4839	0.4977	0.5115	0.5253	0.5391
4	0.5532	0.5670	0.5808	0.5946	0.6084	0.6222	0.6360	0.6498	0.6636	0.6774
5	0.6915	0.7053	0.7191	0.7329	0.7467	0.7605	0.7743	0.7881	0.8019	0.8157
6	0.8298	0.8436	0.8574	0.8712	0.8850	0.8988	0.9126	0.9264	0.9402	0.9540
7	0.9681	0.9819	0.9957	1.0095	1.0233	1.0371	1.0509	1.0647	1.0785	1.0923
8	1.1064	1.1202	1.1340	1.1478	1.1616	1.1754	1.1892	1.2030	1.2168	1.2306
9	1.2447	1.2585	1.2723	1.2861	1.2990	1.3137	1.3275	1.3413	1.3551	1.3689

TABLE IX.

FOR CONVERTING KILOGRAMS PER SQ. CENTIMETER INTO POUNDS PER SQ. INCH.

Kgrs. per sq. cent.	0	1	2	3	4	5	6	7	8	9
0	0	14.2228	28.4456	42.6684	56.8912	71.1140	85.3368	99.5596	113 7824	128.0052
10	142.228	156.451	170.674	184.897	199.119	213.342	227.565	241.788	256 011	270.234
20	284.456	298.679	312.902	327.125	341.348	355.571	369.794	384 017	398.240	412.463
30	426.684	440.907	455.130	469.353	483.576	497.599	512.022	526.245	540.468	554.691
40	568.912	583.135	597.358	611.581	625.804	640 027	654.250	668.473	682.696	696.919
50	711.140	725.363	739.586	753.809	768.032	782.255	796.478	810.701	824.924	839.147
60	853.368	867.591	881.814	896.037	910.260	924.483	938.706	952.929	967.152	981.375
70	995.596	1009.819	1024.042	1038.265	1052.488	1066.711	1080.934	1095.167	1109.390	1123.613
80	1137.824	1152.047	1166.270	1180.493	1194.716	1208.939	1223.162	1237.385	1251.608	1265.831
90	1280.052	1294.275	1308.498	1322.721	1336.944	1351.167	1365.390	1379.613	1393.836	1208.059

TABLE X.

FOR CONVERTING POUNDS PER SQ. INCH INTO KILOGRAMS PER SQ. CENTIMETER.

Pounds per sq. inch.	0	1	2	3	4	5	6	7	8	9
0	0	.070309	.140618	.210927	.281236	.351545	.421854	.492163	.562472	.632781
10	.70309	.77340	.84371	.91402	.98433	1.05464	1.12493	1.19526	1.26557	1.33588
20	1.40618	1.47649	1.54680	1.61711	1.68742	1.75773	1.82804	1.89835	1 96866	2.03897
30	2.10927	2.17958	2.24989	2.32020	2.39051	2.46082	2.53113	2.60144	2 67175	2.74206
40	2.81236	2.88267	2 95298	3.02329	3.09360	3.16391	3.23422	3.30453	3.37484	3.44515
50	3.52545	3.59576	3.66607	3.73638	3.79669	3.86700	3.93731	4 00762	4.07793	4.14824
60	4.21854	4.28885	4.35916	4.42947	4.49978	4.57009	4.64040	4.71071	4.78102	6 85133
70	4.92163	4.99194	5.06225	5.13256	5.20287	5.27318	5.34349	5 41380	5.48411	5.55442
80	5.62472	5 69503	5.76534	5.83565	5.90596	5.97627	6.04658	6.11689	6.18720	6.25751
90	6.32781	6.39812	6.46843	6.53874	6.60905	6.67936	6.74967	6.81998	6 89029	6.96060

TABLE XI.

FOR CONVERTING ATMOSPHERES INTO POUNDS AND KILOGRAMS.

Atmospheres.	Height of mercury in cent.	Height of mercury in inches.	Pressure per sq. cent. in kilogrs.	Pressure per sq. inch in pounds.
1.............	76	29.922	1.0333	14.696
2.............	152	59.844	2.0666	29.392
3	228	89.766	3.0999	44.088
4.............	304	119.688	4.1332	58 784
5.............	380	149.610	5.1665	73.480
6.............	456	179.532	6.1998	88.176
7.............	532	209.454	7.2331	102.872
8	608	239.376	8.2664	117.568
9.............	684	269.298	9.2997	132.264

TABLE XII.

FOR CONVERTING CALORIES INTO ENGLISH (FAHRENHEIT) HEAT UNITS.

Calories.	0	1	2	3	4	5	6	7	8	9
0	0	3.969	7.938	11.907	15,876	19.845	23.814	27.783	31.752	35.721
10	39.69	43.659	47.628	51.597	55.566	59.535	63.504	67.473	71.442	75.411
20	79.38	83.349	87.318	91.287	95.256	99.225	103.194	107.163	111.132	115.101
30	119.07	123.039	127.008	130.977	134.946	138.915	142.884	146.853	150.822	154.791
40	158.76	162.729	166.698	170.667	174.636	178.605	182.574	186.543	190.512	194.481
50	198.45	201.419	205.388	209.357	213.326	217.295	221.264	225.233	229.202	233.171
60	238.14	242.109	246.078	250.047	254.016	257.985	261.954	265.923	269.892	273.861
70	277.83	281.799	285.768	289.737	293.706	297.675	301.644	305.613	309.582	313.551
80	317.52	331.489	325.458	329.427	333.396	337.365	341.334	345.303	349.272	353.241
90	357.21	361.179	365.148	369.117	373.086	377.055	381.024	384.993	388.962	392.931

TABLE XIII.

FOR CONVERTING ENGLISH (FAHRENHEIT) HEAT UNITS INTO CALORIES.

Heat Units.	0	1	2	3	4	5	6	7	8	9
0	0	0.252	0.504	0.756	1.008	1.260	1.512	1.764	2.016	2.268
10	2.52	2.772	3.024	3.276	3.528	3.780	4.032	4.284	4.536	4.788
20	5.04	5.292	5.544	5.796	6.048	6.300	6.552	6.804	7.056	7.308
30	7.56	7.812	8.064	8.316	8.568	8.820	9.072	9.324	9.576	9.828
40	10.08	10.332	10.584	10.836	11.088	11.340	11.592	11.844	12.096	12.348
50	12.60	12.852	13.104	13.356	13.608	13.860	14.112	14.364	14.616	14.868
60	15.12	15.372	15.624	15.876	16.128	16.380	16.632	16.884	17.136	17.388
70	17.64	17.892	18.144	18.396	18.648	18.900	19.152	19.404	19.656	19.908
80	20.16	20.412	20.664	20.916	21.168	21.420	21.672	21.924	22.176	22.428
90	22.68	22.932	23.184	23.436	23.688	23.940	24.192	24.444	24.696	24.948

THERMODYNAMICS.

PART SECOND.

STEAM AND THE STEAM ENGINE

CHAPTER XIV.

THE ACTION OF HEAT IN EVAPORATION.—GENERAL PROPERTIES OF STEAM.—PRESSURE OF SATURATED STEAM.

The different Effects of Heat in Evaporation.—Of all the investigations thus far made upon the deportment of bodies, none are of greater importance than those which relate to the properties of steam. An exact knowledge of these properties lies at the foundation of the construction of the steam engine.

Although much in the way of investigation was done from the time of Watt, who gave us the steam engine almost in its present state of perfection, down to the time of Regnault, one of the most skillful experimenters of the present day, still, much had to remain unknown, because during that time the principles of the mechanical theory of heat were either unknown or but little regarded.

It will not be difficult to call attention to that which must during that time have remained hidden, the key to the explanation of which is, however, given by a number of phenomena of great scientific and technical interest. Let us return to the experiment which we have already described in Chapter VIII. of Part First.

Again, let *ABCD*, Fig. 58, be a hollow cylinder of 1 sq. meter cross-section. Upon the bottom *CD* is 1 cubic decimeter=0.001 cubic meter = 1 liter of water, whose weight is therefore 1 kilogram at 0°. In contact with the surface of the water is the piston *KK*, which we assume perfectly tight and without weight. Above the piston is a vacuum.

We assume the piston loaded with 10334 kilograms. This is the same as the pressure of the atmosphere per square meter for 0° and 760 millimeters (30 inches) barometric height.

Now let us impart heat to the water until the temperature rises to 100°. Up to this point no steam is formed. This only happens when the addition of heat is carried still farther. The piston will then be gradually raised, as steam is formed, and the weight 10334 kilograms will be lifted through a certain distance. When all the water is converted into steam, the piston will stand at a height of about 1.650 meters above the bottom.

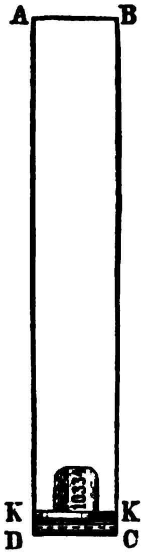

Fig. 58.

In this apparently simple process we can recognize, from the standpoint of the mechanical theory of heat, several different effects.

First, the water is heated, therefore the *vibration work* of the molecules is increased. When the water attains the temperature of 100°, further rise of temperature does not occur. The velocity of the molecules is now so great, however, that, according to our theoretical views, any further addition of heat not only separates the molecules beyond the influence of their cohesion, which constituted the *liquid*, but also gives them a rectilinear motion such as the molecules of a gas possess. The molecules then impinge upon the piston and raise it, until all the water is converted into steam. For this transformation of the water into steam, this overcoming of the forces which bind the molecules of the liquid mass together, or "*disgregation work,*" there is, as we know, a certain amount of heat necessary.

Further, the weight of 10334 kilograms is raised about 1.65 meters. This requires a mechanical work of 10334 × 1.65 = 17051 meter-kilograms, and this work must evidently be also furnished by the heat. Since a work of 424 meter-kilograms is equivalent to one heat unit, the work of 17051 meter-kilograms requires $\frac{17051}{424} = 40.2$ heat units. At the end of the experiment, the heat actually existing in the steam *as heat*, or energy of vibration, must then be less than the total amount imparted, by just this amount, which is required for the performance of outer work. It is this circumstance to which we wish here to call special attention. From it flow a number of other facts as to the deportment of steam, which without it either cannot be recognized or else are wrongly explained. Thus, for example, the steam used per stroke by the steam engine possesses at the end of the stroke no longer the heat which it had in the boiler, and the heat which thus disappears is the equivalent of the work done. This has been verified, as we have seen, by Hirn's experiment. If the water had expanded greatly, while heated from 0° to 100°—which, as we know, is not the case—then the outer work of 10334 × 1.65 meter-kilograms would have been partly performed by this expansion.

If we compare the outer work of 17051 meter-kilograms, which we may call the outer work of steam, with that necessary for overcoming the molecular forces, we shall find considerable difference. Thus in the present case, 496.3 heat units are necessary for this purpose alone. This heat represents a mechanical work of $496.3 \times 424 = 210431$ meter-kilograms, and this is required to overcome the molecular forces of only *one kilogram* of water. Indeed, in comparison, the force of gravity is but slight. If, for example, a mass of water of one cubic meter = 1000 kilograms is required to perform this work, it must sink through a distance of $\frac{210431}{1000} = 210.431$ meters.

The question arises, How can the heat be determined which is necessary for the different effects in this transformation of water into steam? It is not possible to determine with exactness the heat imparted during the heating and vaporization of the water, by direct determination of the heat furnished by the fuel. The opposite method, of determining the heat units set free when the steam condenses, is far more exact. If we use for the condensation a large quantity of cold water, the condensation is rapid, and all the heat is given up to the water, while the amount lost by radiation and conduction is very slight. If we use 2545 kilograms of water at 0°, which is raised in temperature ¼°, while the temperature of the condensed steam is also diminished ¼°, we shall have, evidently, the following equation for the heat contained by the steam:

$$x - \tfrac{1}{4} = \tfrac{1}{4} \times 2545,$$

where x is the number of units of heat possessed by the steam. Hence

$$x = 636.5 \text{ heat units.}$$

It is evident that in this experiment, the steam being condensed by cold water, that heat *reappears* which disappeared in the raising of the weight. This heat is therefore contained in the total heat of the water.

If we assume that for the heating of the water from zero to 100° 100 heat units are necessary (as we shall see hereafter this number is somewhat too small), then, for the disgregation work and the outer work, 636.5 – 100 = 536.5 heat units are required. Accordingly the total heat in our experiment is divided among the various processes as follows:

1. Heating of the water from 0° to 100° (vibration work)	100	heat units.
2. Overcoming the molecular forces (disgregation work)	496.3	" "
3. Raising 10334 kil. 1.65 meters (outer work)	40.2	" "
Total	636.5	" "

In calling attention thus to the circumstances which, in ignorance of the principles of the mechanical theory of heat, must have escaped those physicists who have investigated the phenomena of vaporization, the question arises whether their determinations are therefore worthless, or have their experiments been so conducted that we can make use of them? The question is to be answered decidedly in the affirmative, especially as regards the comprehensive and careful experiments of Regnault, made with large and accurate apparatus, described on page 381, and surpassing in accuracy all other experiments of the same character by other physicists. The experiments of Regnault upon steam, as also upon other vapors, depend not only upon the exact determination of the heat necessary to raise 1 kilogram of water, under a given pressure, from 0° to any other temperature, or to convert, under the same circumstances, 1 kilogram of water at 0° into steam, but also upon the careful determination of the pressure of the steam generated from water at any temperature. It is not the place to describe in detail the apparatus used in these remarkable experiments, nor to notice here the very ingenious methods of investigation. This belongs to experimental physics. We shall only cite some of his experiments, in order to show that they answer well the purposes of our investigations.

Before doing this, we shall call attention to the principal properties of steam and the different kinds of steam.

General Properties of Steam.—Saturated and Superheated Steam. —Let *I.*, *II.*, *III.*, Fig. 59, be prismatic or cylindrical vessels of 1 square meter cross-section, each holding 1 kilogram of water, the air-tight piston *KK* being in contact with the surface of the water. Let the piston in *I.* be loaded with 10334, that in *II.* with 2 × 10334, and in *III.* with 3 × 10334 kilograms, the space above being a vacuum.

In order that the piston in *I.* shall begin to rise, or vaporization begin, experiments show that the water must be heated up to 100°. In *II.* the water must be heated up to 120.6°. In *III.*, up to 134°.

If all the water is converted into steam in all three cylinders, the piston in *I.* will be raised, as shown by experiment, to 1.65 m., in *II.* to 0.86, and in *III.* to 0.59 meters. In *I.*, therefore, the 1 kilogram = 0.001 cubic meters of water has become 1.65 cubic meters, in *II.* the same water volume becomes 0.86, and in *III.* 0.59 cubic meters of steam. In *I.*, then, from 1 cubic meter of water, we should obtain 1650 cubic meters of steam of 100°; in *II.*, 860 cubic meters at 120.6°; and in *III.*, 590 cubic meters at 134°. The number which denotes how many cubic meters of steam are generated from 1 cubic meter of water, or generally, the ratio of the volume of the steam to that of the water from which it is generated, is called the "*specific steam volume.*" In *I.* it is 1650; in *II.*, 860; in *III.*, 590.

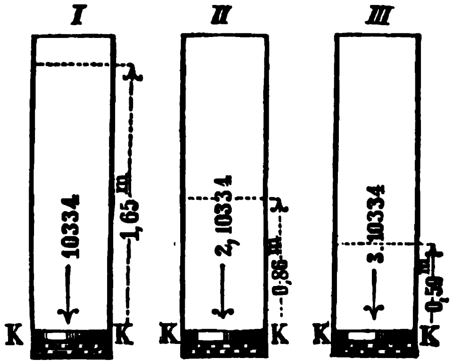

Fig. 59.

Since in *II.* the same weight of steam (1 kilogr.) is contained in about half the space that it occupies in *I.*, the *density* (weight of unit of volume) is about twice as great. In *III.* we have in about ⅓d of the volume the same weight as in *I.*, hence the density is about 3 times as great as in *I.*

We can recognize, then, the following principles:

1. *The greater the pressure the higher the temperature at which vaporization begins.*

2. *Since the steam holds the outer pressure in equilibrium, the higher the temperature the greater the pressure of the steam generated.*

3. *From a certain volume of water there is generated for any given temperature a definite and invariable steam volume.*

(The ratio of this volume to the water volume, or the specific steam volume, must be less the greater the temperature.)

4. *The density is greater for high temperatures than for lower.*

5. *The greater pressure of the steam at high temperatures depends less upon the difference of temperature than upon the greater density.*

We have here assumed that the steam is formed *directly* from the water, and that steam and water are in contact up to the moment when the last drop of water is evaporated. Such steam, which for reasons we shall soon learn we call "*saturated*," must be distinguished from other kinds of steam which we shall have occasion to speak of.

If, when all the water is just converted into steam, and the pistons are at their highest positions, we fix the pistons immovably and force steam into the spaces already filled with steam, condensation will occur and an amount of steam will be condensed equal in weight to the amount of fresh steam forced in. The same will be the case if the pistons are forced down, provided the temperature remains constant. We say, therefore, that the spaces are filled with "*saturated steam*," that is, steam just at the point of condensation. As long as steam is in contact with the water from which it is being generated, it must at any moment be at its point of condensation, and therefore "*saturated.*" Saturated steam differs, then, essentially from a permanent gas, in that it cannot be compressed like gas under constant temperature. If thus compressed, a portion is condensed while the pressure remains the same.

If, for example, *Os*, in Fig. 60, is the steam volume of 1.65 cubic meters which is generated in *I.* from 1 kilogram of water, and $sB = p$ is the pressure (10,334 kil. per sq. meter), and if by forcing down the piston the steam is compressed gradually, then, *under the condition that the temperature is kept constant*, the pressure p remains the same. If all the steam is converted into water, we have finally one kilogram of water at 100°, which occupies the space Os_1 of 0.001 cubic meter. *The straight line* BA,* *parallel to* OX, *represents the change of condition of the steam when compressed under constant temperature.* If, inversely, we evaporate the 1 kilogram of water of 100° under the constant pressure p, the point A passes from A to B. It is evident that in compressing the steam we must abstract, in order to keep the temperature constant, as much heat as must be imparted during its generation.

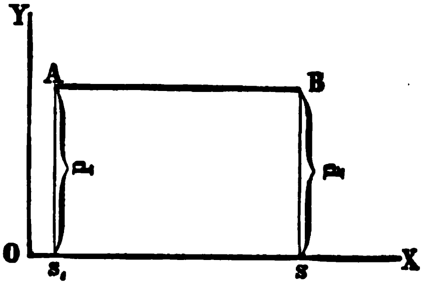

FIG. 60.

If now we consider the steam in *I.* still further heated, then if the pressure remains constant it expands while its temperature increases. Suppose that from the moment of expansion it is no longer in contact with water. The steam is now no longer saturated, no longer just at the point of condensation, and since, with the same pressure, it has a higher temperature, we call it "*superheated.*"

Superheated steam,† *then, is steam which for the same pressure has a greater temperature and a greater specific volume than saturated steam.*

The more the steam is superheated, the more it approaches the condition of a permanent gas, but only at a very considerable distance from the point of saturation are its properties essentially the same.

If the pistons in *I.*, *II.*, and *III.* are held fast, and then more heat added, both the temperature and pressure increase. The steam is superheated. If, for example, in *I.* the pressure is about twice as great, the temperature is considerably greater than 120.6°, which is that of saturated steam of the same pressure. We also understand, therefore, by superheated steam,

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* [" Isopiestic line."]

† [Also sometimes called " steam gas."]

steam *which for the same specific volume has a higher pressure and a higher temperature than saturated steam.*

Suppose we have below the piston *KK*, Fig. 61, highly superheated steam. We now force the piston gradually down. The deportment of the steam is at first that of a permanent gas. If, therefore, we keep the temperature constant, the pressure increases inversely with the volume, and the change of condition follows Mariotte's law. If heat is not abstracted, the pressure increases in a greater ratio according to the exponential law of Poisson, or "adiabatically," only we have now no longer the exponent 1.41, which was for air, but another value for the exponent. The more we force down the piston, under constant temperature, the more we approach a point where Mariotte's law no longer holds good. This point lies near the point of saturation or of condensation. When this point is reached, further compression simply causes condensation, the pressure remains constant and Mariotte's law entirely ceases to apply.

FIG. 61

That which has been said concerning steam applies to all "vapors," that is, to all gaseous bodies generated from liquids, and which by ordinary compression or cooling can be reconverted into liquids. Such are the steam from spirits of wine, ether, carbonic acid, mercury, etc. Only the numerical relations between temperature, pressure, specific volume, etc., are different.

We pass now to a subject which is also of special interest in the mechanical theory of heat, that is, to the exact determination of the relation between the pressure of steam and its temperature.

Empirical Formulæ for the Pressure and Temperature of Saturated Steam.—Very many experiments have been made in order to determine the pressure of saturated steam at different temperatures. None of them possess greater reliability and exactness, and none are more comprehensive than those made by Regnault. The method of his investigations depends upon the principle which holds good for every liquid, *that the* *temperature at which water boils remains constant so long as the pressure upon the liquid is constant,* and that, therefore, the pressure of the steam is in equilibrium with this pressure. Experimenters before Regnault measured the steam pressure directly. Now it is very difficult to maintain, even for only a few minutes, the temperature of the water, and hence the expansive force of the steam, constant, in a vessel closed, and exposed to fire, because the heat of the fire varies. Yet this is necessary while noting the temperature and pressure. In order to avoid the inaccuracies arising from such method of observation, Regnault compressed the air which surrounded his vessel. In order that the steam, when generated, should not act to increase the outer pressure, it was condensed in a special vessel just as soon as it was generated. We give a sketch of Regnault's apparatus, Fig. 62. *C* is a small copper boiler, into which four thermometers enter; two enter the water and two the steam only, in order to determine if the water and steam have always the same temperature. The steam space of *C* is connected by a pipe, *TT*, with a large globe, *A*, of about 24 liters capacity, which is contained in a vessel, *VV*, filled with water.

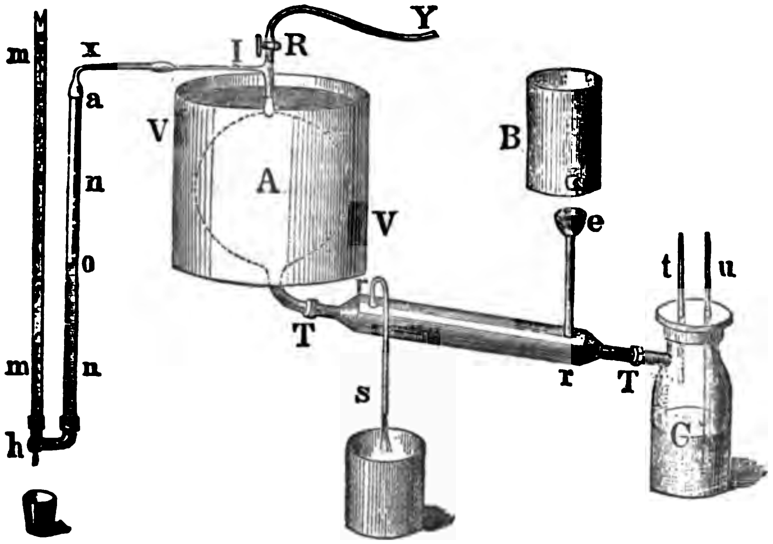

Fig. 62.

From this globe leads a pipe with cock, *R*, to an air-pump, so that the air can be compressed at will in the globe and boiler. The pressure of the air in both globe and boiler is shown by the manometer, *nn*, *mn*, which communicates with the globe by the pipe *Ix*. The pipe *TT* is surrounded by a jacket *rr*, through which cold water continually circulates. The steam generated in *C* must, therefore, since the air is kept cool, be at once condensed, and cannot therefore contribute to increase the expansive force of the air. When, now, the air in *A* and in *C* has been compressed, and the pressure exactly noted on the manometer, the water in *C* is heated until the thermometer shows a constant temperature. This we may be sure is the temperature under which, for the given pressure, the water boils. And the expansive force of the steam at this temperature is exactly equal to the pressure as indicated by the manometer.

Regnault used in his experiments a large and a small apparatus. The first served especially for the determination of pressures for temperatures between 170° and 230°, the other between 0° to 170°.

The experiments of Regnault have thus far furnished no exact law as to the relation of pressure and temperature of saturated steam. We have only empirical formulæ which give this relation with more or less accuracy. Of all these formulæ, none agree with the results of experiment better than that of Regnault himself, as also the formula of Magnus and Röntgen.

Regnault gives three formulæ, the first for temperatures from − 32° to 0° C., the other for from 0° to 100° C., and the third for temperatures above 100° up to 230° C.

The first formula has the form

$$(-32^\circ \text{ to } 0^\circ) \qquad p = a + b\alpha^x$$

where $a = -0.08038$, $\log b = \bar{1}.6024724$, $\log \alpha = 0.0333980$,

$$x = t + 32$$

where t is the temperature. This, as well as all the other formulæ of Regnault, gives the pressure p in millimeters of barometric height, 760 millimeters to one atmosphere.

The second formula of Regnault has the form

$$(0^\circ \text{ to } 100^\circ) \qquad \log p = a + b\alpha^t - c\beta^t$$

where $a = 4.7393707$, $\log b = \bar{2}.131990711$, $\log \alpha = 0.006864937$, $\log c = 0.611740767$, $\log \beta = \bar{1}.996725536$, and t is again the temperature.

The third formula is

$$(100^\circ \text{ to } 230^\circ) \qquad \log. p = a - b\alpha^x - c\beta^x$$

where $a = 6.2640348$, $\log b = 0.1397743$, $\log \alpha = \bar{1}.994049292$, $\log c = 0.6924351$, $\log \beta = \bar{1}.99834862$, $x = t + 20$.

Before giving other formulæ, let us see how, from the preceding, the pressure of steam may be found from the temperature.

EXAMPLE.

What is the pressure of saturated steam whose temperature is 20° C.?

We must use here the second formula.

First, $\log \alpha^t = 0.006864937 \times 20 = 0.13729874$ and $\log b + \log \alpha^t = \bar{2}.131090711 + 0.13729874 = \bar{2}.269289 =$ the number 0.0185903. We have for $\log \beta^t$, $\bar{1}.996725536 \times 20 = \bar{1}.93451072$, and $\log c + \log \beta^t = 0.611740767 + \bar{1}.93451072 = 0.54625149 =$ the number 3.5177. Then, $a - c\beta^t = 4.73937 - 3.5177 = 1.22167$. Finally, $\log p = 0.0185903 + 1.22167 = 1.24026$. Hence $p = 17.388$ millimeters. More exact calculation would have given 17.371. Table I. at the end of this work gives the pressure, according to Regnault's calculations, from -32° to 230°.

The formula of Magnus is

$$p = 4.525 \times 10^{\frac{7.4475\, t}{234.69 + t}}$$

in which t is the temperature and p the pressure in millimeters. It gives, for temperatures below 100°, excellent results as compared with experiment. Above 100° the agreement is not so good. By a change of coefficients, however, great exactness may be attained here also. The formula is, moreover, very convenient for calculation, as an example will show.

EXAMPLE.

What is the pressure of saturated steam at 130° ?

Here $t = 130$, hence $p = 4.525 \times 10^{\frac{7.4475 \times 130}{234.69 + 130}} = 4.525 \times 10^{\frac{968.175}{364.69}} = 4.525 \times 10^{2.6548}$. Now the log of $10^{2.6548}$ is 2.6548, and the corresponding number is 451.65. Hence $p = 4.525 \times 451.65 = 2042.716$ millimeters $= \frac{2042.716}{760} = 2.688$ atmospheres. According to Regnault's formula we should have 2.671 atmospheres.

The formula also has the advantage that we may find inversely the temperature from the pressure.

Thus we have

$$\log p = \log 4.525 + \frac{7.4475 \times t}{234.69 + t},$$

or

$$234.69 \log p + t \log p = 234.69 \log 4.525 + t \log 4.525 + 7.4475t,$$

hence

$$t = \frac{234.69\ (\log p - \log 4.525)}{\log 4.525 - \log p + 7.4475} = \frac{234.69 \log p - 153.867}{8.10312 - \log p}.$$

EXAMPLE.

If the pressure of steam is 10mm. what is its temperature?

Here $p = 10$, hence $\log p = 1$, and we have

$$t = \frac{234.69 - 153.867}{7.103} = \frac{80.823}{7.103} = 11.36^\circ.$$

The formula of Röntgen, for temperatures from $-32°$ to $100°$, is

$$\log p = \log 760 - (a + bx + cx^2 + dx^4)\,x,$$

where $a = 0.015432,\qquad b = 0.0000542,\qquad c = 0.0000000704,$
$d = 0.0000000000066,\qquad x = 100 - t.$

Here t is the temperature, and p is given in millimeters.

If p is given in atmospheres, we have more simply

$$\log p = -(a + bx + cx^2 + dx^4)\,x,$$

the coefficients being the same.

Above 100° the formula is

$$\log p = \log 760 - (a + bx + cx^2)\,x.$$

Where $a = 0.015432$, $b = 0.00004265$, $c = 0.0000000704$, and $x = 100 - t$.

Here again, for the pressure in atmospheres, we have

$$\log p = -(a + bx + cx^2)\,x.$$

EXAMPLE.

What is the tension of saturated steam when the temperature is 105.08°?

Here $t = 105.08$, hence $100 - t = -5.08$. Therefore, in the equation $\log p = \log 760 - (a + bx + cx^2)\,x$, the first and third term in the parenthesis will be positive, and the second negative.

We have

$$\begin{array}{lcl}
a \ldots\ldots\ldots\ldots\ldots\ldots & = & 0.015432 \\
cx^2 = 0.0000000704 \times 25.806 \ldots\ldots & = & \underline{0.00000182} \\
 & & 0.01543382 \\
bx = 0.00004265 \times -5.08 \ldots\ldots & = - & \underline{0.00021666} \\
a + bx + cx^2 \ldots\ldots\ldots\ldots & = & 0.01521716 \\
-(a + bx + cx^2)x \ldots\ldots\ldots & = & 0.0773024 \\
\log 760 \ldots\ldots\ldots\ldots\ldots & = & \underline{2.8808136} \\
\log p \ldots\ldots\ldots\ldots\ldots\ldots & = & 2.9581160
\end{array}$$

$p = 908.06$ millimeters.

Regnault found by experiment 904.87 millimeters.

The following table gives the pressures for other liquids, according to Regnault.

Temperature C.	Ether. ($C_4H_{10}O$)	Alcohol. (C_2H_6O)	Acetone. (C_3H_6O)	Chloroform. ($CHCl_3$)	Chloride of Carbon. (CCl_4)	Bisulphide of Carbon. (CS_2)
0°	184.38	12.70	63.33	59.72	32.95	127.91
10	286.83	24.23	110.32	100.47	55.97	198.46
20	432.78	44.46	180.08	160.47	90.99	298.03
30	634.80	78.52	280.05	247.51	142.27	443.62
40	907.04	133.69	419.35	369.26	214.81	617.53
50	1264.83	219.90	608.81	535.05	314.38	857.07
60	1725.01	350.21	860.96	755.44	447.43	1164.51
70	2304.90	541.15	1189.90	1042.11	621.15	1552.09
80	3022.79	812.91	1611.05	1407.64	843.29	2032.53
90	3898.26	1189.30	2140.82	1865.22	1122.26	2619.08
100	4953.30	1697.55	2796.20	2428.54	1467.09	3325.19
110	6214.63	2367.64	3594.33	3110.99	1887.44	4164.06
120	7719.30	3231.73	4551.95	3925.74	2398.67	5148.79
130	....	4323.00	5684.90	4885.10	2996.88	6291.60
140	....	5674.59	7007.64	6000.16	3709.04	7603.96

QUESTIONS FOR EXAMINATION.

1 cu. dec

10334

What pressure in Kilograms per square meter is equal to one atmosphere? What is a liter? How many millimeters of the barometer correspond to one atmosphere? When water is heated in a vessel under constant pressure, describe the different effects of the heat imparted. In each kilogram of water, how many heat units go to perform the outer work? How many to perform disgregation work? How many to perform vibration work? What then is the total heat imparted in heat units? Define what you mean by heat unit.

760

402

427

36

What is the relation between pressure and temperature at which vaporization begins? Illustrate. What is "specific" steam volume? Illustrate. How does this volume vary with the temperature? How does the density of steam vary with the temperature? Upon what does the increased pressure of steam at high temperatures mainly depend?

What do you understand by saturated steam? Illustrate. How does it differ from permanent gas? What do you mean by superheated steam? Why is it called *super*-heated? What is its specific volume as compared with saturated? For the same specific volume how do the pressure and temperature compare with saturated? If saturated steam is compressed under constant temperature, what happens? If superheated steam is compressed, what happens? What is a "vapor?"

Upon what principle do the experiments of Regnault depend? What was the object of them? Why cannot the steam pressure be measured directly? How did Regnault avoid these inaccuracies? Describe his apparatus and its method of working. Have his experiments given any exact law between pressure and temperature of saturated steam? If saturated steam has a certain temperature, has it a definite pressure? Below what limit of temperature does the formula of Magnus give good results? What limits are included by Regnault's formulæ? Are these formulæ within their limits reliable? How were they deduced? Between what limits does Röntgen's formula hold good? Do any of these formulæ hold good for superheated steam? Why not? What is given by Table I.?

NOTE.—Rankine gives for the relation between pressure and temperature of saturated steam

$$\log p = A - \frac{B}{T} - \frac{C}{T^2} \text{ or } T = 1 + \left\{ \sqrt{\left(\frac{A - \log p}{C} + \frac{B^2}{4C^2}\right)} - \frac{B}{2C} \right\}$$

where T = absolute temperature = t + 461.2° Fah.,
p = pressure in pounds per *square foot*,
and the values of the constants are as follows:

	A	$\log B$	$\log C$	$\frac{B}{2C}$	$\frac{B^2}{4C^2}$
Water............	8.2591	3.43642	5.59873	0.003441	0.00001184
Alcohol...........	7.9707	3.31233	5.75323	0.001812	0.000008383
Ether.............	7.5732	3.31492	5.21706	0.006264	0.00008924
Bisulph. of Carbon.	7.3438	3.30728	5.21839	0.006136	0.00008765
Mercury...........	7.9691	3.72284			

For inches of mercury at 32°, subtract from A, 1.8496.
For pounds per sq. inch, subtract from A, 2.1584.
For the Centigrade scale, subtract from $\log B$, 0.25527.
For the Centigrade scale, subtract from $\log C$, 0.51059.

For the Centigrade scale, multiply $\frac{B}{2C}$ by 1.8.

For the Centigrade scale, multiply $\frac{B^2}{4C^2}$ by 3.24.

CHAPTER XV.

HEAT OF THE LIQUID.—TOTAL HEAT.—INNER AND OUTER HEAT OF VAPORIZATION.—HEAT OF THE STEAM.

Specific Heat and Heat of the Liquid.—Regnault has also investigated, with the same careful accuracy, whether the amount of heat required by water for a certain given rise of temperature is the same at high temperatures as at low. In other words, whether the specific heat of water is constant for all temperatures—if, for example, the heating of one kilogram of water from 0° to 10°, 20°, etc., requires ten or twenty times as much heat as from 0° to 1°. Regarded from the standpoint of the mechanical theory of heat, this involves the following questions:

1. Is the work required for a certain rise of temperature of the water greater or less for high temperatures than for low?
2. Is the disgregation work and outer work greater or less?

This last would be the case, if the water at high temperatures expanded more or less than at low. Numerous experiments have, however, shown that the expansion of water is almost the same, relatively, under the ordinary pressure of the atmosphere, for all temperatures. Since, as we have seen already in Part I., the disgregation work is extremely small in comparison with the vibration work, we can also neglect its influence, and have only to consider that heat which is requisite for the rise of temperature.

The apparatus used by Regnault consisted in part of that which he made use of for the determination of steam pressure, viz., of a boiler, in which the pressure and temperature of the steam and water were carefully determined. By means of a pipe, closed by a cock, the water-space of the boiler was connected with a calorimeter. The water passing from the boiler to this calorimeter could be exactly determined, and, by thermometers, the temperature of the cooling water was determined at every instant. By a suitable apparatus the hot water was made to mix quickly with the cold water of the calorimeter, so that as little heat as possible was lost by radiation and conduction.

After the air pressing upon the water in the boiler had been compressed to a given point, and the water brought to the boiling point, the cock was opened for a short time, so that a certain quantity of water passed into the calorimeter, under almost constant pressure.

If, now, the water while heated up to boiling had expanded much, it would have overcome the air pressure through a certain distance, performed a certain amount of mechanical work, and a certain quantity of heat would have disappeared. On entrance into the calorimeter, the water would then have contracted under the same pressure, work would have been received by it, and heat would have reappeared. We should, therefore, have recovered the heat necessary for the expansion of the water. Further, the water rushes into the calorimeter with a certain velocity. For the generation of this velocity a certain amount of heat must be expended. But neither can this be lost, because the water comes gradually to rest in the calorimeter, so that its living force is transformed into heat again. Therefore the heat appearing in the calorimeter is precisely that which the water received in the boiler.

From his experiments Regnault found by calculation that the specific heat of water between the temperatures t_1 and t, for example, between 50° and 40°, or between 90° and 30°, is given by the equation

$$C_{t_1-t} = 1 + a(t_1 + t) + b(t_1^2 + tt_1 + t^2) \quad . \quad . \quad (1).$$

in which t_1 is the higher and t the lower temperature. For from 0° to t_1° we have, therefore,

$$C_{t_1} = 1 + at_1 + bt_1^2 \; . \; . \; . \; . \; . \; . \; (2).$$

In order to determine the coefficients a and b, Regnault found first the mean specific heat of water between 0° and 100°, or that heat which in the mean is required by 1 kilogram of water between 0° and 100° to raise its temperature 1°. He found for this 1.005 heat units, so that we have from (2)

$$C_{100} = 1.005 = 1 + a \times 100 + b \times 100^2 \quad . \quad . \quad (3).$$

For the specific heat between 0° and 200°, he found $C_{200} =$ 1.016. Hence we have

$$C_{200} = 1.016 = 1 + a \times 200 + b \times 200^2 \quad . \quad . \quad (4).$$

From (3) and (4) a and b can be readily found. We have

$$a = 0.00002 \quad \text{and} \quad b = 0.0000003;$$

hence equation (1) becomes

$$C_{t_1-t} = 1 + 0.00002\,(t + t_1) + 0.0000003\,(t^2 + tt_1 + t_1^2). \quad (5).$$

For Fahrenheit degrees this becomes

$$C_{t_1-t}=1+0.0000111\,[(t_1-32)+(t-32)]+0.0000000926\,[(t_1-32)^2 + (t_1-32)(t-32)+(t-32)^2].$$

EXAMPLE.

What is the mean specific heat of water between 15° and 25°, and how much heat is necessary to raise 1 kilogram of water from 15° to 25°?

Here $t_1 = 25$, and $t = 15$, and $t + t_1 = 40$. Hence

$$\begin{aligned} C_{25-15} &= 1 + 0.00002 \times 40 + 0.0000003 \times (15^2 + 15 \times 25 + 25^2) \\ &= 1 + 0.0008 + 0.0000003 \times 1225 \\ &= 1.0011675 \text{ heat units.} \end{aligned}$$

Therefore the heat necessary to raise 1 kilogram of water from 15° to 25° is $1.0011675\,(25 - 15) = 1.0011675 \times 10 = 10.011675$ heat units. If the specific heat of water were constant for all temperatures, the heat required would have been simply 10 heat units.

If in (5) we put 0 in place of t, we have the specific heat of water between 0° and t°. If we denote this mean specific heat by C_m, we have

$$C_m = 1 + 0.00002t + 0.0000003t^2 \quad . \quad . \quad . \quad (I.)$$

For Fahrenheit degrees

$$C_m = 1 + 0.0000111\,(t - 32) + 0.0000000926\,(t - 32)^2.$$

Thus the mean specific heat between 0° and 150° is

$$C_m = 1 + 0.00002 \times 150 + 0.0000003 \times 150^2 = 1.00975.$$

The amount of heat necessary to raise 1 kilogram of water from 0° to t° is evidently

$$\begin{aligned} C_m t &= (1 + 0.00002t + 0.0000003t^2)\,t \\ &= t + 0.00002t^2 + 0.0000003t^3. \end{aligned}$$

This heat is very appropriately called the "heat of the liquid," and denoted by q, so that

$$q = t + 0.00002t^2 + 0.0000003t^3 \quad . \quad . \quad . \quad \text{(II.)}$$

or for Fahrenheit degrees for 1 pound,

$$q = (t - 32) + 0.0000111\,(t - 32)^2 + 0.0000000926\,(t - 32)^3.$$

This very important equation gives, therefore, the quantity of heat in heat units necessary to raise 1 kilogram of water from 0° to $t°$.

The values in column 5 of Table II. are calculated from this formula.

Regnault has found the heat of the liquid, for other liquids also, and given empirical formulæ, as follows:

For Ether................ $q = 0.52901t + 0.0002959t^2$.
Alcohol............. $q = 0.54754t + 0.001122t^2 + 0.0000022t^3$.
Acetone............. $q = 0.50643t + 0.000397t^2$.
Chloroform.......... $q = 0.23235t + 0.0000507t^2$.
Chloride of carbon... $q = 0.19798t + 0.0000906t^2$.
Bisulphide of carbon.. $q = 0.23523t + 0.000082t^2$.

To reduce these formulæ to English units, divide the second term on right by 1.8, and divide the third by 3.24. The first remains unchanged. Put $t - 32$ in place of t.

We have thus far spoken of the *mean* specific heat of water, that is, of the mean amount of heat required to raise 1 kilogram of water one degree, between the limits $t°$ and $t_1°$ or 0° to $t°$. But this is not the heat necessary to raise the temperature of water 1° above a given point, or, more properly, a very little above that point. Since in this case t_1 is but little greater than t, we find the *actual* specific heat at $t°$ by making $t_1 = t$ in (5).

If we denote this "actual specific heat" by C simply, we have

$$C = 1 + 0.00002\,(2t) + 0.0000003\,(3t^2),$$

or

$$C = 1 + 0.00004t + 0.0000009t^2 \quad . \quad . \quad . \quad \text{(III.)}$$

For Fahrenheit degrees

$$C = 1 + 0.0000222\,(t - 32) + 0.0000002778\,(t - 32)^2.$$

EXAMPLE 1.—What is the actual specific heat of water at 100° and at 180°?
For 100° we have

$$C = 1 + 0.00004 \times 100 + 0.0000009 \times 10000$$
$$= 1 + 0.004 + 0.009 = 1.013.$$

If, then, we denote the heat necessary to raise 1 kilogram of water at 0° a very little higher, by x, the heat necessary to raise 1 kilogram at 100°, the same small amount is $1.013x$.

The specific heat at 130° is

$$\begin{aligned} C &= 1 + 0.00004 \times 130 + 0.0000009 \times 16900 \\ &= 1 + 0.00520 + 0.01521 \\ &= 1.02041. \end{aligned}$$

EXAMPLE 2.—How many heat units are necessary to raise 100 kilograms of water from 0° to 100°?

From (II.) we have

$$\begin{aligned} q &= 100 + 0.00002 \times 10000 + 0.0000003 \times 1000000 \\ &= 100 + 0.2 + 0.3 = 100.5 \text{ heat units,} \end{aligned}$$

hence 100 kilograms will require, in order to raise the temperature from 0° to 100°, $100.5 \times 100 = 10050$ heat units.

The heat necessary to raise one kilogram of water from t to t_1 degrees, is given by

$$q_1 - q = t_1 - t + 0.00002\,(t_1^2 - t^2) + 0.0000003\,(t_1^3 - t^3).$$

Thus, to raise one kilogram of water from 10° to 100°, we require

$$\begin{aligned} q_1 - q &= 100 - 10 + 0.00002\,(100^2 - 10^2) + 0.0000003\,(100^3 - 10^3) \\ &= 90 + 0.198 + 2997 = 90.4977 \text{ heat units.} \end{aligned}$$

Total Heat and Heat of Vaporization.—The "total heat," that is, the amount of heat necessary in order to raise 1 kilogram of water gradually from 0° to *saturated steam* of a given temperature and pressure, has also been determined with great care by Regnault. A part of the apparatus used in these experiments consisted, as before, of the boiler and globe already described. The steam generated was conducted into a calorimeter, which was also connected with the globe. Since, therefore, the water was evaporated under the same pressure at which it was condensed, the heat obtained in the calorimeter was equal to that imparted to the steam in the boiler.

The observations showed that the heat imparted to 1 kilogram of water, in order to convert it, at various pressures, into saturated steam, increased slowly with the temperature, and was given by the following empirical formulæ:

$$W = 606.5 + 0.305t \quad . \; . \; . \; . \; . \quad \text{(IV.)}$$

For Fahrenheit degrees and English units,

$$W = 1091.7 + 0.305\,(t - 32).$$

where W is the total heat and t the temperature of the water or steam.

Thus in order to convert 1 kilogram of water into steam at 10°, 20°, or 100°, we require

$$W = 606.5 + 0.305 \times 10 = 609.55 \text{ heat units.}$$
$$W = 606.5 + 0.305 \times 20 = 613.60 \quad \text{"} \quad \text{"}$$
$$W = 606.5 + 0.305 \times 100 = 637 \quad \text{"} \quad \text{"}$$

For the total heat of other liquids, Regnault found for

Ether	$W = 94.00 + 0.45000t - 0.00055556t^2.$
Acetone	$W = 140.50 + 0.36644t - 0.000516t^2.$
Chloroform...........	$W = 67.00 + 0.1375t.$
Chloride of carbon....	$W = 52.00 + 0.14625t - 0.000172t^2.$
Bisulphide of carbon..	$W = 90.00 + 0.14601t - 0.0004123t^2.$

To reduce these formulæ to English units, multiply the first term on right by 1.8, and divide the third by 1.8. The second remains unchanged. Put $t - 32$ for t.

If we subtract the "heat of the liquid" from the total heat, we have not only the heat necessary for the transformation of the water into steam, but also that required for the outer work. This heat, therefore, we call the "*total heat of vaporization*," and denote it by r.

Accordingly we have

$$r = W - q \quad . \; . \; . \; . \; . \; . \; . \quad \text{(V.)}$$

or putting for W and q their values,

$$r = 606.50 - 0.695t - 0.00002t^2 - 0.0000003t^3 . \quad \text{(VI.)}$$

For Fahrenheit degrees

$$r = 1091.7 - 0.695\,(t - 32) - 0.0000111\,(t - 32)^2$$
$$- 0.0000000926\,(t - 32)^3.$$

From this formula we see that the "total heat of vaporization" diminishes as the temperature increases; is less, for example, at 200° than for 100°.

The following formulæ give the total heat of vaporization for other liquids for Centigrade degrees.

For Ether ....$r = 94.00 - 0.07901t - 0.0008514t^2$.
Acetone ....$r = 140.5 - 0.13999t - 0.0009125t^2$.
Chloroform ....$r = 67.00 - 0.09485t - 0.0000507t^2$.
Chloride of carbon....$r = 52.00 - 0.05173t - 0.0002526t^2$.
Bisulphide of carbon..$r = 90.00 - 0.08922t - 0.0004938t^2$.

To reduce these formulæ to English units, multiply the first term on right by 1.8, and divide the third by 1.8. The second term remains unchanged. Put $t - 32$ for t.

Instead of the above complicated formulæ for the vaporization heat of water, Clausius gives

$$r = 607 - 0.708t \quad . \quad . \quad . \quad . \quad . \quad \text{(VII.)}$$

or for Fahrenheit degrees $r = 1092.6 - 0.708(t - 32)$.

According to this, the total heat of vaporization for 100°, 150°, and 200°, is

$r = 607 - 0.708 \times 100 = 607 - 70.8 = 536.2$ heat units.
$r = 607 - 0.708 \times 150 = 607 - 96.2 = 500.8$ " "
$r = 607 - 0.708 \times 200 = 607 - 141.6 = 465.4$ " "

These vary somewhat from the values found from the more exact formula.

The facts cited seem to indicate that liquids are converted into steam sooner, the higher the temperature at which evaporation takes place, or, what is the same thing, the greater the pressure under which the steam is generated. The reason is evident, for the more heat the liquid contains, the further apart are the molecules, and the less is the mutual attraction between them. The "heat of the liquid" is evidently the so-called "sensible heat." The "total heat of vaporization" is the so-called "latent heat."

Thus far reach the extended and famous experiments of Regnault, to whom our science is greatly indebted.

We pass now to that which the mechanical theory of heat deduces from these experiments.

Inner and Outer Heat of Vaporization.—Heat of the Steam.—Of the *total* heat imparted when 1 kilogram of water at 0° is converted into saturated steam at any given temperature, a portion goes to increase the temperature of the water. This we have called the "heat of the liquid," or the *sensible heat*. This is the heat which takes effect as vibration work of the particles, and exists in the water *as heat.* Another portion goes to change the water into steam. This we call the "total heat of vaporization" or the *latent heat.** Of this last portion a part is required for the change of state of the liquid, that is, for the disgregation work, and another part is required for the outer work. The first part we call the "inner vaporization heat," or the *inner latent heat*, and the second part is the "outer vaporization heat," or the *outer latent heat.* The total heat imparted, then, during the whole process of the conversion of one kilogram of water into steam, consists of three parts—the *sensible heat* of the liquid at the boiling point, the *inner latent* heat, and the *outer* latent heat. The two last, taken together, comprise the *total latent* heat, and the two first, the "*heat of the steam*," since it is the heat which remains after subtracting from the total heat imparted the heat required for the performance of the outer work. The rest remains in the steam, part as heat or actual energy of motion, in the shape of vibration work or sensible heat, and part as energy of position, or potential energy, in the shape of disgregation work, or change of state, or "latent heat."

Since the pressures are known under which water is vaporized at different temperatures, and since, as we shall see, we can calculate the steam volume produced from one kilogram of steam at different temperatures, we can easily find for every temperature the outer latent heat, or "outer heat of vaporization."

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* ["Latent heat" has become part of the vocabulary of our subject, and cannot now well be gotten rid of, but the term is objectionable unless properly understood. "It should be remembered that *heat* is a kind of actual or kinetic energy, consisting in the invisible *motions* of the particles of a body, and hence that heat is not potential energy; for its ability to perform work depends only upon the heat *motions* of the particles, and not on their relative *positions.*" Heat, therefore, which is expended in performing outer work, ceases entirely to exist in the body *as heat* at all. The term "latent heat" has come down to us from a time when heat was supposed to be a substance, and consequently indestructible, and although we now know that heat, as such, can be put out of existence by transformation into other forms of energy which do not affect the thermometer, still the term remains, and continues to lead many to believe that the heat which has been absorbed or disappeared in doing work still lurks concealed somewhere in the working substance *as heat.* This is less often the case when external work only is considered, for the equivalent of the disappearing heat is then seen in visible external work; but when the internal work done in separating the molecules and changing their arrangement is under consideration, the beginner is apt to think that this kind of work "don't count," and that the heat which has been expended in accomplishing it still exists in the body as heat, instead of recognizing that in this case also it has *disappeared* by being transformed into another kind of energy—that of position—which also does not affect the thermometer.

No error can arise from the use of the term "latent heat," however, if understood as defined by Maxwell: "Latent heat is the quantity of heat which must be communicated to a body in a given state in order to convert it into another state without changing its temperature." (See article by J. F. Klein, Journal of Franklin Inst., April, 1879.)]

Then by subtracting this outer latent heat from the total latent heat we obtain the inner latent heat which goes to transform the water into steam, or to perform disgregation work. This inner latent heat we denote by the Greek letter ρ, while the total latent heat is, as before, denoted by r.

As we have seen in Chapter XIV., page 375, at the temperature of 100° the inner latent heat is 496.3, the outer latent heat, page 376, is 40.2, and the sensible heat is 100 heat units when one kilogram of water at 0° is converted into saturated steam at 100°.

Zeuner has found that the inner latent heat is given by the formula

$$\rho = a - bt - ct^2,$$

where the coefficients a, b, c have special values for each different liquid, and t is the temperature for which the inner latent heat is required. For water we have

$$a = 575.40, \qquad b = 0.791, \qquad c = 0, \qquad \text{hence}$$

$$\rho = 575.40 - 0.791t. \quad . \quad . \quad . \quad \text{(VIII.)}$$

For Fahrenheit degrees $\rho = 1035.72 - 0.791\ (t - 32)$.

EXAMPLE.

What is the inner latent heat for saturated steam at 130° and at 150°?

For 130° we have

$$\rho = 575.40 - 0.791 \times 130 = 472.57.$$

For 150°,

$$\rho = 575.40 - 0.791 \times 150 = 456.75.$$

Zeuner has found, also,

For

Ether................$\rho = 86.54 - 0.10648t - 0.0007160t^2$.
Acetone ....$\rho = 131.63 - 0.20184t - 0.0006280t^2$.
Chloroform ....$\rho = 62.44 - 0.11282t - 0.0000140t^2$.
Chloride of carbon....$\rho = 48.57 - 0.06844t - 0.0002080t^2$.
Bisulphide of carbon..$\rho = 82.79 - 0.11446t - 0.0004020t^2$.

We can now easily deduce a general expression for the outer latent heat.

Let, again, *ABCD*, Fig. 63, be a hollow cylinder. Upon the bottom lies one kilogram of water at 0°, and the piston *KK* rests upon the surface. Upon the piston we have the pressure p. As soon as the water is heated to $t°$, let steam formation commence. Let u be the height to which the piston is forced when all the water is just evaporated. Then the work performed by the steam is

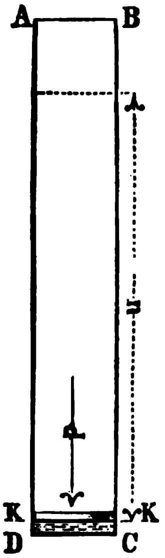


Fig. 63.

$$pu \text{ meter-kilograms.}$$

Since a mechanical work of 1 meter-kilogram corresponds to $A = \frac{1}{424}$ heat units, this work represents

$$Apu \text{ heat units.}$$

This is, therefore, the general expression for the outer latent heat. Since, now, the total latent heat (r) consists of the inner (ρ) and the outer (Apu), we have

$$r = \rho + Apu \quad . \quad . \quad . \quad . \quad . \quad (\text{IX.})$$

From Table II., therefore, we can easily find r for different pressures and temperatures. We have only to add the values of ρ and Apu. Thus, for example, for a pressure of 1.5 atmospheres, therefore for a temperature of 111.7°, $r = 487.01 + 41.16 = 528.17$ heat units.

Under the assumption that Apu can be calculated for every pressure and temperature, and that r is given by observation, we have

$$\rho = r - Apu,$$

and from this the values of ρ in the table are calculated.

Following Zeuner, we have called that work which heat causes in a body when it raises its temperature, and changes the aggregation of the molecules, "inner work;" and that work necessary to overcome the outer pressure, "outer work." In the formation of steam we can call that heat which raises the temperature of the water to the boiling point, and converts it then into steam, the heat of the steam or "*steam heat.*" This we denote by J. The "steam heat" consists, therefore, of the sensible heat (q) and the inner latent heat (ρ), and hence

$$J = q + \rho \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (X.)$$

Thus we can find J from Table II., for different pressures. Thus, for 2 atmospheres, the steam heat J is 121.42 + 480.00 = 601.42 heat units.

We can also obtain the steam heat by subtracting the outer latent heat (Apu) from the total heat imparted (W). Hence we have also

$$J = W - Apu \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (XI.)$$

This expression evidently follows, from the preceding, when we substitute $r - Apu$ for ρ. Thus,

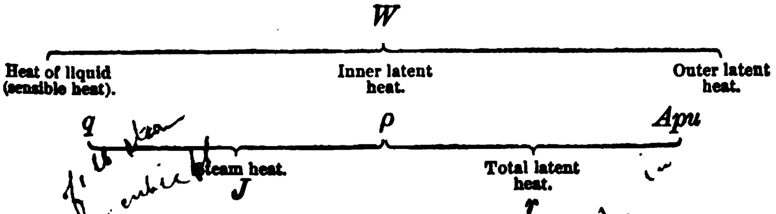
$$J = q + r - Apu,$$

and since $q + r = W$

$$J = W - Apu.$$

We give below a scheme of the manner in which the heat imparted is divided up, together with the notation employed. The student should make himself thoroughly familiar with the exact significance of each letter, and their mutual relations.

TOTAL HEAT.



QUESTIONS FOR EXAMINATION.

Define specific heat. Is the specific heat of water constant? What two separate questions are involved in the preceding? Does the coefficient of expansion of water vary at different temperatures? Why can we neglect the disgregation work? If, then, the disgregation work and variations of outer work can be disregarded, to what must any variation in specific heat of water be ascribed? Describe Regnault's apparatus for investigating this question. Why must the heat appearing in the calorimeter be precisely that received by the water in the boiler? What is Regnault's experimental formula for the specific heat of water between t_1 and $t°$? What does it reduce to between 0° and $t_1°$? How did he determine the value of the coefficients? What is meant by the "mean specific heat?" What is the "heat of the liquid?" What letter in our notation denotes it? What formula gives it? Is the "heat of the liquid" always greater numerically than the temperature? What is "actual specific heat?" How does it differ from "mean?"

What do you understand by "total heat?" Describe Regnault's apparatus for determining it. What is the empirical formula for the total heat? What letter in our notation denotes it? If you subtract the heat of the liquid from the total heat, what remains? Define total heat of vaporization. What letter denotes it in our notation? What is the formula of Clausius for total heat of vaporization? Does it give exact results? What do you understand by sensible heat? What by latent heat? What is inner latent heat? Outer latent heat? If the total latent heat and the outer latent heat are given, how can you find the inner latent heat? What effect does this heat perform? What letter in our notation denotes it? What is Zeuner's formula for it? What does the expression Apu denote? Give the exact significance of each letter. What does r denote? ρ? What is the relation between r, ρ, and Apu? What do you understand by steam heat? What letter denotes it? What is the relation between J, q, and ρ? Construct scheme which shows the manner in which the total heat W is divided up. What is the relation between J, W, and Apu? Give the exact significance of each letter.

NOTE.—Rankine gives the following empirical formulæ for specific heat of water for Fahrenheit scale and point of maximum density of water, 39.1°.

For *mean specific heat* between t_1 and t,

$$C_{t_1-t} = 1 + 0.000000103\,[(t_1 - 39.1)^2 + (t_1 - 39.1)\,(t - 39.1) + (t - 39.1)^2].$$

For mean specific heat between point of maximum density and t,

$$C_m = 1 + 0.000000103\,(t - 39.1)^2.$$

For heat of liquid for one pound between point of maximum density and t,

$$q = (t - 39.1) + 0.000000103\,(t - 39.1)^3.$$

For actual specific heat

$$C = 1 + 0.000000309\,(t - 39.1)^2.$$

To reduce these formulæ to Centigrade scale, put

0.000001 for 0.000000309
0.00000033 for 0.000000103
$t - 4$ for $t - 39.1$.

CHAPTER XVI.

CALCULATION OF SPECIFIC STEAM VOLUME.—EMPIRICAL FORMULÆ FOR THE INNER AND OUTER LATENT HEAT, AS ALSO FOR THE DENSITY OF STEAM.

Calculation of Specific Steam Volume.—The question now arises, How can we calculate, from the temperature and pressure of the steam, the outer latent heat (Apu), or, what amounts to the same thing, how can we find the specific steam volume? As soon as we know this we can easily determine the outer and inner latent heat. At first the specific volume was found by the combined law of Mariotte and Gay-Lussac.

It was *assumed*, therefore, that saturated steam behaved precisely like a permanent gas,* and hence that its specific volume could be easily found from its temperature and tension. Thus if p is the expansive force of a permanent gas, T the absolute temperature, and v the specific volume, we have, as seen already in Part I,

$$pv = RT,$$

where $R = 29.272$ for air. We have, therefore,

$$v = \frac{RT}{p}.$$

Gay-Lussac concluded from his experiments that the volume of *steam*, at the same temperature and tension, was always 1.6064 times as great as that of air. Hence for steam we should have

$$v = 1.6064 \times 29.272 \frac{T}{p} = 47.023 \frac{T}{p}.$$

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* Since now all the so-called "permanent gases" have been liquefied, the term is only to be taken as meaning those gases removed so far from their point of liquefaction that the disgregation work is very slight, or null, and which, under ordinary pressure and temperature, remain approximately perfect gases.

From the preceding we can find upon these assumptions the pressure p for any given value of $T (= 273 + t)$, or for p given, can find the temperature. (This, at least, can be found easily from the formula of Magnus.)

EXAMPLE.

What is the specific volume of steam, according to the above (incorrect) formula, for the temperature of 100° and 144°?

For 100° we have, from Table, $p = 10334$ kilograms, and $T = 273 + 100 = 373°$. Hence $v = 47.023 \frac{373}{10334} = 1.6928$ cubic meters. The volume for 144° is $47.023 \frac{273 + 144}{4 \times 10334}$ (since for 144° the pressure is 4 atmospheres), or 0.4749 cubic meters.

The specific weight, or the weight of 1 cubic meter, must evidently, according to Gay-Lussac, be always $\frac{1}{1.6064} = 0.622$ of that of air at the same temperature and pressure.

Later investigations, especially the calculations of the mechanical theory of heat, have shown that the experiments of Gay-Lussac are *not exact*, and that the conclusions drawn from them are incorrect. The determination of steam volume based upon these experiments and assumptions cannot, therefore, lay claim to much accuracy. The exact determination is, however, essential to any reliable theory of the steam engine, and for this reason all such theories having such incorrect basis are inexact. To Clausius belongs the credit of being the first to show how steam volumes may be found, by the aid of the mechanical theory of heat, with far greater accuracy than according to the earlier methods. Before we give his method, we would call attention to the following customary terms and notations of the mechanical heat theory.

Customary Terms and Notation for Steam.—We call the volume occupied by 1 kilogram of water at 0° the "*specific water volume*," and denote it by σ. The volume of 1 kilogram of water at 0° is, then, σ cubic meters. Let us again assume this specific water volume inclosed in a cylinder with a movable piston. When heat is imparted we have a gradual evaporation of the water. Suppose that at any moment, of the 1 kilogram of water, x kilograms are steam (x being then less than 1), then $1-x$ kilograms are still water. Since now 1 kilogram of water occupies the space σ, if we disregard the slight increase of bulk of the water when heated, $1-x$ kilograms will occupy the space $(1-x)\,\sigma$ cubic meters. When the entire kilogram of water is just converted into steam, let its volume, that is, the specific steam volume, be s. The volume of x kilograms will then be sx cubic meters. Therefore the entire volume of the $1-x$ kilograms of water, and of the x kilograms of steam will be

$$(1-x)\,\sigma + xs \text{ cubic meters.}$$

This volume, whose weight is still one kilogram, and which consists partly of steam and partly of liquid, we denote by v so that we have

$$v = (1-x)\,\sigma + xs,$$

or

$$v = \sigma - x\sigma + xs = \sigma + (s-\sigma)\,x.$$

That is, the specific volume of the mixture of steam and water is equal to the specific water volume (σ) plus the product of the steam weight (x) into the difference of the specific steam and water volumes. Clausius denotes this difference $(s-\sigma)$ by u, so that

$$v = \sigma + ux.$$

The value of σ is readily determined by experiment for different liquids. Thus it has been found

For		
Water	$\sigma=0.001$ cubic met.	$=0.0160$ cub. ft.
Ether ($C_4H_{10}O$)	$\sigma=0.0013$ " "	$=0.0210$ " "
Alcohol (C_2H_6O)	$\sigma=0.0013$ " "	$=0.0210$ " "
Acetone (C_3H_6O)	$\sigma=0.0012$ " "	$=0.0192$ " "
Chloroform ($CHCl_3$)	$\sigma=0.0006$ " "	$=0.0096$ " "
Chloride of carbon (CCl_4)	$\sigma=0.0006$ " "	$=0.0096$ " "
Bisulphide of carbon (CS_2)	$\sigma=0.0008$ " "	$=0.0128$ " "

Hence we see σ is so small with regard to ux that it may be disregarded, and we have simply

$$v = ux.$$

EXAMPLE.

What is the volume of a quantity of steam and water at 100°, whose weight is 1 kilogram, which consists of 0.3 kilograms steam, and 0.7 kilograms water?

According to Table II., for 100° $u = 1.649$ cubic meters, hence $ux = 1.649 \times 0.3 = 0.4947$ cubic meters, and $\sigma + ux = 0.001 + 0.4947 = 0.496$ cubic meters.

For one pound we have for 212° Fah. $u = 26.4216$ cubic feet, hence $ux = 26.4216 \times 0.3 = 7.926$ cubic feet, and $\sigma + ux = 0.0160 + 7.926 = 7.942$ cubic feet.

EXAMPLE 2.—One kilogram of steam and water has a temperature of 130.8°, of which 0.5 cubic meter is steam. How much does it weigh?

Since at the temperature 130.8° $u = 0.647$ cubic meter, and σ is very small compared to s, we have

$$0.5 = 0.001 + 0.647x. \quad \text{Hence } x = \frac{0.499}{0.647} = 0.77 \text{ kilograms.}$$

The water is therefore $1 - 0.771 = 0.229$ kilograms.

For one pound at a temperature of 266.63° Fah. of which 0.5 cubic feet is steam, we have $u = 10.3722$ and $0.5 = 0.0160 + 10.3722x$. Hence $x = 0.046$ pound.

Steam Volume, calculated according to the Principles of Thermodynamics.—Let us now see how the specific steam volume may be calculated.

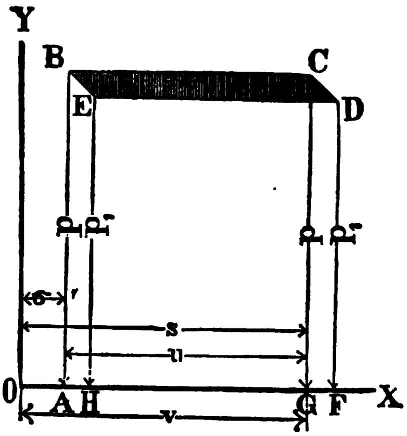

FIG. 64.

Let OA, Fig. 64, be the specific water volume (σ). We conceive it again inclosed in a cylinder of 1 square meter, cross-section, and the piston loaded with p kilograms. In such case, then, σ is the distance of the surface of the water, or of the piston, from bottom of the cylinder. After the water is heated to $t°$, let the steam generated just have the pressure p. Therefore $t°$ is the boiling point corresponding to the pressure p. By further addition of heat steam is formed, and the piston will be lifted. When all the water is converted into steam, let the specific steam volume be $OG = s$. In other words, the piston now stands at the height OG from the bottom. The distance passed through by the piston is then $OG - OA = s - \sigma = u$. During this the pressure p is constant.

Let us now suppose the specific steam volume s to expand adiabatically until the temperature has sunk by the very small quantity τ, and the pressure p has become p_1. For this small decrease of temperature we may assume that the saturated steam acts as a permanent gas, and hence the line *CD*, which is an adiabatic, for this very short distance, is a straight line.

Now let the steam at pressure p_1 be compressed, and heat at the same time abstracted, so that the pressure p_1 remains constant until the specific volume of water and steam is *OH*, the volume of the mixture, or the position of *H*, being so chosen that when the remaining steam is compressed adiabatically along *BE*, it shall all be condensed, and come back to its original temperature and its original volume.

We have in this way completed a simple cycle process, and the outer work performed is given by the shaded area *BCDE*, or by the product

$$(p - p_1)\,BC = (p - p_1)\,u.$$

The heat imparted from *A* to *G*, or *B* to *C*, is greater than that abstracted from *D* to *E*, and the difference must equal the outer work.

If we denote the heat imparted by Q, the work performed is (page 186, Part I.)

$$\frac{Q}{AT}[T - (T - \tau)]$$

where $T = 273 + t$ is the absolute temperature of the steam from *B* to *C*, $T - \tau = 273 + t - \tau$ that from *D* to *E*. We have therefore

$$(p - p_1)\,u = \frac{Q}{AT}\,\tau.$$

Denoting, for the sake of brevity, $p - p_1$ by π,

$$u = \frac{Q}{AT} \cdot \frac{\tau}{\pi}.$$

Now Q is evidently that heat which must be imparted to 1 kilogram of water at the temperature t, in order to convert it, under constant pressure, into steam of t°, or it is the total latent heat of vaporization, r. Hence

$$u = \frac{r}{AT} \cdot \frac{\tau}{\pi} = \frac{424r}{T} \cdot \frac{\tau}{\pi} \quad . \quad . \quad . \quad . \quad \text{(XII.)}$$

This formula is the most important hitherto deduced from the application of the mechanical theory of heat to steam. From it and the preceding equations, those which follow can be easily derived. For English measures we have 772 in place of 424.

The fact that the differences of pressure for the temperatures $t + 1$, t, and $t - 1$ are nearly the same, furnishes the means of determining the ratio $\frac{\tau}{\pi}$ in an elementary manner with all necessary exactness. Thus, for example, if we wish to determine u for a temperature of 80°, and take the difference of pressure (π) for 80° and 79°, this difference will be relatively too small. If we take the difference of pressures for 80° and 81°, it will be relatively too large. If we take the mean of both differences, it will be very closely the increase or decrease of pressure for a very small change of temperature. If we denote, therefore, the pressure at 81° by p, and at 79° by p_1, we have for τ about, say, $\frac{1}{10}$ of a degree.

$$\tau : \pi = \tfrac{1}{10} : \tfrac{1}{10}\,\frac{p - p_1}{2},$$

or generally

$$\frac{\tau}{\pi} = \frac{1}{\dfrac{p - p_1}{2}} = \frac{2}{p - p_1}.$$

Here p and p_1 denote the pressure in kilograms per square meter, or in pounds per square foot. Table I. gives them in centimeters or inches of barometer. To reduce those in the table to kilograms per square meter we have to multiply the tabular values in millimeters by $\frac{10334}{760}$, and to reduce to pounds per square inch, we multiply the tabular value in inches by $\frac{14.6954 \times 144}{29.9215}$.

We have then

$$\frac{\tau}{\pi} = \frac{2 \times 760}{10334\,(p - p_1)} = \frac{0.147}{p - p_1},$$

where p and p_1 are given in millimeters of mercury column;

or

$$\frac{\tau}{\pi} = \frac{2 \times 29.9215}{144 \times 14.6954(p - p_1)} = \frac{0.02828}{p - p_1},$$

where p and p_1 are given in inches of the mercury column.

The formula for u becomes then for French units

$$u = \frac{424 \times 0.147 r}{T(p - p_1)} = \frac{62.328}{p - p_1} \cdot \frac{r}{T} \quad . \quad . \quad \text{(XIII.)}$$

and for English units $u = \dfrac{772 \times 0.02828}{T(p - p_1)} = \dfrac{21.832}{p - p_1} \cdot \dfrac{r}{T}$.

The specific steam volume s we can at once obtain from u by adding the specific water volume $\sigma = 0.001$. Thus

$$s = \frac{62.328}{p - p_1} \frac{r}{T} + 0.001 \text{ cubic meters.}$$

EXAMPLE.

What is the value of u for 0° and for 80°?

According to Röntgen's formula

$$u = \frac{62.328}{4.91 - 4.25} \times \frac{606.5}{273} = 209.78 \text{ cubic meters.}$$

By the aid of the calculus, and using Regnault's values for the pressures, we should obtain $u = 210.66$. The difference is then very small.

Again, for 81°, we have from the table, $p = 369.258$, and for 79°, $p_1 = 340.464$ millimeters. Since for 80° $r = 550.618$,

$$u = \frac{62.328}{369.258 - 340.464} \times \frac{550.618}{273 + 80} = 3.376 \text{ cubic meters.}$$

For one pound at 176° Fah. we have for 177 and 175°, $p = 14.2824$ and $p_1 = 13.6524$, and $r = 991.112$; hence

$$u = \frac{21.832}{0.63} \times \frac{991.112}{635.4} = 54.077 \text{ cubic feet.}$$

Column 8 of Table II. gives the accurate values of u from 0.1 to 14 atmospheres.

More recent investigations of Tate and Fairbairn have shown that the specific steam volumes calculated according to thermodynamic principles agree quite closely with the results of observation, and far better than those found by the combined law of Mariotte and Gay-Lussac.

If the pressures of the other liquids had been determined from degree to degree, we could find with the same exactness the value of u for their vapors also. As this is not the case, we can only find approximate values of u, but values, nevertheless, quite close to the actual.

EXAMPLE 1.—What is the steam volume of 1 kilogram of ether when evaporated at 40°?

The pressure of such vapor at 50° has been found to be 1264.83 millimeters, and at 30°, 634.80 millimeters. The difference 630.03 corresponds to 20°. The difference for 2°, that is, for $t+1$ and $t-1$ degrees, or $p - p_1$, is 63.003 millimeters. Since, from the empirical formulæ already given, r for ether at 40° is 89.48, and since $T = 273 + 40 = 313$, we have

$$u = \frac{62.828}{63.003} \cdot \frac{89.48}{313} = 0.283 \text{ cubic meters.}$$

Zeuner finds by calculus, 0.285.

EXAMPLE 2.—What is the steam volume of alcohol at 50°?

For 60° the pressure has been found 350.21 millimeters, and for 40°, 133.69. Difference for 20°, 216.52, for 2°, 21.652. For 50°, r is 233.79, and $T = 323$, hence

$$u = \frac{62.328}{21.652} \times \frac{233.79}{323} = 2.084.$$

Calculation of the Outer and Inner Latent Heat.—Since we can now find u for every temperature and pressure, it is easy to determine the outer work. Thus if p is the constant pressure, this outer work is simply pu meter-kilograms.

EXAMPLE.

What outer work will be performed by the steam generated from 1 kilogram of water at 150°, the constant pressure being equal to its own tension?

For 150° the pressure, from Table I., is 3581.23 millimeters, or $\frac{10334 \times 3581.23}{760} = 48673.14$ kilograms per square meter. We find from Equation XIII., $u = 0.384$ cubic meters, hence $pu = 48673.14 \times 0.384 = 18690.43$ meter-kilograms. For $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$ of a kilogram converted into steam we should obtain only $\frac{1}{2}$ or $\frac{1}{3}$, etc., of 18690.43 meter-kilograms.

The heat required for this work, which we call the outer latent heat, is

$$Apu = \frac{pu}{424} \text{ heat units.}$$

Therefore the outer latent heat in the vaporization of 1 kilogram of water at 150° is

$$\frac{18690.43}{424} = 44.08 \text{ heat units.}$$

In this way the outer latent heat has been calculated for Table II., column 7, for from 0.1 to 14 atmospheres. We see that it increases with the temperature or expansive force of the steam.

Since, now, we know the total latent heat r from Regnault's experiments and empirical formulæ, and the outer latent heat can be calculated as above, we can find the inner latent heat ρ from the equation

$$\rho = r - Apu.$$

It has been found thus, and is given in column 6 of Table II. It is evident that it must decrease as the temperature increases, since the outer latent heat increases.

Finally, the steam heat J can be found from the equation

$$J = W - Apu.$$

Since for water u differs from s only by about 0.001 cubic meters, and s is very great with reference to 0.001, we can put u in place of s. In such case, u cubic meters require the inner latent heat ρ, and 1 cubic meter requires

$$\frac{\rho}{u} \text{ heat units.}$$

The expression $\frac{\rho}{u}$ gives us, therefore, the heat units necessary for the inner latent heat of 1 *cubic meter* of steam under various pressures. Since we shall have frequent occasion to make use of it, it is given in Table II., column 9.

Empirical Formulæ for the Calculation of the Inner and Outer Latent Heat and of the Specific Steam Volume.—For the mechanical engineer, to whom the easy and accurate determination of u and s is of great importance, empirical formulæ are very desirable, so that even without tables he can find these quantities for different pressures with sufficient exactness. The calculation of u, already given, is too involved for practical men. Zeuner was the first to meet this want. As we have already remarked, he has given for the inner latent heat ρ, the simple formula

$$\rho = 575.40 - 0.791t.$$

Now we have from IX.

$$Apu = r - \rho,$$

$$Apu = r - \rho, = W - \rho - q.$$

For W we have, according to Regnault, $606.5 + 0.305t$, and for q, $t + 0.00002t^2 + 0.0000003t^3$, hence

$$\begin{aligned} Apu = {} & 606.5 + 0.305t - (575.40 - 0.791t) \\ & - (t + 0.00002t^2 + 0.0000003t^3), \end{aligned}$$

or, after reduction, $Apu = 31.1 + 0.096t - 0.00002t^2 - 0.0000003t^3$, or for English units and Fahrenheit degrees $Apu = 55.98 + 0.096 (t-32) - 0.0000111\ (t-32)^2 - 0.0000000926\ (t-32)^3$.

This expression enables us to determine the outer latent heat (Apu) from the temperature alone.

If we divide the last question by Ap, we have

$$u = \frac{31.1 + 0.096t - 0.00002t^2 - 0.0000003t^3}{Ap}. \qquad \text{(XIV.)}$$

or for English units

$$u = \frac{55.98 + 0.096(t-32) - 0.0000111(t-32)^2 - 0.0000000926(t-32)^3}{Apu}.$$

Since A is known, we only need to know the variation of p with the temperature in order to find u. For this we can either make use of Table I. or II., or lacking these, of some one of the expansion formulæ already given.

EXAMPLE.

What is the difference between the specific steam and water volume, or what is the value of u, for 130° and for 200° ?

According to Table I., the pressure at 130° is 2030.28mm., hence the pressure in kilograms per square meter, p, is $\frac{10334}{760} + 2030.28 = 27602$ kil. Now $t = 130$, $t^2 = 16900$, $t^3 = 2197000$.

Hence

$$u = \frac{424\,(31.1 + 0.096 \times 130 - 0.00002 \times 16900 - 0.0000003 \times 219700)}{27602};$$

$$u = \frac{424\,(31.1 + 12.48 - 0.338 - 0.6591)}{27602};$$

$$u = \frac{424\,(43.58 - 0.9971)}{27602} = \frac{424 \times 42.583}{27602} = 0.6548.$$

For 200° the pressure is 11688.96mm. = 158937 kil. $t = 200$, $t^2 = 40000$, $t^3 = 8000000$, and

$$u = \frac{424\,(31.1 + 0.096 \times 200 - 0.00002 \times 40000 - 0.0000003 \times 8000000)}{158937};$$

$$= \frac{424\,(31.1 + 19.2 - 0.8 - 24)}{158937} = \frac{424\,(50.3 - 3.2)}{158937};$$

$$= \frac{19970}{158937} = 0.1262.$$

In the lack of tables, we may use for the calculation of p, for temperatures less than 100°, the formula of Magnus. Above 100°, that of Röntgen.

We have also the following empirical formulæ given by Zeuner for the other liquids already named :

Ether.................... $Apu = 7.46 + 0.02747t - 0.0001354t^2$.
Acetone.................. $Apu = 8.87 + 0.06185t - 0.0002845t^2$.
Chloroform .... $Apu = 4.56 + 0.01797t - 0.0000367t^2$.
Chloride of carbon........ $Apu = 3.43 + 0.01671t - 0.0000546t^2$.
Bichloride of carbon...... $Apu = 7.21 + 0.02524t - 0.0000918t^2$.

EXAMPLE.

What volume of steam is generated from 1 kilogram of ether when evaporated at 40° ?

The pressure at 40° is 907.04mm., or $\frac{10334}{760} \times 907.04 = 12297.5$ kilograms. $t = 40$, $t^2 = 1600$, hence

$$A \times 12297.5 \times u = 7.46 + 0.02747 \times 40 - 0.0001354 \times 1600,$$

or

$$\frac{1}{424} \times 12297.5 \times u = 8.3422, \qquad \text{or} \qquad u = \frac{8.3422 \times 424}{12297},$$

or

$$u = 0.287 \text{ cubic meters.}$$

On page 392 we found for u 0 283 cubic meters, or but little different from the empirical formulæ.

Density of Saturated Steam.—The preceding is sufficient to show that the view of Gay-Lussac, that the density of saturated steam is always 0.6225 of that of air, is not correct.

Since the specific steam volume, that is, the volume of 1 kilogram of steam $s = u + \sigma$, the weight γ of one cubic meter of steam, which we call the "*specific weight,*" will be

$$\gamma = \frac{1}{s} = \frac{1}{u + \sigma} = \frac{1}{u + 0.001} \text{ kilograms per cub. met.,}$$

or for English units $\gamma = \frac{1}{u + 0.016}$ pounds per cubic foot.

Thus, for example, for 150°, the weight of one cubic meter of steam is

$$\gamma = \frac{1}{0.384 + 0.001} = \frac{1}{0.385} = 2.597 \text{ kil.}$$

For one cubic foot at 302° Fahrenheit we have

$$\gamma = \frac{1}{6.149 + 0.016} = \frac{1}{6.165} = 0.162 \text{ pounds.}$$

The value of γ is given in column 11 of Table II. for from 0.1 to 14 atmospheres. For air we have always

$$pv = RT = 29.272\ T,$$

or for English units $pv = RT = 53.354\ T$,

therefore the specific volume is

$$v = \frac{29.272\ T}{p},$$

and the specific weight

$$\gamma_1 = \frac{1}{v} = \frac{p}{29.272\ T}.$$

For example, for 150° the pressure of steam is 4.71 atmospheres, hence the pressure p in kilograms per square meter is $4.71 \times 10334 = 48673$ kilograms. Since now $T = 273 + 150$

$= 423$, for this temperature and pressure, the weight of one cubic meter of air would be

$$\gamma_1 = \frac{48673}{29.272 \times 423} = \frac{48673}{12382} = 3.930 \text{ kil.}$$

On the other hand, the weight γ of one cubic meter of saturated steam, under the same conditions, is really 2.597 kilograms, as already computed. Accordingly, the *density* δ of the steam, that is, the ratio of its weight to that of an equal amount of air under the same conditions, is

$$\delta = \frac{\gamma}{\gamma_1} = \frac{2.597}{3.930} = 0.661.$$

If we calculate in this way the specific weights of steam and air for different temperatures and pressures, we find for the density,

For

	0.1	0.5	1	2	5	10 atmospheres
$\delta =$	0.621	0.633	0.640	0.648	0.662	0.676,

from which we see that the density increases tolerably rapidly with increasing temperature. Hence

$$\frac{pv}{T}$$

cannot be a constant quantity as with gases is the case. Saturated steam follows some other law than this.*

[* The relation $pv = RT$ therefore holds good only for those gases so far removed from their point of saturation that they may be considered as perfect. Zeuner has recently shown that for steam, whether saturated or superheated,

$$pv = BT - Cp^{\frac{k-1}{k}}$$

in which $B = \frac{c_p(k-1)}{Ak}$, and $c_p = 0.4805$, and $k = 1.333$, hence $B = 50.933$, and $C = 192.50$, p being in kilograms, or if p is in atmospheres, then for both saturated and superheated steam $pv = BT - C\sqrt[4]{p}$, where $B = 0.0049287$, and $C = 0.187815$. See Appendix to Chap. XXIII.]

QUESTIONS FOR EXAMINATION.

Define specific steam volume. How was this first calculated? Upon what assumption was this calculation founded? Give the relation between pressure, volume, and temperature for a gas. What does R stand for? What did Gay-Lussac conclude from his experiments? Was this conclusion correct? Is the old method of calculation of specific volume correct? Why not? How does saturated steam differ from a perfect gas?

Define what is meant by specific water volume? What letter denotes it in our notation? What is specific steam volume? What letter denotes it? What does u denote in our notation? What relation exists between v, σ, and u, if x is the weight of steam in one kilogram of steam and water? What does v denote? σ? u?

Deduce the expression $u = \frac{r}{AT} \cdot \frac{\tau}{\pi}$. What is the exact significance of each of the letters? Is this formula important? Why? Show how to find in an elementary manner the approximate value of the ratio $\frac{\tau}{\pi}$ in any given case. Explain now the use of Tables I. and II.

If u is given, how can you find the outer work? The outer latent heat? How can you find the inner latent heat? How can we find the total latent heat? What is the steam heat? What does $\frac{\rho}{u}$ denote? What is Zeuner's formula for the inner latent heat? How can you find from it the outer latent heat and the specific steam volume?

What is specific weight? What letter in our notation denotes it? What is meant by density of steam? How does this vary for different temperatures and pressures?

CHAPTER XVII.

CURVE OF CONSTANT STEAM WEIGHT.—EMPIRICAL FORMULÆ.—DEPORTMENT OF STEAM WHEN IT EXPANDS PERFORMING WORK.

Curve of Constant Steam Weight.—If we lay off the volumes of, say, 1 kilogram of steam for successive pressures, as abscissas, and the corresponding pressures as ordinates, we obtain a curve (Fig. 65) which represents the law according to which the volume changes with the pressure. We may call this the "*curve of constant steam weight.*" For a pressure of 1 atmosphere the volume of 1 kilogram of steam is, from Table II., 1.649 + 0.001 = 1.650 cub. m. Taking only 2 decimal places, we have $s = 1.65$. Hence $OA = 1.65$ units to any given scale, and the perpendicular AB is laid off according to another given scale. The volume of 1 kilogram of steam at 2 atmospheres is about 0.86 cubic meters. Therefore, $OC = 0.86$, and $CD = 2$, and so on.

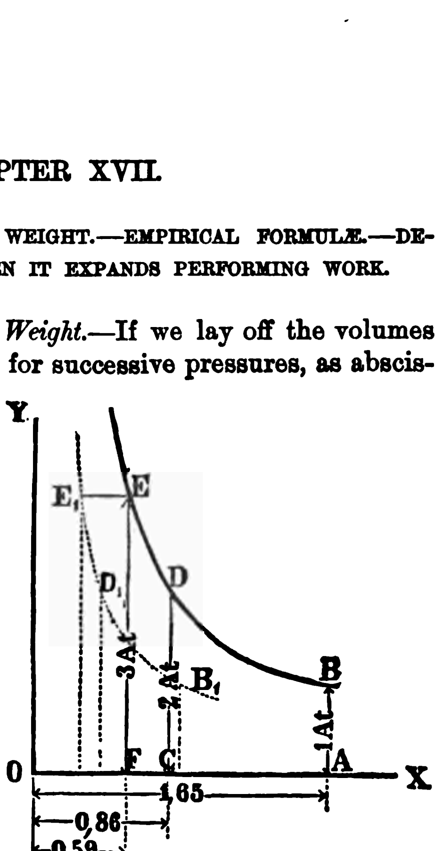

Fig. 65.

We see that the volumes decrease nearly inversely as the pressures, that is, that the volumes are 2, 3, 4 times less when the pressures are 2, 3, 4 times greater. If this were accurately the case, the relation between the pressures and the specific volumes would be

$$ps = p_1s_1 = p_2s_2, \text{ etc.}$$

As, however, is seen from the Figure, this is not exactly the case. Zeuner has found by calculation that the law for the curve of constant steam weight is given very closely by the formula

$$ps^{1.0646} = p_1 s_1^{1.0646} = p_2 s_2^{1.0646} \quad . \quad . \quad . \quad . \quad \text{(XV.)}$$

where the exponent of s, s_1, etc., is only 0.0646 greater than in the preceding formula which gives the law of Mariotte for permanent gases.

Evidently these products, since they are all equal to each other, must equal a constant value, and this value is, according to Zeuner, 1.704, so that

$$1.704 = ps^{1.0646} = p_1 s_1^{1.0646} \quad . \quad . \quad . \quad . \quad \text{(XVI.)}$$

or for English units $32.653 = ps^{1.0646} = p_1 s_1^{1.0646}$ where p is taken in atmospheres.

EXAMPLE.

What is, according to this formula, the specific volume of saturated steam at a pressure of two atmospheres ?

Since here $p = 2$, we have $s = \sqrt[1.0646]{\frac{1.704}{2}} = \sqrt[1.0646]{0.854}$, or $\log s = \frac{\log 0.854}{1.0646} = \frac{\bar{1}.9314579}{1.0646}$, or $s = 0.8622$ cubic meters per kil.

For English units we have $\log s = \frac{\log 16.3265}{1.0646}$ or $s = 13.765$ cub. ft. per pound.

From this formula we obtain s as well as u with great exactness. It is also more convenient for calculation than Equation XIV. We may also obtain from it the specific weight γ of the steam. Thus $\gamma = \frac{1}{s}$ and $s = \frac{1}{\gamma}$, hence

$$1.704 = p\left(\frac{1}{\gamma}\right)^{1.0646},$$

or

$$\gamma^{1.0646} = \frac{1}{1.704}p = 0.5868p,$$

or

$$\gamma = p^{\frac{1}{1.0646}} \times \sqrt[1.0646]{0.5868}, \quad \text{or finally}$$

$$\gamma = 0.6061 \times p^{0.9393} \text{ kilograms per cubic meter.}$$

For English units we have $\gamma = 0.037839 \times p^{0.9393}$ pounds per cubic foot.

By means of this formula Zeuner has found the specific weight of steam for different pressures, and compared with the values obtained by previous calculations. The coincidence is so great that only occasionally is there a deviation of one unit in the third decimal place. Hence the last formula is of great practical use.

We have assumed above that the volumes and pressures for *one* kilogram of steam are taken as abscissas and ordinates. Instead of this, we might have taken the volumes of one-half, one-third, etc., kilogram of steam, together with the corresponding pressures, and thus obtained a curve. This new curve would have the same law as the above, and will only differ in having, with reference to the same axes, a different beginning and end. It is represented in the Figure by the dotted line $E_1D_1B_1$.

[*Curve of Saturation.—Critical Temperature.*—Let a series of isothermals be drawn, as $A_1B_1S_1T_1$, $A_2B_2S_2T_2$, etc., as in the following Figure, of which the portions A_1B_1, A_2B_2 represent the changes of pressure and volume of the fluid at constant temperature in the liquid state; B_1S_1, B_2S_2, etc., the process of evaporation, and S_1T_1, S_2T_2, etc., the expansion of the superheated vapor at constant temperature. A curve drawn through the points S_1, S_2, etc., will represent the changes which may be undergone by the fluid while it remains entirely in the state of saturated vapor. It is, therefore, called the *curve of saturation*. The volume of all fluids in the state of saturated vapor decreases as the pressure and temperature increase, and thus the curve of saturation slopes downward from left to right, as shown in the Figure.

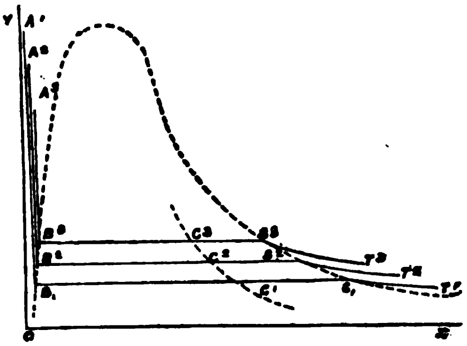

FIG. 66.

On the other hand, the volume of every liquid at the boiling point increases with pressure and temperature. Therefore a curve drawn through the series of points B_1, B_2, etc., will slope in the opposite direction to the curve of saturation, and the two will approach each other as the pressure increases, and at length meet. The physical interpretation of this is that at a certain temperature the liquid and gaseous states become continuous, there being no marked separation, such as that observed in the ordinary processes of evaporation and condensation, between them. This is called the *critical* temperature of the fluid. Above this temperature the fluid retains the properties of a gas under any pressure however great.

It is supposed that the so-called permanent gases resist condensation into the liquid form so greatly because the lowest temperatures which we are able to produce ordinarily are still above their critical temperatures.

For certain substances the critical temperature has been accurately determined. For instance, that of carbonic acid is at 87.7° Fahr., or 31.17° C., and the corresponding pressure of saturation is about 74 atmospheres.

There are a few substances, however, which can readily be brought to the critical temperature. Water reaches it at about 720.6° Fahr., or 382.55° C. The corresponding pressure of saturation has not been determined. But both temperature and pressure are far higher than those met with in the practical applications of steam.]

Deportment of Steam when it Expands Performing Work.—A knowledge of the deportment of saturated steam, when it expands while performing work, is of especial importance in practice, as, by means of it, we are in a position to estimate more exactly the action of steam in the steam engine. It was formerly assumed that steam in expanding not only remained saturated, but also that the steam weight did not change; that, therefore, the expansion took place along the curve of constant steam weight. Pambour especially, to whom we owe the first complete and systematic theory of the steam engine, assumed this principle in his development, and after him all writers down to recent times accepted it as correct. Although now the saturated steam, under the given conditions, remains saturated, as is indicated by the older observations of Pambour, and the more recent observations of Hirn, still *the steam weight is not constant*, in other words, expansion does not take place according to the curve of constant steam weight. This fact can only be made apparent by the aid of the mechanical theory of heat, as was done in 1851, almost simultaneously, by Clausius and Rankine. It is easy to show that the expansion of steam in a steam engine does not follow the curve of constant steam weight.

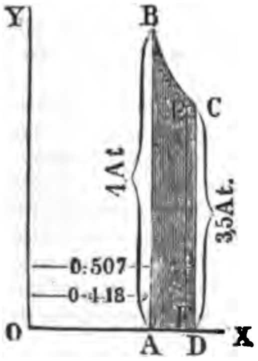

Fig. 67.

Let OA, Fig. 67, be the steam volume behind the piston, its temperature $t = 144°$, and pressure $p = 4$ atmospheres $= 41336$ kilograms per square meter. Let the volume OA of 1 kilogram be 0.447 cubic meters. If this steam expands to the volume $OD = 0.507$ cubic meters, the temperature sinks to 140.44° and the pressure to $CD = 3.5$ atmospheres $= 36169$ kilograms per square meter. If now the steam during this expansion remains saturated, and the steam weight constant, the curve BC is a portion of the curve of constant steam weight, and the shaded area $ABCD$ denotes the mechanical work performed during expansion. The contents of this area are, considering it as a trapezoid,

$$\frac{AB + CD}{2} \times AD,$$

or

$$\frac{41336 + 36169}{2} \times (0.507 - 0.447) = 2325$$

meter-kilograms. This work represents

$$\frac{2325}{424} = 5.49 \text{ heat units.}$$

Now the steam heat at 4 atmospheres of 1 kilogram of steam is

$$J = 145.31 + 461.50 = 606.81 \text{ heat units,}$$

and at 3.5 atmospheres

$$J = 140.44 + 465.26 = 605.70 \text{ heat units.}$$

The difference, 1.11 heat units, is not sufficient to perform the work of 2325 meter-kilograms. For this purpose 4.38 heat units more are necessary.

Since now, according to our assumption, no heat is imparted from without, we must conclude that the steam condenses, and that the condensation supplies the lack of heat of 4.38 heat units.

Since steam condenses during the expansion, the work done cannot be so great as when the steam weight is constant, and hence the curve of expansion must approach the axis OX more rapidly than BC, which is a portion of the curve of constant steam weight. If, then, the end pressure is the same, the end volume cannot be OD, but must be less than OD. If OF is this volume, and $FE = CD$ the final pressure, the work during expansion is given by *ABEF*. The volume *OF* and the pressure *EF* must correspond to the volume and pressure of the remaining saturated steam. The point *E* must therefore lie in a curve of constant steam weight, where the weight is less than 1 kilogram. Since the expansion takes place without heat being added from without, the curve *BE* must be part of an adiabatic.

Our example shows plainly that during expansion steam is condensed, or else that heat must be imparted, but it does not give the exact value of this heat, since the work during expansion is not given by *ABCD* but by the less area *ABEF*. If, therefore, we wish to find this heat we must adopt another method. This has been done by Clausius, in his "*Abhandlung über Mechanische Wärmetheorie*," 1864, and we shall now proceed to point it out.

Suppose in a prismatic vessel a mixture of steam and water of the temperature t and pressure *AB*, Fig. 68. Of this mixture let M kilograms be liquid and m kilograms steam. Upon the steam presses a piston whose pressure is *AB*.

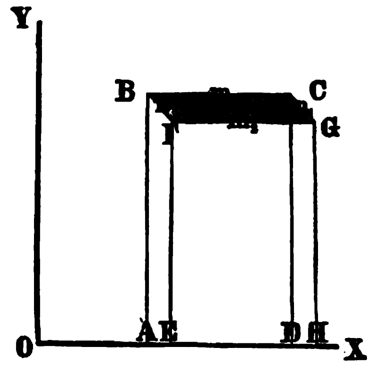

Fig. 68.

We impart heat to the water while assuming the pressure remains constant. In this case all the heat goes to form steam, and is therefore latent. Suppose that thus m_1 kilograms of water are vaporized, so that we now have in all $m + m_1$ kilograms of steam. If now r denotes the latent heat when 1 kilogram of water at the temperature t, and under the constant pressure *AB*, is evaporated, the heat imparted is

$$m_1 r_1 \text{ heat units} \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

Now let the entire steam volume $m + m_1$ expand adiabatically. The expansion is then at the expense of the heat of the mixture, and the temperature sinks. If we suppose the expansion CG to be very small, the decrease of temperature is slight. Denote it by τ, so that at G the temperature of the mixture is

$t - \tau$. It is clear that CG is a portion of an adiabatic. We may suppose now, in opposition to our calculations, that from C to G, n_1 kilograms of steam are formed.

Now we compress the steam, assuming that it is always saturated, from the volume OH, temperature $t - \tau$, and pressure HG, so that pressure and temperature remain constant. We must then abstract heat during compression. This compression is carried to a point F, so chosen that when from there on the steam is compressed adiabatically the mixture will retake its original condition, and we shall have again M kilograms of water, and m of steam at the temperature t. Then the temperature from F to B has been increased by τ, and the work which the steam performed by expansion from C to G has been received again from F to B. We have thus here a simple cycle process. Inner work has been neither gained nor lost.

Let now r_2 be the heat which must be abstracted from 1 kilogram of steam at the temperature $t - \tau$, in order to obtain 1 kilogram of water at the temperature $t - \tau$, then if from G to F, m_2 kilograms are condensed, the heat abstracted is

$$m_2 r_2 \text{ heat units} \quad . \quad . \quad . \quad . \quad . \quad . \quad (2).$$

The excess of the heat imparted over that abstracted is

$$m_1 r_1 - m_2 r_2 \text{ heat units} \quad . \quad . \quad . \quad . \quad . \quad (3).$$

By this excess a certain mechanical work is obtained, represented by the area $BCGF$, which, since CG and BF are very small, we may regard as a parallelogram. The area is then

$$BC \times (AB - FE).$$

If we denote the difference $AB - FE$ by π, we have

$$BC \times \pi.$$

If now the volume of 1 kilogram of steam at t° is u, that of m_1 kilograms is $m_1 u$. Hence $BC = m_1 u$, and

$$BC \times \pi = m_1 u \pi.$$

The heat corresponding to this work is

$$A m_1 u \pi \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

This heat must be equal to (3), hence

$$m_1 r_1 - m_2 r_2 = A m_1 u \pi \quad . \quad . \quad . \quad . \quad . \quad (5).$$

Now the steam weight generated on the path *BCG* is

$$m_1 + n_1,$$

and that condensed on the path *GFB* is

$$m_2 + n_2,$$

assuming that during the compression *BF*, n_2 kilograms are deposited. Since at the end of the process we have the original quantity of water and steam,

$$m_1 + n_1 = m_2 + n_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

Hence

$$m_2 = m_1 + n_1 - n_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

Substituting this value in (5),

$$m_1 r_1 - m_1 r_2 - n_1 r_2 + n_2 r_2 = A m_1 u \pi \quad . \quad . \quad . \quad (8).$$

We can eliminate n_1 and n_2 from this equation as follows:

We have assumed during the expansion *CG* steam to be formed, therefore heat amounting to

$$n_1 r_2 \text{ heat units}$$

taken from the existing water and steam in order to form the n_1 kilograms. If the specific heat of the water is c, then the heat abstracted from the water $M - m_1$ is

$$(M - m_1) c\tau \text{ heat units.}$$

But heat is also taken from the existing steam mass $m + m_1$. If we suppose that 1 kilogram of saturated steam at $t°$ must give up h heat units in order to remain saturated at $t - 1$ degrees,* then the $m + m_1$ kilograms of steam lose

$$(m + m_1) h\tau \text{ heat units.}$$

---

* [We see therefore that h plays the part of a specific heat. We may consider it as the "specific heat of saturated steam for constant steam quantity."]

(We shall see hereafter that h is negative, so that during expansion the steam does not lose heat, but gains it, as should be the case.)

We have then

$$n_1r_2 = (M - m_1)c\tau + (m + m_1)h\tau \quad . \; . \; . \quad (9).$$

From F to B the steam is compressed adiabatically. If, now, on the way CG heat is abstracted from the existing mass, on the way BF it is given back. Since, by supposition, n_2 kilograms are condensed, the heat set free is

n_2r_2 heat units.

This is divided among M kilograms of water, and m of steam. The first accordingly receives

$Mc\tau$ heat units,

and the second

$mh\tau$ heat units.

Hence

$$n_2r_2 = Mc\tau + mh\tau \quad . \; . \; . \; . \; . \quad (10).$$

Substituting (9) and (10) in (8)

$$m_1r_1 - m_1r_2 - (M - m_1)c\tau - (m + m_1)h\tau + Mc\tau + mh\tau = Am_1u\pi,$$

or reducing

$$r_1 - r_2 + c\tau - h\tau = Au\pi \quad . \; . \; . \; . \quad (11).$$

Now the total heat of 1 kilogram of steam of the temperature t is $606.5 + 0.305t$ heat units. Or if the specific heat of the water from which the steam is generated is c, and the latent heat r_1,

$$606.5 + 0.305t = ct + r_1 \quad . \; . \; . \; . \quad (12).$$

For 1 kilogram of steam of the temperature $t - \tau$ we have, in like manner,

$$606.5 + 0.305(t - \tau) = r_2 + c(t - \tau) \quad . \; . \quad (13).$$

Subtracting, we have

$$0.305\tau = r_1 - r_2 + c\tau.$$

Substituting in (11)

$$0.305\tau - h\tau = Au\pi \quad . \quad . \quad . \quad . \quad . \quad (14).$$

From Equation (XII.), page 390, we have

$$r\,\frac{\tau}{T} = Au\pi \, . \quad . \quad . \quad . \quad . \quad . \quad (15).$$

and from this and (14) we have

$$h = 0.305 - \frac{r}{T} . \quad . \quad . \quad . \quad . \quad (\text{XVII.})$$

or $$h = 0.305 - \frac{r}{273 + t},$$

or for English units $h = 0.305 - \dfrac{r}{459.4 + t}$.

Since $\dfrac{r}{T} = \dfrac{r}{273 + t}$ is always greater than 0.305, h is *negative.* We see, therefore, that *when saturated steam expands performing work, so that the temperature sinks, we have not to abstract but to add heat in order to keep it saturated. And if saturated steam is compressed, heat must not be added but abstracted in order to keep it saturated.* Otherwise the steam is superheated, and has a higher pressure than saturated steam of the same volume. The heat imparted in the first case, and abstracted in the second, is for 1 kilogram, for a rise or fall of 1 degree,

$$h = 0.305 - \frac{r}{273 + t}.$$

As $r = W - q$, or $r = 606.5 - 0.695t - 0.00002t^2 - 0.0000003t^3$,

$$h = 0.305 - \frac{606.5 - 0.695t - 0.00002t^2 - 0.0000003t^3}{273 + t}.$$

or for English units $h = 0.305 -$

$$\frac{1091.7 - 0.695\,(t - 32) - 0.0000111\,(t - 32)^2 - 0.0000000\,(t - 32)^3}{459.4 + t}.$$

Since, according to Clausius, we have with good exactness

$$r = 607 - 0.708t,$$

or for English units $\quad r = 1092.6 = 0.708\,(t - 32).$

we may also write $h = 0.305 - \frac{607 - 0.708t}{273 + t}$,

or finally, $$h = 1.013 - \frac{800.3}{273 + t} \quad . \quad . \quad . \quad . \quad (\text{XVIII.})$$

or for English units $h = 1.013 - \frac{1440.5112}{459.4 + t}$.

EXAMPLE.

How many heat units must be imparted to 1 kilogram of saturated steam at 100°, when it expands performing work, and the temperature sinks 1° ?

From Table II. we have, since $r' = \rho + Apu$, for 100°, $496.30 + 40.20 = 536.5$ heat units, hence

$$h = 0.305 - \frac{536.5}{273 + 100} = 0.305 - \frac{536.5}{373} = 0.305 - 1.438 = -1.133 \text{ heat units.}$$

For one pound at 212° Fah. we have $h = 0.305 - \frac{965.7}{671.4} = -1.133$ Eng. heat units.

The problem which we have discussed can be solved in a simpler manner.

Suppose the cycle process completed. The work performed is given by the shaded area $BCGF$. Suppose now that the expansion on the path CG had extended until the temperature had sunk 1° instead of τ. Then the work L would be

$$L = \frac{r}{AT}[T - (T - 1)] = \frac{r}{AT}$$

where r is the latent heat from B to C. We know that this work can only be gained when the heat imparted along BC is greater than that abstracted along GF, because the work which is performed by expansion CG, by reason of inner heat of the steam, is equal to that which is expended in the compression FB.

The work performed expressed in heat units is therefore

$$Q = AL = \frac{r}{T}.$$

Hence the volume OC of the saturated steam of t degrees contains $\frac{r}{T}$ heat units more than the volume OF, of the saturated steam of $t - 1$ degrees. Now, 1 kilogram of saturated steam of t° has only 0.305 heat units more than 1 kilogram of $t-1$ degrees. Since $\frac{r}{T} > 0.305$, we have to impart during expansion $\frac{r}{T} - 0.305$ heat units, or to abstract during compression BE, the same amount, in order to keep the steam saturated. This is, therefore, the heat added or abstracted, which we have denoted by h, so that

$$h = \frac{r}{T} - 0.305.$$

Or, if we consider the heat imparted during expansion as negative,

$$h = 0.305 - \frac{r}{T}.$$

Our formula shows that h is variable with the temperature. We see, especially from our equation, page 409, that h is greater, that is, is nearer zero, the greater t, since the quotient $\frac{800.3}{273+t}$ diminishes with increasing temperature.

Heat Imparted or Abstracted for Great Differences of Temperature.—In the following tabulation we have given the heat necessary in order to keep one kilogram or one pound of steam of 10, 20, 30, to 120 degrees Centigrade, saturated and uncondensed during its expansion and cooling of one degree. The same heat is requisite to keep the same quantities of steam saturated at the same temperatures, when the temperature is raised 1 degree, but the heat must be then abstracted.

Temperature	0	10	20	30	40	50	60
Value of h	-1.917	-1.814	-1.718	-1.628	-1.544	-1.465	-1.391

Temperature	70	80	90	100	110	120
Value of h	-1.321	-1.255	-1.192	-1.133	-1.077	-1.024

If, now, it is required to determine, for example, what heat must be imparted, in order that 1 kilogram of saturated steam at 100° may expand gradually to 1 kilogram of saturated steam at 0°, we must determine the mean of h between 0° and 100°, and multiply by the number of terms by which the mean was determined. We may find the mean best by Simpson's rule.

If we have a number of quantities occurring at equal intervals, and denote them by P_0, P_1, $P_2 \ldots P_{n-1}$, so that n is the number of intervals, the mean is

$$P = (\tfrac{1}{2}P_0 + P_1 + P_2 + P_3 + \ldots P_{n-1} + \tfrac{1}{2}P_n) \div n.$$

If the number of intervals n is even, *viz.*, 2, 4, 6, 8, etc., the rule gives for the mean

$$P = (P_0 + 4P_1 + 2P_2 + 4P_3 + \ldots 4P_{n-1} + P_n) \div 3n.$$

If we wish, then, to find the mean of h between 0° and 100°, according to the first formula, we must put for P_0 1.917, for P_1 1.814, for P_2 1.718, finally, for P_n 1.133. Then $\tfrac{1}{2}P_0 = 0.958$, and $\tfrac{1}{2}P_n = 0.567$, hence

$P = (0.958 + 1.814 + 1.718 + 1.628 + 1.544 + 1.465 + 1.391 + 1.321 + 1.255 + 1.192 + 0.567) \div 10 = 14.853 \div 10 = 1.485$; or since h is generally negative, $h = -1.485$.

If, then, 1 kilogram of saturated steam expands, performing work, from 100° to 0°, and still remains saturated and uncondensed, we must impart on the average, for each degree of cooling, 1.485 heat units. The entire heat imparted is then

$$Q = 1.485 \times 100 = 148.5 \text{ heat units.}$$

For one pound from 212° to 32° Fah. we have

$$Q = 1.485 \times 180 = 267.3 \text{ English heat units.}$$

In the same way we may find for 1 kilogram of steam whose temperature sinks during expansion from 80° to 0°,

$$Q = 1.558 \times 80 = 124.64 \text{ heat units.}$$

For one pound from 176° to 32° Fah. we have

$$Q = 1.558 \times 144 = 224.352 \text{ English heat units.}$$

Zeuner has given a table, which gives the amount of heat which must be imparted when 1 kilogram of saturated steam of 1, 2, 3, etc., atmospheres cools by expansion to 0° and remains all steam. In Table III. we have given these values of Q, as well as the corresponding values of $\frac{r}{T}$.

EXAMPLE.

How many units of heat must be imparted to 1 kilogram of saturated steam of 5 atmospheres, when it expands in the cylinder of an engine, performing work, down to 1 atmosphere, and yet still remains saturated and uncondensed?

From Table III., for a pressure of 5 atmospheres.. $Q = 200.46$
And for 1 atmosphere............................ $Q_1 = 148.47$

The heat imparted from 5 to 1 atmos. is then..... 51.99 heat units.
For one pound under the same circumstances we have 93.576 English heat units.

This amount of heat is too great to be supplied by the hot cylinder sides, as has been assumed by the followers of Pambour. If therefore no heat is imparted from without, so much steam must be condensed as will furnish the necessary heat. This amount of steam can indeed be relatively very small, since the latent heat of steam is great with respect to the heat required.

Deportment of other Vapors during Expansion.—The formula for h was

$$h = 0.305 - \frac{r}{T}.$$

In this 0.305 is the amount of heat which one kilogram of steam of $t + 1$ degrees possesses *more* than 1 kilogram of t degrees, because 1 kilogram of $t + 1$ degrees has

$$606.5 + 0.305\,(t + 1) \text{ heat units,}$$

and one of t degrees has

$$606.5 + 0.305t \text{ heat units,}$$

and the difference is 0.305 heat units.

The total heat of 1 kilogram of ether steam of $t + 1$ degrees is, as we have given it,

$$94 + 0.45\,(t + 1) - 0.00055556\,(t + 1)^2,$$

or

$$94.449444 + 0.4489t - 0.000556t^2.$$

For t° it is $\quad 94 + 0.45t - 0.000556t^2.$

The difference is $\quad 0.449444 - 0.0011t.$

Hence the expression for h for ether steam is

$$h = 0.449444 - 0.0011t - \frac{r}{T^r}$$

Since, however (page 392), $r = 94 - 0.079t - 0.00085t^2$, we have

$$h = 0.449444 - 0.0011t - \frac{94 - 0.079t - 0.00085t^2}{273 + t},$$

$$hT = (273 + t)\,0.449444 - 0.0011t\,(273 + t) - 94 + 0.079t + 0.00085t^2,$$

or
$$hT = 28.6982 + 0.225t - 0.00026t^2.$$

For English units

$$hT = 51.7783 + 0.2254\,(t-32) - 0.0001443\,(t-32)^2.$$

If we find the formula for water steam in a similar manner, we have

$$hT = -523.23 + t + 0.00002\,t^2 + 0.0000003t^3,$$

or for English units

$$hT = -941.843 + (t-32) + 0.0000111(t-32)^2 + 0.0000000926(t-32)^3.$$

We see from this formula, that even for very high temperatures h is still negative, as we have already concluded from the form of other formulæ. We see from the formula for ether steam that h is *positive* even when the temperature is very great. This kind of steam therefore, must have heat extracted from it during expansion, if no part of it is condensed. This peculiar deportment of ether steam was first pointed out by Hirn. All other vapors which we have named, in fact, all for which Regnault has determined the sensible heat and latent heat of vaporization, comport themselves like water steam, and for them h is therefore *negative*.

QUESTIONS FOR EXAMINATION.

What is the curve of constant steam weight? When saturated steam expands, performing work, does it remain saturated? Is the steam weight constant? If not, can you prove that it is not? If saturated steam is compressed, and heat at the same time abstracted so that the temperature is kept constant, what takes place? If no heat is abstracted? If it expands performing work, and heat is not added? How many heat units must be imparted to 1 kilogram of saturated steam at 100° to keep it saturated and uncondensed, when it expands performing work, till the temperature is 90°? Suppose no heat is added, how much steam will be condensed?

H_t = total heat = W

H_h = Heat of the liquid = q

H_i = Internal work heat = ρ

H_E = External " " = APu

H_l = Latent heat of evaporation = r

H = Steam heat = J

[illegible] = Admission + expansion work of 1 pound steam

CHAPTER XVIII.

HEAT CURVES OF STEAM AND LIQUID MIXTURES.—CONSTRUCTION OF THE SAME.—TECHNICAL APPLICATIONS.

A. ISOTHERMAL CURVE.

Form of the Curve.—The isothermal curve has been defined as that which gives the change of condition of a body when the temperature is kept constant. For gases this was a curved line which made apparent the law of Mariotte. Now we know that if for saturated steam the pressure is constant, the temperature is constant also. If, then, $AB = p$ (Fig. 69) is the pressure of the steam in a mixture of steam and water, this pressure remains constant so long as the temperature is the same. Heat added to the water simply vaporizes some of it, the volume increases, and the isothermal for the mixture is *a straight line* parallel to OX.* Since for a higher or lower temperature the pressure p is greater or less, the line BC will be at a greater or less distance from OX.

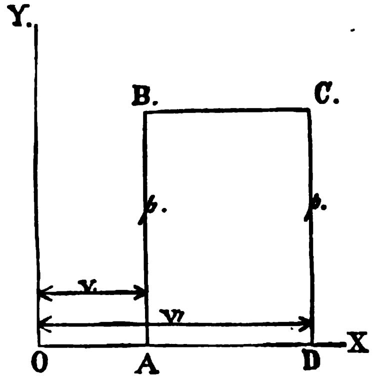

Fig. 69.

Outer and Inner Work during Expansion.—Let $OA = v$ be the initial specific volume of the mixture (volume of 1 kilogram), then, as already proved, page 401,

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* [Here therefore the isothermal and isopiestic lines coincide.]

$$v = xu + \sigma,$$

where u can be taken from Table II. for given pressure and temperature. If now we heat the mixture till the volume is v_1, we have

$$v_1 = x_1 u + \sigma.$$

Since the ordinate $AB = p$ describes the rectangle $ABCD$, the outer work performed is

$$L = p(v_1 - v) = pu(x_1 - x);$$

hence

$$x_1 - x = \frac{v_1 - v}{u}.$$

In practice $v_1 - v$ is generally given, u can be taken from Table II., and thus the weight of water vaporized $(x_1 - x)$ can be found.

Now what is the entire amount of heat imparted? All this heat, as we know, goes to vaporize the water. Of this the outer latent heat is

$$AL = Ap(v_1 - v) = Apu(x_1 - x),$$

while the inner latent heat is

$$\rho(x_1 - x) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (XX.)$$

The total amount of heat is then

$$Q = r(x_1 - x) = r\frac{v_1 - v}{u} \quad . \quad . \quad . \quad (XXI.)$$

or

$$Q = (Apu + \rho)(x_1 - x)$$

$$= (Apu + \rho)\frac{v_1 - v}{u} . \quad . \quad . \quad . \quad (XXII.)$$

The value of $Apu + \rho$, as also u, can be found from Table II.

Example 1.—The cylinder of a non-condensing engine, working with full pressure, has a cross-section of 0.174 square meter (or 1.873 sq. feet), and a stroke of 1.048 meters (or 3.44 feet). The steam pressure p is 3½ atmospheres, and the number of revolutions per minute is 24. What is the theoretical work per second, and how much heat is required?

The steam quantity per stroke is 0.174 × 1.048 = 0.182 cubic meter. Hence

$$v_1 - v = 0.182 \text{ cubic meters or } 6.443 \text{ cubic feet,}$$

and the work per stroke is

$$L = p(v_1 - v) = 10334 \times 2\tfrac{1}{2} \times 0.182 = 4702 \text{ m. kil., or } 34085.7 \text{ foot lbs.}$$

The work per second is

$$\frac{2 \times 24}{60} \times 4702 = 3761 \text{ meter-kilograms, or } 27268.58 \text{ ft. lbs.}$$

or

$$\frac{3761}{75} = 50 \text{ horse power, or } 49.57 \text{ horse power English.}$$

For the heat required per stroke,

$$Q = (Apu + \rho)\frac{v_1 - v}{u}.$$

Since $Apu + \rho$ for 3½ atmospheres is, from Table II., = 465.26 + 43.27 = 508.53, and $u = 0.507$, we have

$$Q = 508.53 \frac{0.182}{0.507} = 182.56 \text{ heat units.}$$

$$\text{In English units,} \quad Q = 915.354 \frac{6.443}{8.1248} = 725.87 \text{ heat units.}$$

Hence the heat per second is

$$182.56 \times \tfrac{24}{30} = 146.05 \text{ heat units.}$$

If all this heat had been converted into outer work, we should have had 424 × 146.05 = 61925 meter-kilograms, while in reality we have only 3761, or hardly the 16th part. Now, perhaps, only half the heat of the fuel acts to vaporize the water, so that we utilize only the 32d part of the heat of the fuel. Finally, even this is but the total work of the engine, and from it we must subtract the prejudicial resistances, in order to find the useful work. Since these resistances take about 50 per cent. from the total work, we have only $\frac{1}{64}$th of the heat of the fuel actually utilized.

Example 2.—A condensing engine sends 0.182 cubic meter (or 6.443 cubic feet) of steam, at a pressure of $\frac{1}{10}$th of an atmosphere, into the condenser, where the pressure is constant. What work is necessary, and how much heat is taken from the steam? The work required is

$$10334 \times \tfrac{1}{10} \times 0.182 = 188.08 \text{ meter-kilograms},$$

or

$$2116.14 \times \tfrac{1}{10} \times 6.443 = 1363.429 \text{ foot-pounds.}$$

From Table II., $Apu + \rho$ for $\frac{1}{10}$ atmosphere is $538.85 + 35.46 = 574.31$, and $u = 14.55$. Hence

$$Q = 574.31 \frac{0.182}{14.55} = 7.179 \text{ heat units},$$

or

$$Q = 1033.76 \frac{6.443}{233.088} = 28.57 \text{ heat units English.}$$

B. Isodynamic Curve.

The isodynamic curve gives the law of change of p and v, when the inner work is constant.

Equation and Construction of the Curve.—Suppose, as before, one kilogram of mixture to consist of x kilograms of steam and $1 - x$ of water. The sensible heat of the mixture is q, and hence the steam heat is

$$q + x\rho,$$

and the inner work is

$$\frac{1}{A}(q + x\rho).$$

If now we have, after adding heat, x_1 kilograms of steam and $1 - x_1$ of water, and the sensible heat q_1, and the inner latent heat ρ_1, we have for the "steam heat" in the new state

$$q_1 + x_1\rho_1,$$

and the inner work

$$\frac{1}{A}(q_1 + x_1\rho_1).$$

For the isodynamic curve then

$$\frac{1}{A}(q + x\rho) = \frac{1}{A}(q_1 + x_1\rho_1),$$

or

$$q + x\rho = q_1 + x_1\rho_1 \quad . \quad . \quad . \quad . \quad \text{(XXIII.)}$$

This is the equation of the isodynamic curve. In order to construct the curve we must know the abscissa and ordinate for different points. If we assume p and x known for the initial condition, then from p we know t, u, q, and ρ. The corresponding volume is given by

$$v = xu + \sigma \quad \text{or} \quad v = xu.$$

If now, p_1 is given for a second position, we know at once q_1 and ρ_1, and since we know $q + x\rho$ for the initial condition, we have

$$x_1 = \frac{q + x\rho - q_1}{\rho},$$

and then from

$$v_1 = x_1 u_1 + \sigma_1$$

can find $v_1 =$ for the second condition.

Thus let $p = 5$ atmospheres and $x = 0.80$ kilograms, then from Table II.,

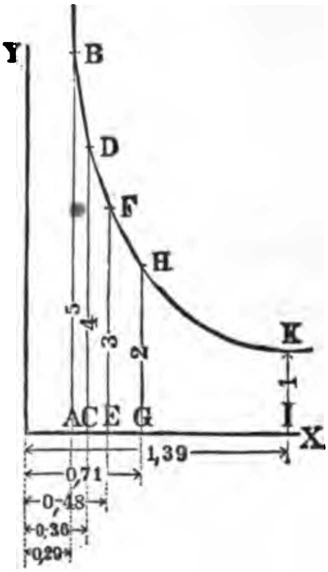

Fig. 70.

$$q = 153.74, \quad \rho = 454.99, \quad \text{and}$$
$$u = 0.363.$$

Hence

$$v = xu + \sigma = 0.8 \times 0.363 + 0.001$$
$$= 0.291 \text{ cubic meter.}$$

Lay off now $OA = v = 0.29$, Fig. 70, and $AB = 5$. Then B is a point of the isodynamic curve. We can now calculate v_1 for $p_1 = 4$ atmospheres. For this, $q_1 = 145.31$, $\rho_1 = 461.5$, and $u_1 = 0.447$, hence

$$x_1 = \frac{q + x\rho - q_1}{\rho_1}$$
$$= \frac{153.74 + 0.8 \times 454.99 - 145.31}{461.5}$$
$$= 0.807 \text{ kilogram.}$$

Therefore 0.007 kilogram of water are vaporized. For v_1 we have

$$v_1 = x_1 u_1 + \sigma = 0.807 \times 0.447 + 0.001 = 0.362 \text{ cubic meter.}$$

Lay off $OC = 0.362$ and $CD = 4$, and D is a second point on the curve. In the same way we can determine the volumes for pressures of 3 atmospheres, 2, and 1 atmosphere, as shown in Fig. 70.

The curve joining all the points thus found is the isodynamic curve for a mixture of steam and water.

We see here also, that as in the curve of constant steam weight, the volumes increase as the pressures decrease.

The curve can be represented then by an equation of the form

$$pv^n = p_1v_1^n = p_2v_2^n, \text{ etc.}$$

Zeuner found that when x is originally $= 1$ kilogram, and then the steam compressed according to the isodynamic curve, $n = 1.0456$. For the curve of constant steam weight, $n = 1.0646$. The curve of constant steam weight approaches the axis of X more rapidly therefore than the isodynamic curve, and lies therefore between the latter and the isothermal.

From the preceding we see, that *during expansion of steam along the isodynamic curve, water is vaporized, and during compression, is condensed.* Thus, as we have seen, for a pressure of 4 atmospheres, $x_1 = 0.807$ kilogram, while for 5 atmospheres, x was 0.8, and for 3 atmospheres, 0.815, etc.

Outer Work.—Heat Required.—In order to determine the work performed during expansion, we determine the area of $ACDB$, then of $CDFE$, etc., considering them as trapezoids. Thus for example, for the outer work during expansion from $p = 5$ to $p_1 = 4$ atmospheres, we have

$$L = \frac{p + p_1}{2}(v_1 - v) = 10334 \times 4.5\ (0.362 - 0.291) = 3302 \text{ met.-kil.}$$

Since further, the inner work is constant, all the heat imparted goes to outer work. This heat is then

$$Q = AL = \tfrac{1}{424} \times 3302 = 7.79 \text{ heat units.}$$

This curve is of little value in practice, hence we will not discuss it further.

C. ADIABATIC CURVE.

This curve gives the law of variation of p and v, when no heat is either imparted or abstracted during the change of condition.

Equation and Construction of the Curve.—To construct this curve we must find from a given pressure or temperature the corresponding volume. In this connection we refer to what has been said in the *Appendix* to this chapter, and advise that it be read before the following:

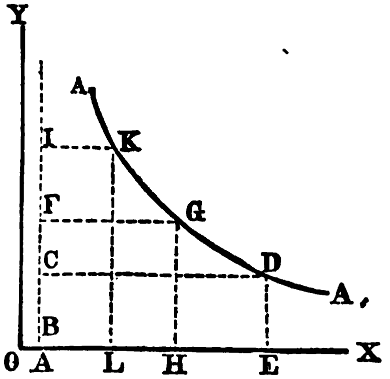

FIG. 71.

Let OA, Fig. 71, be the volume of 1 kilogram of water at 0°. If this water is not partly vaporized when heat is imparted to it, it must be loaded with a certain weight or subjected to a certain pressure. Call this pressure AB. Suppose now the temperature of this water is raised gradually to 1, 2, 3, etc., degrees. To prevent vaporization the pressure AB must be correspondingly increased. When the temperature of the water is 100°, the pressure is 10334 kilograms. The imparting of heat and increase of pressure is thus conducted in the same manner as for permanent gases in the Appendix. We have then here a certain "heat weight." If c is the mean specific heat of water between 0 and t degrees, the heat weight imparted for this rise of temperature is approximately, taking c as constant,

$$\frac{c}{A}\log\text{ nat}\frac{273+t}{273}=\frac{c}{A}\log\text{ nat}\frac{T}{273}.$$

If we denote this by $\frac{\tau}{A}$, we have

$$\tau = c\log\text{ nat}\frac{T}{273} = 2.3026\,c\log\frac{T}{273} \quad . \quad \text{(XXIV.)}$$

The exact value of τ, for French units and temperature Centigrade, is $\tau = 2.4318892\log\frac{T}{273} - 0.0002057\,t + 0.00000045\,t^2$.

For English units and temperature Fahrenheit,

$$\tau = 2.414100458\log\frac{T}{459.4} - 0.0001054(t-32)$$
$$+\ 0.0000001889\,(t-32)^2.$$

The value of τ is given in Table III., for different pressures.

Let us assume that the water is heated under these conditions up to 100°, and that the pressure is $AC = p$. Now let heat be still further imparted, while the pressure remains unchanged. Vaporization then takes place under constant pressure and temperature. Suppose we thus allow a certain weight of water, x, to be vaporized. The volume is increased, and Ac is carried to DE. Through this point D let an adiabatic curve be constructed. The heat weight necessary for vaporization, which must be imparted to the x kilograms of water is $\frac{xr}{AT}$. Hence the total heat weight imparted both to water and steam is

$$\frac{\tau}{A} + \frac{xr}{AT} = \frac{c}{A} \log \text{nat} \frac{T}{273} + \frac{xr}{AT}.$$

In other words, this equation gives the heat weight necessary to raise 1 kilogram of water from 0° and the corresponding pressure, into water of t degrees and the corresponding pressure, and *then* to convert x kilograms of this water into steam. As soon as x is known, we can find the corresponding volume,

$$v = xu + \sigma,$$

and can then lay off OE and ED.

Suppose again, we raise the water from 0° to t_1°, for which the pressure is p_1. The heat weight is then, under the assumption that between 0° and t_1° the mean specific heat of water is the same,

$$\frac{\tau_1}{A} = \frac{c}{A} \log \text{nat} \frac{273 + t_1}{273} = \frac{c}{A} \log \text{nat} \frac{T_1}{273}.$$

Now let a certain weight x_1 of this water be vaporized under constant pressure p_1, so that AF passes to GH, and the point G is in the adiabatic curve. The heat weight imparted to the steam is $\frac{x_1 r_1}{AT_1}$.

We have, therefore, for the entire heat weight imparted,

$$\frac{\tau_1}{A} + \frac{x_1 r_1}{AT_1} = \frac{c}{A} \log \text{nat} \frac{T_1}{273} + \frac{x_1 r_1}{AT_1}.$$

Since G is a point in the adiabatic curve, this heat weight must be equal to the first.
Hence

$$\frac{\tau}{A}+\frac{xr}{AT}=\frac{\tau_1}{A}+\frac{x_1r_1}{AT_1},$$

or

$$\tau+\frac{xr}{T}=\tau_1+\frac{x_1r_1}{T_1}. \quad . \quad . \quad . \quad . \quad \text{(XXV.)}$$

The values of τ, τ_1, and $\frac{r}{T}$, $\frac{r_1}{T_1}$, are given in Table III., so that x_1 can be easily found when x is known. We have

$$x_1=\left(\tau-\tau_1+\frac{xr}{T}\right)\frac{T_1}{r_1}.$$

If x_1 is found, the volume v_1 is given by

$$v_1=x_1u_1+\sigma,$$

where u_1 is given by p_1 and t_1. If therefore only the point D is given, we can construct the point G on the adiabatic through D.

In similar manner, if we raise the kilogram of water from 0° to t_2°, for which the pressure is $AI=p_2$, the heat weight added to the water is

$$\frac{\tau_2}{A}=\frac{c}{A}\log\text{nat}\,\frac{T_2}{273}.$$

If then we evaporate x_2 kilograms under constant pressure, so that AI passes to K, the heat weight is $\frac{x_2r_2}{AT_2}$, and we have for the total heat weight

$$\frac{\tau_2}{A}+\frac{x_2r_2}{AT_2}.$$

If K is on the adiabatic,

$$\tau+\frac{xr}{T}=\tau_2+\frac{x_2r_2}{T_2},$$

so that we can find x_2 as also the corresponding volume $v_2 = oL$. We can thus construct the point K. Generally, we see that by the principles given in the Appendix, we can easily find different points on an adiabatic. For the sake of illustration, let us take a special example.

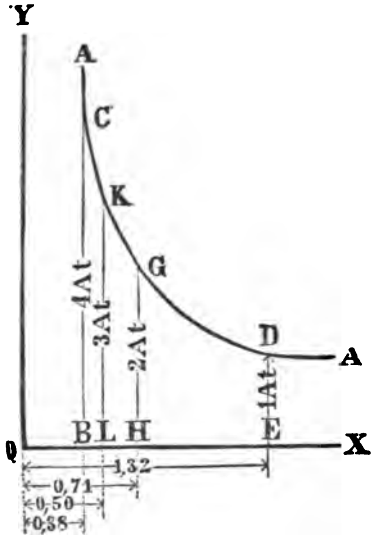

Fig. 72.

Let us assume that we have to start with $x = 0.80$ kilograms of steam, and hence $1 - x = 0.20$ of water. The pressure p is 1 atmosphere. Then we have for OE

$$OE = v = xu + \sigma.$$

Since for $p = 1$, $u = 1.65$,

$$v = 0.80 \times 1.65 + 0.001,$$

or, disregarding σ,

$$v = 0.8 \times 1.65 = 1.320 \text{ cub. m.}$$

Lay off then, Fig. 72, $OE = 1.32$ and $ED = 1$ atmos., and D is a point in the adiabatic. We may construct a second point for $p = 2ED = 2$ atmos.

For this we have

$$\tau + \frac{xr}{T} = \tau_1 + \frac{x_1 r_1}{T_1}.$$

In Table III, we have the values of τ, r, and $\frac{r}{T}$ for different pressures.

For $p = 1$

$$\tau = 0.31, \quad \frac{r}{T} = 1.44,$$

hence

$$\tau + \frac{xr}{T} = 0.31 + 0.8 \times 1.44 = 1.46,$$

and

$$1.46 = \tau_1 + x_1 \frac{r_1}{T_1}.$$

For $p_1 = 2$ atmos. $\tau_1 = 0.37$, $\frac{r_1}{T_1} = 1.33$, hence

$$1.46 = 0.37 + x_1 \times 1.33,$$

and

$$x_1 = \frac{1.46 - 0.37}{1.33} = 0.82.$$

By the rise of temperature, 0.02 kilograms of water are thus vaporized. Now $v_1 = x_1 u_1$, and since for $p_1 = 2$ atmospheres, $u_1 = 0.86$, we have

$$v_1 = 0.82 \times 0.86 = 0.71 \text{ cubic meters.}$$

Make, then, $OH = 0.71$, and $HG = 2$, and we have the point G of the curve.

Let $p_2 = 3$ atmospheres. Then

$$\tau + \frac{xr}{T} = \tau_2 + \frac{x_2 r_2}{T_2},$$

or

$$1.46 = \tau_2 + x_2 \frac{r_2}{T_2}.$$

According to Table III., for $p_2 = 3$ atmospheres,

$$\tau_2 = 0.40, \quad \frac{r_2}{T_2} = 1.26$$

hence

$$1.46 = 0.40 + x_2 \times 1.26,$$

and

$$x_2 = \frac{1.46 - 0.40}{1.26} = 0.84 \text{ kilograms.}$$

Hence, since $u_2 = 0.59$,

$$v_2 = x_2 u_2 = 0.84 \times 0.59 = 0.50 \text{ cubic meters.}$$

Lay off then, $OL = 0.5$, and $LK = 3$, and K is a third point on the curve.

For $p_3 = 4$ atmospheres, we find in similar manner, $v_3 = 0.38$ cubic meters. In our Fig. 72, $OB = 0.38$, and $BC = 4$, and thus we have a fourth point C. The curve joining these points is the adiabatic.

We see from the preceding, that when a mixture of steam and water is compressed adiabatically, water is vaporized, and at the end there is more steam and less water than at first. If there were at first saturated steam only, without water, by compression the steam would be superheated, and the adiabatic curve for this superheated steam would be different. If the saturated steam expands performing work, we have inversely, condensation of steam, as has already been proved elsewhere. The deportment of saturated steam by adiabatic expansion or compression is thus the reverse of that for the isodynamic curve.

Since a knowledge of the law of the adiabatic curve is of the greatest importance for a reliable and thoroughly scientific theory of the steam engine, we shall proceed to show by an example, how condensation takes place during expansion, and shall then investigate what takes place when we have at first only water of a certain temperature, and then diminish the pressure according to the adiabatic curve.

Suppose in a cylinder, 1 kilogram of pure* saturated steam, without admixture of water, of 4 atmospheres pressure, and therefore at a temperature of 144°. Then here $x = 1$, and from Table III.,

$$\tau = 0.427, \quad \frac{r}{T} = 1.211,$$

and from Table II., $u = 0.447$, hence

$$v = xu = 1 \times 0.447 = 0.447.$$

If now, Fig. 73, $OB = 0.447$ and $BC = 4$, we have the point C as the first point of the curve. We have now

$$\tau + x\frac{r}{T} = 0.427 + 1.211 = 1.638.$$

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* "Pure" *i. e.*, *dry*—no water particles being mechanically suspended in the steam. When this is the case the steam is said to be "*wet.*"

Now let the steam expand adiabatically, until the pressure is $p_1 = DE = 2$ atmospheres. For this case

$$\tau_1 = 0.368, \quad \frac{r}{T} = 1.326, \text{ hence}$$

$$1.638 = 0.368 + x_1 \times 1.326, \text{ or}$$

$$x_1 = 0.958 \text{ kilograms.}$$

Hence $1 - 0.958 = 0.042$ kilograms of steam have condensed. The volume $v_1 = x_1 u_1 = 0.958 \times 0.859 = 0.823$. If then, $OD = 0.823$ and $DE = 2$, E is a second point in the curve.

Let the steam still expand, till its pressure is 1 atmosphere $= FG$.

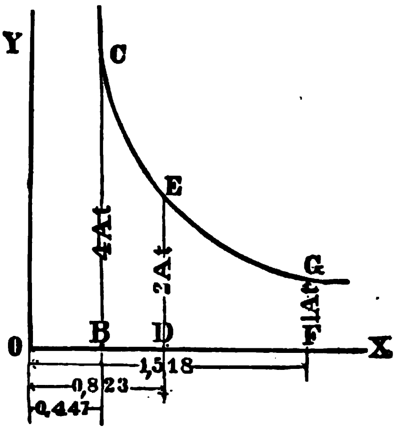

FIG. 73.

Then $\tau_2 = 0.314, \frac{r_2}{T_2} = 1.438, \quad u_2 = 1.649$, and

$$1.638 = 0.314 + x_2 \times 1.438 \quad \text{or} \quad x_2 = 0.920 \text{ kilogram.}$$

Hence by expansion from 4 atmospheres to 1, $1 - 0.920 = 0.08$ kilogram of steam have been condensed.

The specific volume is

$$v_2 = x_2 u_2 = 0.92 \times 1.649 = 1.518 \text{ cubic meters.}$$

If we make $OF = 1.518$ and $FG = 1$, we have a third point in the curve.

We see then, very plainly, that during expansion the steam condenses. If, inversely, we had to start with only 0.920 (x_2) kilogram of steam, and $1 - 0.920 = 0.080$ of water, under a pressure of 1 atmosphere, and compressed the mixture adiabatically to 4 atmospheres, we would have at the end of the process, 1 kilogram of steam, saturated, and the 0.08 kilogram of water, will be completely vaporized.

The expansion ratio in the first case is

$$\frac{v_2}{v} = \frac{1.518}{0.447} = 3.390.$$

The expansion ratio is therefore less than according to the old views as to the properties of steam.

Let us now suppose we have only water to start with, of a given temperature, the pressure being therefore such that there is no vaporization. Now let the pressure diminish gradually, and no heat be imparted or abstracted, and let us see what are the changes.

Suppose the temperature of the water is 144°, and hence the pressure $p = 4$ atmospheres. We have then

$$\tau = 0.427, \quad \frac{r}{T} = 1.211, \quad \text{and} \quad x = 0,$$

hence

$$\tau + x\frac{r}{T} = 0.427.$$

If now the pressure sinks gradually to 2 atmospheres, we have

$$\tau_1 = 0.368, \quad \frac{r_1}{T_1} = 1.326 \quad \text{and} \quad \tau_1 + x_1\frac{r_1}{T_1} = 0.368 + x_1 \times 1.326,$$

hence

$$0.368 + x_1 \times 1.326 = 0.427, \quad \text{and} \quad x_1 = 0.044 \text{ kilogram.}$$

This weight of steam has been formed. If in this, as well as in the previous case, we had used more decimal places and calculated more accurately, we would have found that the same quantity of steam was formed, as in the case of pure saturated steam only was condensed.

We have further

$$v_1 = x_1u_1 = 0.044 \times 0.859 = 0.039 \text{ cubic meter.}$$

If the pressure still falls to 1 atmosphere, we have

$$\tau_2 = 0.314, \quad \frac{r_2}{T_2} = 1.438,$$

$$0.427 = 0.314 + 1.438x_2, \quad \text{or} \quad x_2 = 0.0786 \text{ kilogram.}$$

This steam has been formed, and there is left 1 – 0.0786 = 0.9214 kilogram of water. Here also we should have just the same steam weight produced, as in the first case was condensed.

We see then, that when there is more steam than water, there is partial condensation during expansion. But when there is only water in the beginning, steam is *formed* during expansion. Hence it follows, that there is a certain proportion of steam and water for which, during expansion there is neither condensation nor vaporization, or at least, for which during the first period of the expansion, there is just as much steam generated as during the second is condensed. This mixture can be determined. Since at the beginning and end of the expansion, we have the same amount of steam or water,

$$\tau + x\frac{r}{T} = \tau_1 + x\frac{r_1}{T_1},$$

or

$$x = \frac{\tau - \tau_1}{\dfrac{r}{T} - \dfrac{r_1}{T_1}} \quad . \quad . \quad . \quad . \quad \text{(XXVI.)}$$

If we assume the initial pressure at 4 atmospheres and the end pressure at 1 atmosphere, we have for x almost exactly 0.5 kilogram. We must, therefore, have to start with as much water as steam, by weight, if by expansion between 4 and 1 atmospheres there is to be at the end the same steam and water quantity as at the beginning. For from 10 to 5, and 5 to 1, and 1 to ½ atmospheres, we have respectively

$$x = 0.56, \qquad x = 0.50, \qquad x = 0.46 \text{ kilogram.}$$

The mixture ratio does not vary, therefore, much from 1 to 1.

If we suppose for the extreme pressures 4 and 1 atmospheres, and the mixture ratio 1 to 1, that is, as much steam as water, by weight, the adiabatic curve and also that for constant steam weight constructed, both curves must then cross at the beginning and end of expansion, Fig. 74. Since, further, the steam formed during the first half of expansion is small, both curves vary but little from each other. The adiabatic curve, however, approaches the axis somewhat more rapidly than the curve of constant steam weight. In Fig. 74, the dotted line is the adiabatic curve for the mixture ratio of 1 to 1 and for the limiting pressures of 4 and 1 atmospheres. We see from the preceding how complicated is the phenomenon of expansion or compression adiabatically of saturated steam. We thus arrive at the following general results.

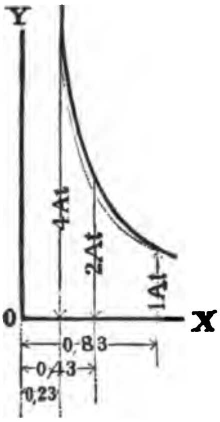

Fig. 74.

(*a.*) *For* EXPANSION, *adiabatic:*

1. *If we start with pure saturated steam, without admixture of water, steam condenses during expansion.*

2. *If there is more steam than water, there is also condensation.*

3. *If there is more water than steam to start with, steam is generated during expansion.*

(*b.*) *For* COMPRESSION, *adiabatic:*

1. *If we start with pure saturated steam, without admixture of water, it will be superheated by compression.*

2. *If the initial steam weight is greater than that of the water, steam is generated by the compression.*

3. *If there is more water than steam, steam is condensed during compression.*

Calculation of the Outer Work.—Since during the expansion or compression according to the adiabatic curve, heat is neither imparted nor abstracted, the outer work performed during expansion must be at the expense of inner work. If now U is the inner work contained by a mixture of steam and water before expansion, and U_1 that after, the outer work is

$$L = U - U_1.$$

Hence the heat disappearing is

$$Q = AL = A(U - U_1).$$

The inner heat, which is equivalent to the inner work, is easily calculated. If we have x kilograms of steam and $1 - x$ of water, the

$$\begin{array}{lll} x \text{ kilograms of steam contain} \ldots\ldots & x(q+\rho) & \text{heat units.} \\ 1-x \quad \text{"} \quad \text{" water} \quad \text{"} \quad \ldots\ldots & \underline{(1-x)q} & \text{"} \\ \text{The mixture contains} \ldots\ldots\ldots & x(q+\rho)+(1-x)q & \text{"} \end{array}$$

or, reducing,

$$q + x\rho \text{ heat units.}$$

For x kilograms of steam and $1 - x$ of water, we have

$$q_1 + x_1\rho_1 \text{ heat units.}$$

Hence the heat disappearing during expansion is

$$Q = AL = A(U - U_1) = q + x\rho - (q_1 + x_1\rho_1). \quad \text{(XXVII.)}$$

where x and x_1 are the steam weights at beginning and end of expansion.

EXAMPLE 1.—What work is performed by 1 kilogram of saturated steam at 4 atmospheres, when it expands adiabatically to 1 atmosphere?

We have in this case $x = 1$, and can find, as on page 439, $x_1 = 0.920$. Further, from Table II., we have

$$q = 145.31, \quad \text{and} \quad \rho = 461.5$$

for 4 atmospheres, and for 1 atmosphere,

$$q_1 = 100.5, \quad \text{and} \quad \rho_1 = 496.8.$$

Inserting these values, we have

$$Q = AL = 145.31 - 100.5 + 461.5 - 496.8 \times 0.920 = 49.7 \text{ heat units, and hence}$$

$$L = 424 \times 49.7 = 21072.8 \text{ meter-kilograms.}$$

For 1 pound we have $q = 261.558$, $\rho = 830.6928$, $q_1 = 180.9$, $\rho_1 = 893.84$, $x_1 = 0.920$. $Q = 89.47$ heat units, $L = 69070.84$ foot-pounds.

EXAMPLE 2.—What would the work be, if to start with, we had only water and no steam?

In this case we have $x = 0$, and find, as on page 442, $x_1 = 0.079$, while q, q_1, ρ, ρ_1, are as before. Hence

$$AL = 145.31 - 100.5 + 0.461.5 - 496.8 \times 0.079 = 44.81 - 39.2 = 5.61 \text{ heat units.}$$

$$L = 424 \times 5.61 = 2378.6 \text{ meter-kilograms.}$$

For 1 pound we have $x_1 = 0.079$.

$$AL = 261.558 - 180.9 - 893.84 \times 0.079 = 10.084.$$

$$L = 772 \times 10.084 = 7784.85 \text{ ft.-lbs.}$$

The work is, therefore, very much less than before, which is easily comprehended from the fact that during expansion *steam is formed*, which consumes a large part of the heat of the liquid, so that the temperature and pressure decrease more rapidly than before.

Approximate Formula for the Adiabatic Curve.—For the isothermal curve for steam and water mixture, we can easily find the volume for any pressure and steam weight. For the isodynamic curve, also, we have given an expression which gives with sufficient exactness the relation between pressure and volume. Since the adiabatic curve is of especial practical importance, it is desirable to find for it, also, such an expression, which shall furnish us with a simpler and less tedious method of calculation.

Rankine found that the law of the adiabatic curve was given by an equation of the form

$$pv^m = p_1v_1^m = p_2v_2^m, \text{ etc.},$$

where $m = 1.11$. But it is not stated by Rankine whether this value of m was found for *every* mixture of steam and water, or only for a certain definite proportion. From the calculations given by Rankine, the first appears very improbable, and Grashof has pointed out that this value is too small for pure saturated steam without admixture of water. He shows that in this case, m should be 1.140. The preceding calculations enable us to determine at once if this value is correct, and at the same time show that the value of m given by Rankine answers only to a certain definite mixture, and not to all others.

From the equation

$$pv^m = p_1v_1^m. \quad . \quad . \quad . \quad . \quad \text{(XXVIII.)}$$

we have

$$\frac{p}{p_1} = \left(\frac{v_1}{v}\right)^m,$$

or

$$m = \frac{\log \frac{p}{p_1}}{\log \frac{v_1}{v}}. \quad . \quad . \quad . \quad . \quad \text{(XXIX.)}$$

Let us take the example on page 439.

Here $p = 4$, $p_1 = 1$, $v = 0.447$, and $v_1 = 1.520$.

Accordingly,

$$m = \frac{\log 4}{\log \dfrac{1.520}{0.447}} = \frac{\log 4}{\log 3.39} = \frac{0.602}{0.530} = 1.136,$$

a value which agrees perfectly with that of Grashof if we take only 2 decimal places.

If, however, we suppose only water at the beginning of expansion, and find m for the case of the example on page 442.

Here $p = 4$, $p_1 = 1$, $v = 0.001$, and $v_1 = 0.1312$, hence

$$m = \frac{\log 4}{\log \dfrac{0.1312}{0.001}} = \frac{\log 4}{\log 131.2} = \frac{0.602}{2.118} = 0.284.$$

It follows, then, that the value of m is entirely dependent upon the original proportion of steam and water. Zeuner has, therefore, calculated m for different mixtures, as follows:

Initial Pressure p in Atmospheres.	Initial Specific Steam Quantity x.	Final Pressure in Atmospheres.		
		0.5	1	2
4	$x = 0.90$	$m = 1.124$	1.127	1.130
	0.80	$= 1.114$	1.116	1.119
	0.70	$= 1.102$	1.104	1.105
2	$x = 0.90$	$m = 1.123$	1.126	
	0.80	$= 1.114$	1.117	
	0.70	$= 1.103$	1.104	
1	$x = 0.90$	$m = 1.122$		
	0.80	$= 1.114$		
	0.70	$= 1.103$		

We see from this tabulation that the value of m depends upon the original steam quantity x; That

1st, it is greater the greater x is.

2d, it depends upon the initial and final pressures. The greater these are the greater is m.

We see also from the Table, that Rankine's value for m belongs to a mixture of about 80 parts steam and 20 parts water.

The dependence of m upon the initial and final pressures is also shown by the following tabulation given by Zeuner, in which the steam is assumed to be at first pure, without admixture of water. The table also gives the expansion ratio e.

Initial Pressure p in Atmospheres.		Final Pressure p_1, in Atmospheres.			
		0.5	1	2	4
8	$x =$	0.854	0.884	0.918	0.956
	$e =$	11.577	6.236	3.375	1.834
	$m =$	1.132	1.136	1.140	1.143
4	$x =$	0.888	0.921	0.958	
	$e =$	6.282	3.390	1.837	
	$m =$	1.132	1.135	1.140	
2	$x =$	0.924	0.960		
	$e =$	3.409	1.842		
	$m =$	1.130	1.134		
1	$x =$	0.961			
	$e =$	1.848			
	$m =$	1.129			

We see from this that for the same initial pressure p, the value of m is greater, the greater the final pressure p_1. We see also that the deviations are slight, and that hence we can take for m the mean value $m = 1.134$.

The expression which gives the adiabatic curve of saturated steam, *originally without admixture of water*, is therefore

$$pv^{1.134} = p_1v_1^{1.134} = p_2v_2^{1.134}, \text{ etc.}$$

Hence

$$\frac{p}{p_1} = \left(\frac{v_1}{v}\right)^{1.134},$$

or the expansion ratio e is

$$e = \frac{v_1}{v} = \left(\frac{p}{p_1}\right)^{\frac{1}{1.134}} = \left(\frac{p}{p_1}\right)^{\frac{1}{m}} \quad . \quad . \quad \text{(XXX.)}$$

Now from the tablé on page 447, we see that for the same initial pressure, m is less the greater the water weight in the mixture. If we take the mean of those cases where the initial steam quantity is 0.90, we have $m = 1.125$.

The mean for $x = 0.80$ is $m = 1.115$ and for $x = 0.70$, 1.103. Hence we have,

for $x = 1$ .... $m = 1.135$
$x = 0.90$................ $m = 1.125$
$x = 0.80$................ $m = 1.115$
$x = 0.70$................ $m = 1.103$.

Zeuner has found that these values are given very closely by the empirical formula,

$$m = 1.035 + 0.100x \quad . \quad . \quad . \qquad \text{(XXXI.)}$$

Grashof has assumed in his investigation of the steam engine, $m = 1.125$, a value which corresponds therefore to a mixture containing ten per cent. of water. We shall refer to this when speaking of the steam engine. We would only remark here, that the steam passing from the boiler to the cylinder has always a certain amount of water suspended in it mechanically; that the amount of this water depends upon the velocity and the fierceness of ebullition; that also in long passages, some steam is condensed and carried into the cylinder. In locomotives the water weight is not unfrequently 25 to 30 per cent.

The indicator diagram of the steam engine confirms rather than contradicts the correctness of the mechanical theory of heat. If we compare the indicator diagram of the steam engine with the isothermal curve for gases, which is given by Mariotte's law, we find that this curve deviates but little from the curve of the diagram. It has thus been asserted that the steam in the cylinder of a steam engine follows, during expansion, Mariotte's law, $pv = p_1v_1$, and that hence the conclusions of the mechanical theory of heat must be incorrect. Properly regarded, the contrary is the case. The value of m in the equation

$$pv^m = p_1v_1^m,$$

approaches unity more nearly, the greater the quantity of water and the greater the expansion. As the steam always carries with it a considerable percentage of water, m cannot differ much from 1, and hence the indicator curve does not vary much from that which gives the law of Mariotte. If, for example, we refer to the figure on page 438, we find that between 4 and 1 atmospheres and for 20 per cent. of water, the volumes are nearly inversely as the pressures. Thus while these last are 4, 2, 1, the former are 0.38, 0.71, 1.32.

Work of Steam Expanding Adiabatically.—We have seen in Part I., that the work of one kilogram of air, when expanding adiabatically, is

$$L = \frac{pv}{k-1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{k-1}{k}}\right]$$

where $k = 1.41$, and the equation of the curve is

$$pv^k = p_1 v_1^k.$$

Since our equation for saturated steam has the same form, we have a similar expression

$$L = \frac{pv}{m-1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right] \quad . \quad . \quad \text{(XXXII.)}$$

If we compute according to this formula the work during expansion of steam for different initial pressures and expansion ratios, and compare the results with those given by the formula on page 431, we find a very satisfactory agreement. We will show this by an example, which will serve at the same time to illustrate the use of the last formula.

EXAMPLE.

What is the work done by 1 kilogram of saturated steam, while expanding adiabatically from 4 atmospheres to 1 atmosphere?

Here $p = 4$, $p_1 = 1$ and $v = xu + \sigma = 0.448$. Hence $\frac{p_1}{p} = 0.25$, and for $\frac{m-1}{m}$, we have $\frac{0.135}{1.135} = 0.122$. Substituting these values we have

$L = 10334 \times \frac{4 \times 0.447}{0.135} [1 - (0.25)^{0.122}]$. Now $\log (0.25)^{0.122} = 0.122 \log 0.25 = \bar{1}.9266$, and $(0.25)^{0.122} = 0.845$. Hence $1 - (0.25)^{0.122} = 0.155$. Further, $10334 \times \frac{4 \times 0.447}{0.135} = 136408$. Therefore

$$L = 136408 \times 0.155 = 21143 \text{ meter-kilograms.}$$

The corresponding heat is

$$Q = AL = \frac{21143}{424} = 49.86 \text{ heat units.}$$

On page 445, we found 49.7 heat units, a very close correspondence.

Zeuner has investigated, by several other examples, how far the results of the present formula agree with those given by the previous. The following tabulation gives the comparison. The product AL_1 indicates the result of our present formula, and AL that of the other, while e is the expansion ratio.

Initial Pressure in Atmospheres.		Final Pressure p_1 in Atmospheres.			
p.		0.5	1	2	4
	$e =$	11.51	6.247	3.392	1.842
8	$AL_1 =$	94.90	74.02	51.35	26.78
	$AL =$	94.93	73.75	51.01	27.57
	$e =$	6.25	3.392	1.842	
4	$AL_1 =$	70.95	49.22	25.63	
	$AL =$	71.14	49.17	25.53	
	$e =$	3.392	1.842		
2	$AL_1 =$	47.19	24.57		
	$AL =$	47.40	24.59		
	$e =$	1.842			
1	$AL_1 =$	23.58			
	$AL =$	23.70			

We see that the values of AL_1 and AL coincide as near as can be desired. We may therefore make use of the approximate formula XXXII., in calculating the work of expansion in the steam engine. This we shall do, but we may give it a more convenient form.

Thus instead of $\left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}$ we can put $\left(\frac{v}{v_1}\right)^{m-1}$, and therefore

$$L = \frac{pv}{m-1}\left[1 - \left(\frac{v}{v_1}\right)^{m-1}\right] \quad . \; . \quad \text{(XXXIII.)}$$

As already remarked, Grashof, in his discussion of the steam engine, has taken $m = 1.125$.

APPENDIX TO CHAPTER XVIII.

THE principle that in every reversible cycle process, the product of the highest and lowest absolute temperatures is equal to the product of the intermediate absolute temperatures, the correctness of which we have shown by an example on page 252, of Part I., can be proved generally in an elementary manner.

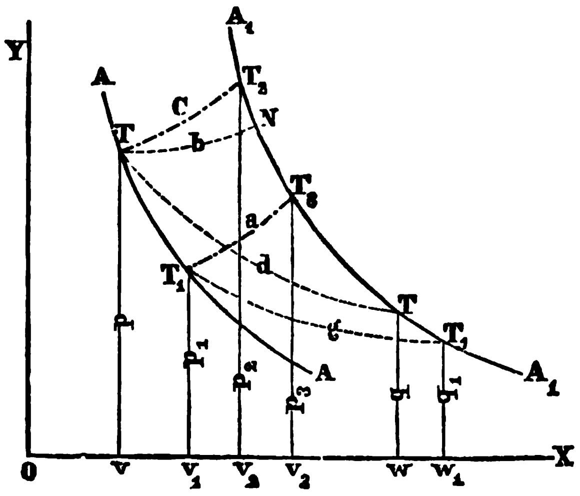

FIG. 75.

Let AA and A_1A_1, Fig. 75, be adiabatic curves; TcT_2 and T_1aT_3 lines which follow the general law

$$pv^{\frac{n}{m}} = p_2v_2^{\frac{n}{m}},$$

or

$$p_1v_1^{\frac{n}{m}} = p_3v_3^{\frac{n}{m}}.$$

Let T and T_1 on AA be the absolute temperatures of 1 kilogram of air, and T_2 T_3 the final absolute temperatures when this weight of air passes along TcT_2 or T_1aT_3. Let the curves TdT and T_1gT_1 be isothermals.

If we put now for w and v their values from (4) and (5), we have

$$P = \frac{c(k-1)}{A} \log \text{nat} \frac{v_2 \left(\frac{T_2}{T}\right)^{\frac{1}{k-1}}}{v_2 \left(\frac{T_2}{T}\right)^{\frac{m}{n-m}}} = \frac{c}{A}(k-1) \log \text{nat} \left(\frac{T_2}{T}\right)^{\frac{n-mk}{(n-m)(k-1)}}$$

or

$$P = \frac{c}{A} \log \text{nat} \left(\frac{T_2}{T}\right)^{\frac{n-mk}{n-m}},$$

or finally,

$$P = \frac{1}{A} \cdot \frac{mk-n}{m-n} c \log \text{nat} \left(\frac{T_2}{T}\right).$$

From Equation XLI., page 200, Part I., $\frac{mk-n}{m-n}c$ is the specific heat s for the law of change

$$pv^{\frac{n}{m}} = p_2 v_2^{\frac{n}{m}}.$$

Therefore we have

$$P = \frac{s}{A} \log \text{nat} \frac{T_2}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

This is, therefore, the heat weight for the path TcT_2.

We can thus find the heat weight for any transference of a body from one condition to another, if we know the initial and final temperatures as well as the specific heat for such transfer. If the initial temperature is t, and the final t_2, then $T = 273 + t$ and $T_2 = 273 + t_2$, so that

$$P = \frac{s}{A} \log \text{nat} \frac{273 + t_2}{273 + t}.$$

If we change the condition T_2, p_2, v_2 along the same curve T_2cT into the condition T, p, v, by abstracting heat, the heat weight abstracted is

$$\frac{s}{A} \log \text{nat} \frac{T_2}{T}$$

If TbN is another curve, which has the same initial point T, and whose end lies in the adiabatic A_1A_1, then, if the change of condition follows this curve, for which the specific heat is, say s_1, and the final temperature T_4, we have

$$P = \frac{s_1}{A} \log \text{nat} \frac{T_4}{T}.$$

Since N is on the adiabatic curve A_1A_1, the heat weight added along TcT_2 is the same as that along TbN, and

$$\frac{s}{A} \log \text{nat} \frac{T_2}{T} = \frac{s_1}{A} \log \text{nat} \frac{T_4}{T}.$$

Therefore, *when a body passes from the same initial condition into different final conditions, the heat weight imparted is always the same, if these final states are in the same adiabatic.*

On page 177, Part I., we have seen that the heat Q and Q_1, which must be imparted to a body, when passing by different isothermals from one adiabatic to another, are as the absolute temperatures T and T_1. Hence

$$\frac{Q}{T} = \frac{Q_1}{T_1} \quad \text{or} \quad \frac{Q}{AT} = \frac{Q_1}{AT_1}.$$

That is, the heat weights are equal. If, now, T_1aT_3 is a curve whose law is $pv^{\frac{n}{m}} = p_2v_2^{\frac{n}{m}} =$ etc., that is, if it is a curve of the same kind with TcT_2, we have again

$$\frac{Q_1}{AT_1} \quad \text{or} \quad R \log \text{nat} \frac{w_1}{v_1} = \frac{s}{A} \log \text{nat} \frac{T_3}{T_1}.$$

For since the curve T_1aT_3 is of the same kind as TcT_2, we have the same specific heat s.

Since now

$$\frac{Q_1}{AT_1} = \frac{Q}{AT} = \frac{s}{A} \log \text{nat} \frac{T_2}{T},$$

we have

$$\frac{s}{A} \log \text{nat} \frac{T_3}{T_1} = \frac{s}{A} \log \text{nat} \frac{T_2}{T} \quad . \quad . \quad . \quad (10).$$

From this equation we have $\frac{T_3}{T_1} = \frac{T_2}{T}$,

or

$$T_3T = T_2T_1,$$

and this is the principle already deduced in Part I. for a special case.

If in the cycle process, TcT_2, T_2T_3, T_3aT_1, T_1T, we consider the heat weight imparted on the way TcT_2, that is, $\frac{s}{A}$ log nat $\frac{T_2}{T}$, as positive, and that abstracted on the way T_3aT_1, viz., $\frac{s}{A}$ log nat $\frac{T_3}{T_1}$, as negative, we can say that

In every reversible cycle process the algebraic sum of the heat weights is zero.

We have thus far spoken only of such reversible cycle process as are inclosed by four curves. We may have a process composed of three, or more than four lines. One of more than four lines is called *compound.* In Fig. 75, TcT_2T_3TdT is a cycle process of only three lines. In such a process, also, the heat weight imparted is equal to that abstracted. For the heat weight on the way TdT is equal to that on the way TcT_2.

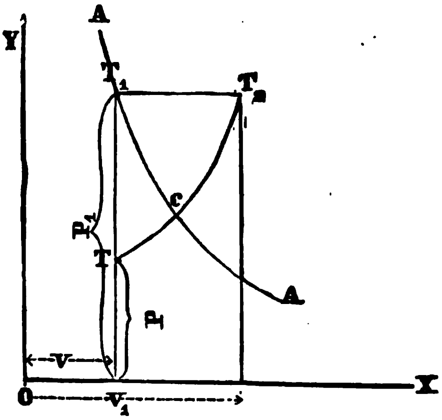

FIG. 76.

Since we shall have frequent occasion to speak of "heat weight," let us by some examples endeavor to get a clear idea of it.

Suppose in a cylinder the unit of weight of air of the volume v, and absolute temperature T, and pressure p, Fig. 76, and let us heat this air under constant volume v, until the pressure is p_1 and temperature T_1.

The heat weight imparted is

$$\frac{c}{A} \log \text{nat} \frac{T_1}{T},$$

where c is the specific heat for constant volume.

Now let the air expand under constant pressure p_1, until the volume is v_1, and temperature T_2. The heat weight added is

$$\frac{ck}{A} \log \text{nat} \frac{T_2}{T_1},$$

where ck is the specific heat for constant pressure.

The sum of the heat weights, during the change of condition from p, v, T to v_1, p_1, T_2, is then

$$\frac{c}{A} \log \text{nat} \frac{T_1}{T} + \frac{ck}{A} \log \text{nat} \frac{T_2}{T_1}.$$

We should have to add the same heat weight when the air is brought in any other way from the initial condition p, v, T to p_1, v_1, T_2. For if we suppose an adiabatic through T_1, as AA, the heat weight from T to c is the same as from T to T_1, and from T_2 to c the same as from T_2 to T_1. Hence the sum along Tc and cT_2 is equal to that along TT_1 and T_1T_2.

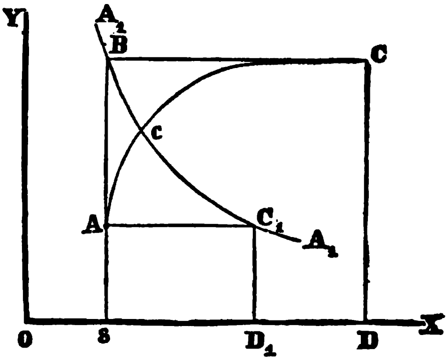

Fig. 77.

Again, let Os, Fig. 77, be the specific water volume (volume of 1 kilogram of water) at 0°. Let sA be the pressure of the steam at 0°. Let now the water be heated until the temperature has risen to $t°$, and at the same time let the pressure be so increased that at every instant it is equal to the steam pressure which *would* exist *without* this outer pressure. Then, if the mean specific heat of the water between 0° and $t°$ is denoted by c, we have for the heat weight imparted

$$P = \frac{c}{A} \log \text{nat} \frac{t + 273}{0 + 273} = \frac{c}{A} \log \text{nat} \frac{T}{273}.$$

If now we suppose the pressure constant and heat the water, then, as we know, steam is formed at constant temperature t. If after a certain time a certain volume of steam, SD, is formed, the line BC represents the law of change. This line is the isothermal line therefore of saturated steam, because it gives the relation between volume and pressure for *constant* temperature. If, then, the amount of heat imparted from B to C is Q, we have for the heat weight $\frac{Q}{AT}$ where T is the constant absolute temperature. This heat Q, is the latent heat. When the water is evaporated at 100° it is 537 heat units per kilogram. If, then, $SD = BC$ represents the volume of only one half of a kilogram of steam, Q would be $\frac{537}{2} = 268.5$ heat units, and the heat weight imparted is

$$\frac{268.5}{AT} = \frac{424 \times 268.5}{100 + 273}.$$

Hence to form from water at 0°, in the manner described, a certain volume of saturated steam of $t°$, requires the heat weight

$$\frac{c}{A} \log \text{nat} \frac{T}{273} + \frac{Q}{AT}.$$

If we suppose the vaporization had taken place according to the law represented by AcC, the heat weight imparted is still the same. For if we pass through B an adiabatic A_1A_1, we have for the heat weight from A to c the same value as from A to B, and from c to C the same value as from B to C. If further, AC_1 is parallel to OX, then AC_1 is the isothermal for vaporization at 0°. The heat weight imparted from A to C_1 must be equal to that from A to B, or A to c.

If $AC_1 = SD_1$ represents a volume whose weight is x_2 kilograms, and if the latent heat of vaporization is r_1, then the heat weight imparted from A to C_1 is

$$\frac{x_2 r_1}{A \times 273}.$$

We have then

$$\frac{c}{A}\log \text{nat}\frac{T}{273}=\frac{x_2 r_1}{A\times 273},$$

or

$$c\log \text{nat}\ \frac{T}{273}=\frac{x_2 r_1}{273}.$$

Since c is known and r_1 can be found for every temperature, we can find from this equation x_2 for every value of T.

QUESTIONS FOR EXAMINATION.

What is an isothermal curve? What is the form of this curve for saturated steam? Define what is meant by saturated steam? What effect has the addition of heat to a mixture of water and steam, when the pressure is constant? If saturated steam is compressed while the temperature is kept constant, what happens? What is the expression for the outer work during isothermal expansion? What for the heat added? What does *u* denote in our notation? What is specific volume of steam?

What is an isodynamic curve? Make out its equation for steam. Show how to construct it. When water and steam expand isodynamically what takes place? What is the outer work during expansion? What is the heat imparted?

What is an adiabatic curve? Make out its equation for steam. Show how to construct it. In adiabatic expansion of saturated steam, if there is no water at the beginning, what takes place? If there is more steam than water, what takes place? Does this mean more by volume or by weight? If there is more water than steam, what takes place? In adiabatic compression, if there is no water at the beginning, what takes place? If the initial steam weight is greater than that of the water, what takes place? If there is more water than steam, what takes place? What is the expression for the outer work? What for the heat? What is the form of the equation for the adiabatic curve for saturated steam? What is the value of the exponent for dry steam alone? What for 0.90 per cent. steam? For 0.80 per cent.? 0.70 per cent.? What is the general equation which gives the value of the exponent for any percentage of steam? What does the indicator diagram seem to show as regards the adiabatic curve of saturated steam? What is the expression for the work of saturated steam expanding adiabatically? What for the heat? What two formulæ have we then for this work? Do these agree in their results?

Can you prove generally that in every reversible cycle process the product of the highest and lowest absolute temperatures is equal to the product of the intermediate absolute temperatures? What do you understand by heat weight? What is the specific heat for any law of relation of pressure and volume? What is the general expression for the heat weight? If a body passes from the same initial condition into different final conditions upon the same adiabatic, what is the relation between the heat weights imparted? In every reversible cycle process, what is the algebraic sum of the heat weights? What is a cycle process? When is it reversible? When simple? When compound? When complete? When incomplete?

APPLICATION OF THE CALCULUS TO SATURATED STEAM.

THE relation between the pressure and temperature of saturated steam is given by the empirical formula of Regnault,

$$\log p = a + b\alpha^{\tau} + c\beta^{\tau}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{where } \tau = t - t_0.$$

In this equation a, b, c, α, and β are constants, the values of which have been determined by Regnault for various steams. The logarithm is the common logarithm, the pressure p is in millimeters, t is the temperature Centigrade, and t_0 is a constant given by Regnault. We have already given the values of these quantities for water in the text, page 388.

In the application of the Calculus we shall find it necessary to know not only the relation of the pressure to the temperature, but also the relation of $\frac{dp}{dt}$ to the temperature.

This we can easily determine from equation (1). Thus, if we multiply both sides by the natural log of 10, or 2.302585093, which we denote by k, we have

$$\text{nat} \log p = ka + kb\alpha^{\tau} + kc\beta^{\tau}.$$

Differentiate, remembering that $\tau = t - t_0$, where t_0 is a constant, and we have

$$\frac{dp}{p\,dt} = kb \text{ nat} \log \alpha \times \alpha^{\tau} + kc \text{ nat} \log \beta \times \beta^{\tau},$$

or replacing nat. by common logarithms,

$$\frac{dp}{p\,dt} = (k^2 b \log \alpha)\,\alpha^{\tau} + (k^2 c \log \beta)\,\beta^{\tau}$$

Since the constants b, c, α, and β are known for different steams, we can calculate the coefficients of α^{τ} and β^{τ}. Denote them by m and n, so that

$$m = k^2 b \log \alpha, \qquad \text{and } n = k^2 c \log \beta,$$

and we have

$$\frac{dp}{p\,dt} = m\alpha^{\tau} + n\beta^{\tau} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

We give in the following Tables I. and II., the values of a, $\log (b\alpha^{\tau})$ and $\log (c\beta^{\tau})$ for equation (1), and of $\log (m\alpha^{\tau})$ and $\log (n\beta^{\tau})$ in equation (2) in terms of the temperature, for various steams, as given by Regnault's values for the constants. This makes it necessary to give also the signs to be prefixed to each of the terms on the right side of equations (1) and (2).

By the aid of these Tables we can calculate for various steams the values of $\frac{dp}{p\,dt}$ and then multiplying by the corresponding values of p, we obtain $\frac{dp}{dt}$. We thus find the values given in the Tables, pages 479 to 488.

TABLE I.

PRESSURE IN MILLIMETERS, TEMPERATURE CENTIGRADE.

$$\log p = a + b\alpha^\tau + c\beta^\tau.$$

Saturated steam of	Value of a.	Sign before $b\alpha^\tau$	Sign before $c\beta^\tau$	Values of $\log(b\alpha^\tau)$	Values of $(\log(c\beta^\tau)$
Water 0—100°	4.7393707	−	+	$+0.6117408-0.003274463t$	$-1.8680093+0.006864937t$
Water 100—200°	6.2640348	−	−	$+0.6593123-0.001656138t$	$+0.0207601-0.005950708t$
Ether ($C_4H_{10}O$)	5.0286298	−	+	$+0.4414817-0.0031223t$	$-3.3497080+0.0145775t$
Alcohol (C_2H_6O)	5.4562028	−	+	$+0.6390301-0.0029148t$	$-2.4949825-0.0590515t$
Acetone (C_3H_6O)	5.3085419	−	−	$+0.5312766-0.0026148t$	$-0.9645222-0.0215592t$
Chloroform ($CHCl_3$)	5.2253893	−	−	$+0.5219943-0.0025856t$	$-0.9111888-0.0131824t$
Chloride of Carbon (CCl_4)	12.0962331	−	−	$+0.9550688-0.0002880t$	$+0.1934744-0.0050220t$
Bisulphide of Carbon (CS_2)	5.4011662	−	−	$+0.4918860-0.0022372t$	$-0.7200868-0.0088008t$
Mercury	5.6640459	−	+	$+0.8890208-0.0012438t$	$-0.4179605-0.0119062t$
Carbonic Acid	6.6771989	−	+	$+0.2175859-0.0052911t$	$-0.3948491-0.0089594t$

For the pressure p in inches and temperature Fahrenheit, simply subtract 1.4048299 *from a, and put* $\frac{5}{9}(t-32)$ *for t.*

TABLE II.

$$\frac{dp}{p\,dt} = m\alpha^\tau + n\beta^\tau.$$

Saturated Steam of	Sign before $m\alpha^\tau$	Sign before $n\beta^\tau$	Values of $\log(m\alpha^\tau)$.	Values of $\log(n\beta^\tau)$.
Water 0—100	+	+	$-1.1486877-0.003274463t$	$-3.3069414+0.006864937t$
Water 100—200	+	+	$-1.3971597-0.001656138t$	$-1.4802398-0.005950708t$
Ether	+	+	$-1.3396624-0.0031223t$	$-4.4616896+0.0145775t$
Alcohol	+	−	$-1.1720041-0.0029148t$	$-2.9992701-0.0590515t$
Acetone	+	+	$-1.3268535-0.0026148t$	$-1.9064582-0.0215592t$
Chloroform	+	+	$-1.3410130-0.0025856t$	$-2.0667124-0.0131824t$
Chloride of Carbon	+	+	$-1.8611078-0.0002880t$	$-1.3812195-0.0050220t$
Bisulph. of Carbon	+	+	$-1.4339778-0.0022372t$	$-2.0511078-0.0088008t$
Mercury	+	−	$-1.2917974-0.0012438t$	$-1.6177651-0.0119062t$
Carbonic Acid	+	−	$-1.3344869-0.0052911t$	$-1.7181390-0.0089594t$

For English units and temperature in Fahrenheit degrees, add 0.2552725 *to the first term in both columns and put* $\frac{5}{9}(t-32)$ *in place of t.*

As the values of $\frac{dp}{p\,dt}$ and $\frac{dp}{dt}$ lie at the basis of further calculation, we will illustrate how to find them by a few examples.

Suppose we wish to find the value of $\frac{dp}{dt}$, and $\frac{dp}{p\,dt}$ and p for saturated steam of acetone at the temperature 100° Centigrade.

From Table I., we have

$$\log (b\alpha^\tau) = +\,0.5812766 - 0.0026148t = 0.2697966,$$

$$\log (c\beta^\tau) = -\,0.9645222 - 0.0215592t = -\,3.1204422 \text{ or } \bar{4}.8795578.$$

Hence $$b\alpha^\tau = 1.8612155 \quad \text{and} \quad c\beta^\tau = 0.0007578.$$

Since $a = 5.3085419$, we have, taking the proper signs from the Table,

$$\log p = 5.3085419 - 1.8612155 - 0.0007578 = 3.4465686,$$

or $$p = 2796.2 \text{ millim.}$$

If we wish the pressure p in inches for 212° Fah., we have

$$\log p = 3.9037120 - 1.8612155 - 0.0007578 = 2.0417387,$$

or $$p = 110.08 \text{ inches.}$$

For the value of $\frac{dp}{p\,dt}$ in French units, we have from Table II.,

$$\log (m\alpha^\tau) = -\,1.3268535 - 0.0026148t = -\,1.5883335 = \bar{2}.4116665,$$

$$\log (n\beta^\tau) = -\,1.9064582 - 0.0215592t = -\,4.0623782 = \bar{5}.9376218.$$

Hence $$m\alpha^\tau = 0.025802, \qquad n\beta^\tau = 0.00008662,$$

and $$\frac{dp}{p\,dt} = 0.025802 + 0.00008662 = 0.025888.$$

Multiply this by 2796.2 and we have $\frac{dp}{dt} = 72.391$.

In this way the values of $\frac{dp}{p\,dt}$ and $\frac{dp}{dt}$ in the Tables, pages 479 to 488, are found and tabulated once for all, for future use.

If we denote the specific heat of any liquid by c, the amount of heat dq required to raise the unit of weight by the amount dt is

$$dq = c\,dt, \qquad \text{hence } c = \frac{dq}{dt} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

Now the heat of the liquid q has already been given, page 390, on the basis of Regnault's experiments. We have then only to differentiate and find $\frac{dq}{dt}$, to find the actual specific heat. The values of W and r have also been given already, page 392.

By the aid of our fundamental equations, page 212, we can now easily determine the outer latent heat, Apu.

Thus if we have a unit weight of mixture, of which x kilog. are steam and $1 - x$ liquid, we have for the volume

$$v = xu + \sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

If we impart under constant pressure, and therefore constant temperature, the heat dQ, an amount of liquid dx is converted into steam, and hence (page 000),

$$dQ = r\,dx.$$

If we differentiate (4) under the assumption that the temperature t, and hence u, is constant, we have

$$dv = \mu\,dx, \qquad \text{or } dx = \frac{dv}{du},$$

hence

$$dQ = \frac{r}{u}\,dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

But we have from our fundamental equations (page 212),

$$dQ = A\,[X\,dp + Y\,dv],$$

$$dQ = \frac{A}{\left(\frac{dt}{dp}\right)}[X\,dt + (a + t)\,dv].$$

Since the temperature and pressure are constant, $dp = 0$, $dt = 0$, and $dQ = A\,Y\,dv$, or $dQ = \frac{A\,(a + t)}{\left(\frac{dt}{dp}\right)}\,dv.$

These equations are identical with (5), hence we have

$$A\,Y = \frac{r}{u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and

$$\frac{r}{u} = \frac{A\,(a + t)}{\frac{dt}{dp}} = \frac{AT}{\frac{dt}{dp}} \quad . \quad . \quad . \quad . \quad . \quad (7)$$

The first equation gives the value of the function Y for steam and liquid mixtures. The second equation gives us

$$\frac{r}{u} = AT\frac{dp}{dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

If we put for r its value $\rho + Apu$, we have

$$\frac{\rho}{u} = A\left(T\frac{dp}{dt} - p\right) \quad . \quad . \quad . \quad . \quad . \quad (9)$$

Now r and ρ are the total latent heat and the inner latent heat for a unit of weight, and, disregarding the very small volume of the liquid as compared with the steam, u is the volume of the unit of weight. Therefore $\frac{r}{u}$ and $\frac{\rho}{u}$ express the total latent heat and inner latent heat per *unit of volume*.

Since we have already seen how $\frac{dp}{dt}$ can be calculated, we can calculate $\frac{r}{u}$ and $\frac{\rho}{u}$ from (8) and (9) by making use of the values of $\frac{dp}{dt}$ already found. We thus obtain the values of $\frac{r}{u}$ in Tables, pages 479 to 488.

Having now the values of $\frac{r}{u}$ and knowing r (page 392), we have

$$u = \frac{r}{\left(\frac{r}{u}\right)},$$

and we thus obtain the values of u as given in column 10 of the steam Tables at the end of this work.

If we divide equation (8) by Ap we have

$$\frac{r}{Apu} = T\frac{dp}{p\,dt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

As we have already found and tabulated the values of $\frac{dp}{p\,dt}$ we have only to multiply by T to obtain the values of $T\frac{dp}{p\,dt}$ in tables, pages 479 to 488.

We have also tabulated the reciprocals of these values

$$\varphi = \frac{Apu}{r},$$

and given them in Tables, pages 479 to 488.

Evidently we have only to multiply these values by r (page 392), to obtain the value of Apu as given in column 7 of the Steam Tables at the end of this work.

We may also find $Apu = r - \rho$, directly in terms of the temperature from the experimental values of r and ρ already given* (pages 392 and 395). Thus,

Water . . . . $Apu = 31.1 + 0.096t - 0.00002t^2 - 0.0000003t^3$.
Ether . . . . $Apu = 7.46 + 0.02747t - 0.0001854t^2$.
Acetone . . . . $Apu = 8.87 + 0.06185t - 0.0002845t^2$.
Chloroform . . . . $Apu = 4.56 + 0.01797t - 0.0000367t^2$.
Chloride of Carbon . . . . . . $Apu = 3.43 + 0.01671t - 0.0000546t^2$.
Bisulphide of Carbon . . . . $Apu = 7.21 + 0.02524t - 0.0000918t^2$.

where t is the temperature in degrees Centigrade.

To reduce to English units and Fahrenheit degrees, multiply the first term on the right by 1.8, divide the third by 1.8, and the fourth (in the case of water) by 3.24. The second term remains unchanged. Put $(t - 32)$ in place of t.

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* The values of ρ given on page 395, are determined from the values of ρ given in column 9 of the Steam Tables at the end of this work. These values are found by subtracting the values of Apu in column (7) from the values of r in column (6).

These formulæ give results agreeing almost exactly with the tabular values, a result most satisfying when we recall the multitude of experiments and the extensive calculations by which the tabular values were obtained.

From our fundamental equations (page 212), we have

$$dQ = A\,(X\,dp + Y\,dv),$$
$$dU = X\,dp + Z\,dv,$$

where the functions Y and Z are connected by the relation $Z + p = Y$.

We have just found by equation (6) for steam and liquid mixtures $AY = \frac{r}{u}$, hence

$$AZ = \frac{r}{u} - Ap,$$

or putting for r its value $\rho + Apu$,

$$AZ = \frac{\rho}{u}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (10)$$

which gives us the value of the function Z for steam and liquid mixtures. Referring to the values of r and ρ, we see that both Y and Z are functions only of the pressure or temperature, and are independent of the volume.

We found the same result for permanent gases (page 213). There we found

$$AY = \frac{c_p}{R}\,p, \qquad \text{and } AZ = \frac{c_v}{R}\,p.$$

These give

$$\frac{Y}{Z} = \frac{c_p}{c_v} = k,$$

and the value of k for the permanent gases was constant. If we should make the same assumption for steam and liquid mixtures we should have

$$\frac{Y}{Z} = \frac{\frac{r}{u}}{\frac{\rho}{u}} = \frac{AT\,\frac{dp}{dt}}{A\left(T\frac{dp}{dt} - p\right)} = k = \text{constant},$$

and hence

$$\frac{T\,dp}{p\,dt} = \frac{k}{k-1} = \text{constant}.$$

Now we see from our Tables, pages 479 to 488, that this is not true for any steam. The assumption therefore must be rejected that saturated steam behaves like a gas.

The density of steam or the weight of a cubic unit is found by

$$\gamma = \frac{1}{u + \sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

We thus find the values of γ in column 11 of Table II. at the end of this work.

FUNDAMENTAL EQUATIONS FOR STEAM AND LIQUID MIXTURES.

In a unit of weight of mixture of steam and water, consisting of x kilog. of steam and $(1 - x)$ water, the inner work is

$$U = \frac{1}{A}(q + x\rho),$$

or differentiating,

$$A\,dU = dq + d(x\rho) \quad . \quad . \quad . \quad . \quad . \quad (12)$$

The corresponding heat imparted is

$$dQ = dq + d(x\rho) + Ap\,dv \quad . \quad . \quad . \quad (13)$$

and

$$v = xu + \sigma \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (14)$$

We can put equation (13) in various forms by the aid of (14) according as we put for v its value in terms of x or for x its value in terms of v, and according as we express q and ρ in terms of t or of p.

First Transformation.—From (14) we have

$$x = \frac{v - \sigma}{u}.$$

Substituting in (13)

$$dQ = dq + d\left[\frac{\rho}{u}(v - \sigma)\right] + Ap\,dv.$$

Differentiating, regarding q and $\frac{\rho}{u}$ as functions of p, we have

$$dQ = \frac{d}{dp}\left[q + \frac{\rho}{u}v - \frac{\rho}{u}\sigma\right]dp + \left(\frac{\rho}{u} + Ap\right)dv.$$

Putting $\rho = r - Apu$

$$dQ = \frac{d}{dp}\left[q + \frac{\rho}{u}v - \frac{\rho}{u}\sigma\right]dp + \frac{r}{u}dv \quad . \quad . \quad . \quad . \quad . \quad (15)$$

Comparing this with the general equation, page 212,

$$dQ = A\,(X\,dp + Y\,dv),$$

we have

$$AX = \frac{d}{dp}\left(q + \frac{\rho}{u}v - \frac{\rho}{u}\sigma\right)$$

or

$$AX = \frac{dq}{dp} - \sigma\frac{d}{dp}\left(\frac{\rho}{u}\right) + v\frac{d}{dp}\left(\frac{\rho}{u}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

and

$$AY = \frac{r}{u} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

This last result we have already found, page 465. We may put for $\frac{dq}{dp}$, $\frac{c}{\frac{dp}{dt}}$, where the value of the specific heat $c = \frac{dq}{dt}$ is easily found from the equation for q, page 390, and $\frac{dp}{dt}$ we have already found and tabulated. From equation (9) we have $\frac{d}{dp}\left(\frac{\rho}{u}\right) = AT\frac{d}{dt}\left[\text{nat log}\,\frac{dp}{dt}\right]$.

Second Transformation.—From (14) we have

$$dv = d\,(xu),$$

and hence

$$Ap\,dv = Ap\,d\,(xu),$$

or

$$Ap\,dv = A\,d\,(pux) - Axu\,dp.$$

Substitute this in (13) and

$$dQ = dq + d\,(x\rho) + A\,d\,(pux) - Axu\,dp.$$

Put $r = \rho + Apu$, and from (7) $Axu\,dp = \frac{xr}{T}\,dt$,

and we have

$$dQ = dq + d(xr) - \frac{xr}{T}\,dt, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (18)$$

which may also be written

$$dQ = dq + T\,d\,\frac{xr}{T} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

Third Transformation.—If we put in (18), $c\,dt$ for dq we have

$$dQ = c\,dt + r\,dx + x\,dr - \frac{xr}{T}\,dt,$$

or adding and subtracting $xc\,dt$,

$$dQ = (1 - x)\,c\,dt + r\,dx + x\left(c + \frac{dr}{dt} - \frac{r}{T}\right)dt.$$

If we denote the term

$$\left(c + \frac{dr}{dt} - \frac{r}{T}\right)$$

by h, where h is evidently a function of t only, we have

$$dQ = (1 - x)\,c\,dt + r\,dx + xh\,dt \quad . \quad . \quad . \quad (20)$$

The term $(1 - x)\,c\,dt$ gives that portion of the heat which goes to raise the temperature of the liquid. The heat to generate the steam is $r\,dx$. The last term $xh\,dt$ represents then the heat imparted to the steam beyond what is necessary to heat the water and generate the steam. For one unit of weight this amount is $h\,dt$.

The quantity

$$h = c + \frac{dr}{dt} - \frac{r}{T}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

is then analogous to specific heat, and gives the amount of heat which must be imparted to the unit weight of steam in order that when the temperature is raised dt, *it shall still be in the saturated condition.*

In applying these equations it must be remembered that they only hold good when the change of state is reversible, or when the steam tension and outer pressure are always equal. If during the change the outer pressure is different from the steam tension, we must have recourse to equations (12) and (13).

It remains only to show the form which the two fundamental equations I. and II., given on page 212, take for steam and liquid mixtures, and to show that they are confirmed by the preceding.

The first fundamental equation I., page 212, is

$$\frac{dY}{dp} - \frac{dX}{dv} = 1.$$

Equations (16) and (17) give Y and X. Differentiating we have

$$\frac{dY}{dp} = \frac{1}{A}\frac{d}{dp}\left(\frac{r}{u}\right) \text{ and } \frac{dX}{dv} = \frac{1}{A}\frac{d}{dp}\left(\frac{\rho}{u}\right).$$

Put $r = \rho + Apu$ and subtract, and the difference is unity as the first fundamental equation requires.

The second fundamental equation II., page 212, is

$$a + t = Y\left(\frac{dt}{dp}\right) - X\left(\frac{dt}{dv}\right).$$

Since for mixtures of steam and liquid the temperature is a function of the pressure only, and is independent of the volume,

$$\frac{dt}{dv} = 0,$$

and hence

$$Y = (a + t)\frac{dp}{dt} = T\frac{dp}{dt}.$$

Put for Y its value from eq. (17), and we have

$$\frac{r}{u} = AT\frac{dp}{dt}.$$

The second fundamental equation takes then the same form as we have already deduced in equation (8).

ISOTHERMAL AND ISODYNAMIC CURVES AND CURVE OF CONSTANT STEAM WEIGHT.

Since when the pressure is constant the temperature of saturated steam is also constant, the isothermal becomes a straight line. Also the functions of the temperature, u, q, ρ, and r are constant.

The outer work performed during expansion from v_1 to v, is

$$L = \int_{v_1}^{v} p\,dv = p\,(v - v_1),$$

or since $v = xu + \sigma$, and $v_1 = x_1 u + \sigma$,

$$L = pu\,(x - x_1). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (21)$$

The change of inner work from equation (12) is, since $dq = 0$ and ρ is constant,

$$U - U_1 = \frac{\rho}{A}\,(x - x_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (22)$$

The heat which must be imparted during expansion is

$$Q = A\,(U - U_1 + L) = (\rho + Apu)\,(x - x_1) = r\,(x - x_1). \quad . \quad . \qquad (23)$$

If we suppose the expansion to continue until all the liquid is converted into steam, we have simply to put $x = 1$ in the preceding formulæ. If heat is imparted after this point is reached, so as to keep the temperature constant, the steam is superheated and the curve is different.

For the isodynamic curve we have $dU = 0$, and therefore equation (12) becomes

$$dq + d\,(x\rho) = 0,$$

or

$$q - q_1 + x\rho - x_1\rho_1 = 0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (24)$$

Combining this with $v = xu + \sigma$ and $v_1 = x_1 u_1 + \sigma$, we can find for any given pressure p the corresponding volume v. Thus if the initial conditions x_1 and t_1 are given, we know at once p_1 and u_1, and can find v_1. Then for any other pressure, p_1, we can find t, u, q, and ρ, and from (24) can find x, and then v.

Solving a number of cases in this manner, Zeuner has determined that for all the steams given in our Tables, if a mixture of steam and liquid expands according to the isodynamic curve, there is an increase of steam, and if compressed, steam is condensed.

If we start with pure saturated steam, or $x_1 = 1$, the curve is given by

$$pv^n = p_1 v_1^n,$$

where for water steam $n = 1.0456$.

If we determine for one unit of weight of steam the specific volume s, by adding to the value of u from our Tables the value of σ, and lay off those values of s as abscissas, with the corresponding values of p as ordinates, we obtain the curve of *constant steam weight*. For water steam Zeuner has thus determined

$$pv^{1.0646} = p_1 v_1^{1.0646}.$$

If in equation (12) we put then $x = x_1 = \text{const.}$, we have

$$A\,dU = dq + x_1\,d\rho; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (25)$$

and since q and ρ are functions of t,

$$A\frac{dU}{dt} = \frac{dq}{dt} + x_1\frac{d\rho}{dt}.$$

For compression, or increase of temperature, we have an *increase* of inner work, because for all the steams in our Tables, $\frac{d\rho}{dt}$ is positive even when $x_1 = 1$.

Integrating, we have for the increase of inner work

$$A\,(U - U_1) = q - q_1 + x_1\,(\rho - \rho_1).$$

As to the heat which must be imparted or abstracted in order that the steam weight shall be constant, we have from equation (20) by making $dx = 0$,

$$dQ = (1 - x_1)\,c\,dt + x_1 h\,dt \quad . \quad . \quad . \quad . \quad . \quad . \quad (26)$$

The first term on the right, since $c\,dt = dq$, gives the heat of the liquid. The second term gives the heat required by the steam, during the rise of temperature dt. One unit weight of steam requires then $h\,dt$ heat units in order to remain saturated. The quantity h may then be defined as *the specific heat of saturated steam for constant steam weight.* Its value from equation (21) is

$$h = c + \frac{dr}{dt} - \frac{r}{T},$$

and it is therefore a function of the temperature. Since $W = q + r$ and $c\,dt = dq$, we have

$$h = \frac{dW}{dt} - \frac{W - q}{T}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

Putting for W and q their values for water steam, page 392, we have

$$h = 0.305 - \frac{606.5 - 0.695t - 0.00002t^2 - 0.0000003t^3}{273 + t}, \quad . \quad . \quad . \quad (28)$$

and from this we have the noteworthy result that for *water steam* the value of h is *negative*. Thus for $t = 0°$, $100°$, and $200°$, we have $h = -1.9166$, -1.1333 and -0.6766. That is, during the compression heat must be *abstracted* in order to keep the steam weight constant, and during expansion heat must be *added*.

If then during expansion, no heat is added, a portion of the steam will be condensed.

Equations (25) and (26) give the change of inner work and the heat which must be added, for constant steam weight compression. As to the outer work we have

$$dQ = A\,dU + A\,dL,$$

or from (25) and (26)

$$A\,dL = x_1\,[h\,dt - dq - d\rho] \quad . \quad . \quad . \quad . \quad . \quad . \quad (29)$$

We have given in column 3 of the following Table III. the value of $-\int_0^t h\,dt$ for water steam. This integral is easily determined from equation (28). We give in the same Table the values of other quantities which will hereafter occur, and which in connection with Table II., at the end of this work, will enable us to solve many examples for saturated water steam. For other steams similar tables can easily be prepared if ever needed.

To illustrate the use of the preceding formulæ, suppose a unit of weight of pure saturated water steam at 5 atmospheres, to expand performing work, and to remain during expansion all steam, until the pressure is 1 atmosphere.

Here we have $x = x_1 = 1$, and from (26)

$$Q = \int_{t_1}^{t_2} h\,dt = -\int_0^{t_1} - \left(-\int_0^{t_2} h\,dt\right)$$

TABLE III.

Auxiliary Table for Saturated Water Steam.

French units and Centigrade degrees.

1 Press. p in atmospheres	2 $\frac{dp}{dt}$ in kilogr. per sq. mt.	3 $-\int_0^t h\,dt$	4 $\tau = \int_0^t \frac{dq}{T}$	5 r	6 $\frac{r}{T}$	7 $\frac{1}{A}\frac{dq}{dp}$	8 $\frac{1}{A}\frac{d}{dp}\left(\frac{\rho}{u}\right)$	9 $\frac{r}{Apu}$	10 $\frac{d}{dt}\left(\frac{\rho}{u}\right)$	1 Press. p in atmospheres
0.5	207.423	126.747	0.26273	549.404	1.54887	2.0631	11.976	14.229	5.859	0.5
1	369.708	148.470	0.31356	536.500	1.43834	1.1618	11.088	13.344	9.668	1
2	653.927	170.639	0.36814	521.866	1.32588	0.6600	10.194	12.453	15.722	2
3	909.307	183.778	0.40205	512.353	1.25913	0.4763	9.675	11.985	20.749	3
4	1146.689	193.163	0.42711	505.110	1.21129	0.3788	9.299	11.568	25.149	4
5	1371.141	200.457	0.44693	499.186	1.17395	0.3175	9.014	11.284	29.160	5
6	1585.431	206.394	0.46392	494.121	1.14322	0.2752	8.774	11.052	32.808	6
7	1791.503	211.481	0.47840	489.686	1.11714	0.2440	8.557	10.856	36.155	7
8	1990.199	215.862	0.49120	485.709	1.09441	0.2201	8.409	10.684	39.471	8
9	2183.469	219.726	0.50270	482.093	1.07425	0.2009	8.264	10.535	42.557	9
10	2371.153	223.178	0.51297	478.776	1.05617	0.1853	8.143	10.401	45.533	10
11	2554.223	226.292	0.52266	475.707	1.03980	0.1723	8.004	10.280	48.217	11
12	2732.873	229.134	0.53150	472.839	1.02477	0.1613	7.896	10.168	50.892	12
13	2907.722	231.752	0.53975	470.141	1.01088	0.1518	7.794	10.066	53.450	13
14	3079.063	234.165	0.54744	467.600	0.99801	0.1435	7.699	9.971	55.910	14

TABLE III.—(*Continued.*)—*English units and Fahrenheit degrees.*

1 Pressure p in atmospheres	2 $\frac{dp}{dt}$ in lbs. per sq. ft.	3 $-\int_0^t h\,dt$	4 $\tau = \int_0^t \frac{dq}{T}$	5 r	6 $\frac{r}{T}$	7 $\frac{1}{A}\frac{dq}{dp}$	8 $\frac{1}{A}\frac{d}{dp}\left(\frac{\rho}{u}\right)$	9 $\frac{r}{Apu}$	10 $\frac{d}{dt}\left(\frac{\rho}{u}\right)$	1 Pressure p in atmospheres
0.5	23.598	228.1446	0.33252	988.9272	1.54887	33.0181	11.976	14.229	0.365755	0.5
1	42.061	267.246	0.38335	965.7000	1.43834	28.5936	11.088	12.344	0.603537	1
2	74.396	307.1502	0.43793	939.3588	1.32588	10.5627	10.194	12.453	0.981465	2
3	103.450	330.8004	0.47184	922.2354	1.25913	7.6227	9.675	11.985	1.29528	3
4	130.456	347.6934	0.49690	909.198	1.21129	6.0624	2.299	11.568	1.56996	4
5	155.992	360.8226	0.51672	898.5348	1.17395	5.0813	9.014	11.284	1.82035	5
6	180.371	371.5092	0.53371	889.4282	1.14322	4.4043	8.774	11.052	2.04808	6
7	203.815	380.6658	0.54819	881.4348	1.11714	3.9050	8.557	10.856	2.25702	7
8	226.421	388.5516	0.56099	874.2762	1.09441	3.5225	8.409	10.684	2.46402	8
9	248.409	395.5068	0.57249	867.7674	1.07425	3.2152	8.264	10.535	2.65667	9
10	269.761	401.7204	0.58276	861.7968	1.05617	2.9655	8.143	10.401	2.84245	10
11	290.539	407.3256	0.59245	856.2726	1.03980	2.7575	8.004	10.280	3.01000	11
12	310.913	412.4412	0.60129	851.1102	1.02477	2.5815	7.896	10.168	3.17699	12
13	330.806	417.1536	0.60954	846.2538	1.01088	2.4294	7.794	10.066	3.33668	13
14	350.299	421.4970	0.61723	841.6800	0.99801	2.2966	7.699	9.971	3.49025	14

Taking the corresponding values from Table III., page 473, column 3, for 5 and 1 atmospheres, we have

$$Q = 200.457 - 148.470 = 51.987 \text{ French heat units}$$

for one kilogram of steam, or

$$Q = 360.8226 - 267.2460 = 93.5766 \text{ English heat units}$$

for one pound of steam.

We see that this amount of heat is quite considerable, and even if we assume that the hot sides of the cylinder impart during expansion a certain amount of heat to the steam, still it is certain that they cannot yield enough to keep all the steam saturated, and hence heat from without must be imparted.

The change of inner work is, from equation (25),

$$A(U_2 - U_1) = q_2 - q_1 + \rho_2 - \rho_1.$$

From Table II., page 666, we have for 5 atmospheres, $q_1 = 153.741$ $\rho_1 = 454.994$, and for 1 atmosphere, $q_2 = 100.5$ $\rho_2 = 496.8$, hence

$$A(U_2 - U_1) = -11.935 \text{ French heat units for one kilogram.}$$

The negative sign shows a decrease of inner work.

Finally the outer work is

$$AL = Q - A(U_2 - U_1) = 63.922 \text{ French heat units,}$$

or,

$$L = 424 \times 63.922 = 27102.9 \text{ meter-kilograms}$$

per kilogram of steam. Only a part therefore of the entire heat goes to outer work.

From equation (27) we have in general

$$hT = (a + t)\frac{dW}{dt} - W + q. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Inserting for W the experimental values as determined by Regnault, page 392, we have for steam of

Water	$hT = -523.23 + t + 0.00002t^2 + 0.0000003t^3.$
Ether	$hT = +28.85 + 0.2257t - 0.0002596t^2.$
Acetone	$hT = -40.473 + 0.2247t - 0.000119t^2.$
Chloroform	$hT = -29.462 + 0.2323t - 0.0000507t^2.$
Chloride of Carbon	$hT = -12.087 + 0.1041t - 0.000081t^2.$
Bisulphide of Carbon	$hT = -50.139 + 0.0101t - 0.0003308t^2.$

For all these steams, within the limits of temperature of Regnault's experiments, except ether, the value of h is negative. That is, heat must be abstracted during compression and added during expansion, to prevent partial condensation, and keep the steam saturated. The only exception is ether steam. This steam alone is condensed by compression and superheated by expansion, if no heat is imparted or abstracted during the change of state. All these facts are confirmed by direct experiment. Alcohol and benzine steams act in this respect like water steam also, but are not included above, because hT is not thus far known as a function of t. For alcohol steam the value of W is not known, and for benzine the value of q is still undetermined by experiment, yet we can decide as to this point.

Thus we see that it is the value of the first term on the right in all our equations which determines the sign of hT. This term is the value of hT for $t = 0$. If then we determine this we can settle the question.

If we put $t = 0$ in equation (30) we have

$$(hT)_0 = a\left(\frac{dW}{dt}\right)_0 - W_0.$$

Regnault gives for W in general

$$W = \alpha + \beta t + \delta t^2,$$

where α, β, γ are constants determined by experiment.

Hence $\frac{dW}{dt} = \beta + 2\,\delta t$, or for $t = 0$, $W = \alpha$, and $\frac{dW}{dt_0} = \beta$. Thus when $t=0$

$$(hT)_0 = \beta a - \alpha.$$

Now for benzine steam Regnault gives

$$W = 109.00 + 0.24429t - 0.0001813t^2.$$

Accordingly $\alpha = 109$, $\beta = 0.24429$, and as $a = 273$, we have

$$(hT)_0 = -42.309.$$

This steam therefore acts like water steam.

For alcohol steam we do not know W, but we know q. From the value of h,

$$h = c + \frac{dr}{dt} - \frac{r}{T}.$$

we have

$$h\,dt = c\,dt + Td\left(\frac{r}{T}\right),$$

and putting $dq = c\,dt$,

$$\int\frac{h\,dt}{T} = \int\frac{dq}{T} + \frac{r}{T}.$$

It is evident that if h is negative, the right side of this equation must decrease as the temperature increases. Our Tables for alcohol show that such is actually the case. Thus for 0°, 40°, 80°, we have, taking the value of r given by the Tables and using the value of q for alcohol, page 390, for the right side of the equation. 3.1418, 3.1172, 2.0447.

Thus ether steam is the only exception thus far, to the general rule, that to keep the steam saturated and uncondensed heat must be added during expansion and abstracted during compression.

THE ADIABATIC CURVE FOR MIXTURES OF STEAM AND LIQUID.

Since for adiabatic change there is no transfer of heat, we have from equation (19), putting $dQ = 0$,

$$0 = dq + Td\left(\frac{xr}{T}\right).$$

Integrating and putting $\int_0^t \frac{dq}{T} = \tau$, we have

$$\frac{xr}{T} + \tau = \text{constant}.$$

The value of τ is easily determined from the value of q as given page 390. We have thus for French units and Centigrade degrees

$$\tau = 2.4318892 \log \frac{T}{273} - 0.0002057t + 0.00000045t^2,$$

and for English units and Fahrenheit degrees

$$\tau = 2.414100458 \log \frac{T}{459.4} - 0.0001054\,(t - 32) + 0.0000001389\,(t - 32)^2.$$

We have found and tabulated the values of τ for the temperatures corresponding to different pressures in column (4) of Table III., page 473, preceding. We have also given r and $\frac{r}{t}$ for the same pressures.

If we denote the initial temperature by t_1, and the final by t, we have for adiabatic change

$$\frac{xr}{T} + \tau = \frac{x_1 r_1}{T_1} + \tau_1 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (31)$$

This is the equation given on page 437, and its use is there explained.

If we put for dq, $c\,dt$, we have also, considering c as constant, the approximate formula given on page 435,

$$\tau = \int_0^t \frac{dq}{T} = \int_0^t \frac{c\,dt}{T} = c \text{ nat } \log \frac{T}{a},$$

where c is the mean specific heat. As there is no difficulty in finding the exact value of τ, we have no use for this equation here.

With the aid of Table III., page 473, we can find x from equation (31), when the initial and final pressures are given. If x is found greater than x_1 it shows that steam has been formed. If x is found to be less than x_1, steam has been condensed. If x is greater than 1, it shows that the steam is superheated, and the formula ceases entirely to apply, because it is based expressly upon the condition that the steam remains saturated.

The method of using equation (31) has been fully illustrated on page 437, *et seq.*

The empirical formula for the adiabatic curve of water steam,

$$pv^m = p_1 v_1^m,$$

where m has the value

$$m = 1.035 + 0.100x$$

has been already deduced, page 449.

DEPORTMENT OF STEAM AND LIQUID MIXTURES WHEN THE CHANGE OF CONDITION IS A REVERSIBLE PROCESS.

The process of change is reversible when at any moment the outer pressure is equal to the steam tension. This assumption is made in all our foregoing investigations of the isothermal, isodynamic and adiabatic curves, as well as the curve of constant steam weight. We can now apply them to problems of technical importance.

PROBLEM.—*Transfer of heat under constant volume.*

Suppose a unit of weight of mixture, x_1 by weight steam and $1 - x$ liquid, at the temperature t_1 and corresponding pressure p_1.

We have then,

$$x = \frac{u_1}{u} x_1.$$

Since the volume is constant, we have $dL = 0$, and hence

$$dQ = A\,dU = dq + d\,(x\rho),$$

or, as on page 472,

$$Q = q - q_1 + x_1 u_1 \left(\frac{\rho}{u} - \frac{\rho_1}{u_1}\right). \quad . \quad . \quad . \quad . \quad (32)$$

For the "*heat weight*" (page 189), we have from eq. (19),

$$P = \int \frac{dQ}{AT} = \int \frac{1}{A}\left[\frac{dq}{T} + d\left(\frac{xr}{T}\right)\right],$$

or

$$P = \frac{1}{A}\left(\tau - \tau_1 + \frac{xr}{T} - \frac{x_1 r_1}{T_1}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (33)$$

As long as $x = \frac{u_1}{u} x_1$ is less than unity, we have to do with saturated steam, and the formulæ hold good.

Equation (15), which holds good for any change of the mixture, is

$$dQ - \frac{d}{dp}\left[q - \sigma\frac{\rho}{u} + v\frac{\rho}{u}\right] dp + \frac{r}{u}\,dv.$$

Inserting p as a function of t and $c = \frac{dq}{dt}$, we have

$$dQ = \left[c - \sigma \frac{d}{dt}\left(\frac{\rho}{u}\right) + v \frac{d}{dt}\left(\frac{\rho}{u}\right)\right] dt + \frac{r}{u} dv. \quad . \quad (84)$$

If now the volume is constant, $dv = 0$, and $v = v_1$, hence

$$\frac{dQ_v}{dt} = c - \sigma \frac{d}{dt}\left(\frac{\rho}{u}\right) + v_1 \frac{d}{dt}\left(\frac{\rho}{u}\right). \quad . \quad . \quad . \quad (85)$$

This value, gives *the specific heat of the mixture for constant volume.*

The value of $c = \frac{dq}{dt}$ is easily determined from the equations, page 376. For steam of water we have given the values of $\frac{d}{dt}\left(\frac{\rho}{u}\right)$ in column 10, of the Table III., page 473.

Thus for water, $c = \frac{dq}{dt} = 1 + 0.00004t + 0.0000009t^2$, and for 5 atmospheres' pressure, $t = 152.32°$ C. Hence $c = 1.0269$. From our Table III., page 473, $\frac{d}{dt}\left(\frac{\rho}{u}\right) = 29.160$, and since $\sigma = 0.001$,

$$\frac{dQ_v}{dt} = 0.9977 + 29.160 v_1.$$

If we have pure saturated steam and no water, $v_1 = u_1 + \sigma = 0.3686$, and

$$\frac{dQ_v}{dt} = 11.595,$$

which of course holds good for heat abstraction, since if heat were added the steam would be superheated. For *mixtures* of steam and liquid, the specific heat for constant volume is always greater than that of the liquid, and therefore for water, greater than unity. This is due to the change of aggregation.

It is worth noting that for water steam the value of $c - \sigma \frac{d}{dt}\left(\frac{\rho}{u}\right)$ in equation (85) may be put with slight error equal to unity.

The application of the preceding to an example will be found on page 449. Also to the problem of determining the time in which the pressure of steam will rise a certain amount in a boiler, on page 451.

SATURATED STEAM OF WATER.

French Units.

For corresponding values of T and p, see Table II. (a), at end of work.

Temperature Centigrade t.	$\frac{dp}{dt}$ in millimeters of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
0	0.3289	0.071502	19.520	2.879	0.051229
5	0.4503	0.068915	19.158	4.014	0.052196
10	0.6088	0.066429	18.799	5.525	0.053198
15	0.8132	0.064041	18.444	7.511	0.054219
20	1.0738	0.061746	18.091	10.090	0.055274
25	1.4022	0.059542	17.743	13.400	0.056358
30	1.8117	0.057427	17.400	17.604	0.057470
35	2.3171	0.055397	17.062	22.886	0.058609
40	2.9347	0.053449	16.730	29.457	0.059774
45	3.6825	0.051582	16.403	37.554	0.060964
50	4.5800	0.049794	16.083	47.442	0.062176
55	5.6484	0.048081	15.771	59.418	0.063409
60	6.9100	0.046443	15.465	73.792	0.064661
65	8.3891	0.044876	15.168	90.933	0.065927
70	10.111	0.043380	14.879	111.220	0 067206
75	12.104	0.041953	14.600	135.077	0.068494
80	14.395	0.040594	14.329	162.959	0.069786
85	17.017	0.039300	14.069	195.369	0.071076
90	20.002	0.038072	13.820	232.852	0.072359
95	23.383	0.036907	13.582	276.009	0.073628
100	27.189	0.035775	13.344	325.229	0.074940
105	31.454	0.034701	13.117	381.286	0.076236
110	36.212	0.033674	12.897	444.778	0.077536
115	41.499	0.032691	12.684	516.362	0.078838
120	47.348	0.031750	12.478	596.786	0.080143
125	53.795	0.030848	12.277	686.610	0.081450
130	60.873	0.029982	12.083	786.716	0.082761
135	68.617	0.029152	11.894	897.803	0.084075
140	77.060	0.028355	11.711	1020.627	0.085391
145	86.234	0.027590	11.533	1155.960	0.086709
150	96.171	0.026854	11.359	1304.583	0.088034
155	106.901	0.026146	11.191	1467.283	0.089361
160	118.455	0.025465	11.026	1644 860	0.090691
165	130.858	0.024809	10.866	1838.076	0.092026
170	144.138	0.024177	10.711	2047.724	0.093365
175	158.819	0.023568	10.559	2274.570	0.094709
180	173.423	0.022981	10.410	2519.373	0.096059
185	189.470	0.022414	10.265	2782.875	0.097413
190	206.478	0.021866	10.124	3065.794	0.098774
195	224.462	0.021337	9.986	3368.820	0.100140
200	243.438	0.020826	9.851	3692.650	0.101514

SATURATED STEAM OF WATER (*Continued*).

English Units.

For corresponding values of T and p, see Table II. (a), at end of work.

Temperature Fahrenheit t.	$\frac{dp}{dt}$ in inches of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
32	0.00719	0.039723	19.520	0.3226	0.051229
41	0.00985	0.038286	19.158	0.4510	0.052196
50	0.01332	0.036905	18 799	0.6208	0.053198
59	0.01779	0.035578	18 444	0.8439	0.054219
68	0.02349	0.034303	18.091	1.1247	0.055274
77	0.03067	0.033079	17.743	1.5057	0 056358
86	0.03963	0.031904	17.400	1.9781	0.057470
95	0.05068	0.030776	17.062	2.5716	0.058609
104	0.06419	0.029694	16.730	3.3100	0.059774
113	0.08054	0.028656	16.403	4.2198	0.060964
122	0.10018	0.027663	16.083	5.2309	0.062176
131	0.12345	0.026711	15.771	6.6761	0.063409
140	0.15114	0 025801	15.465	8.2827	0.064661
149	0.18349	0.024931	15.168	10.1268	0.065927
158	0.22115	0.024100	14.879	12.4975	0.067206
167	0.26474	0.023307	14.600	15.1782	0.068494
176	0.31485	0.022552	14.329	18.2193	0.069786
185	0.37220	0.021833	14.069	21.9521	0.071076
194	0.43749	0.021151	13.820	26.1649	0.072359
203	0.51155	0.020548	13.582	31.0184	0.073628
212	0.59469	0.019875	13.344	36.5441	0.074940
221	0.68797	0.019278	13.117	42.844	0.076236
230	0.79204	0.018707	12.897	49.9784	0.077536
239	0.90768	0.018161	12.684	58.022	0.078838
248	1.03562	0.017639	12.478	67.0585	0.080143
257	1.17663	0.017137	12.277	77.1524	0.081450
266	1.33144	0.016656	12.083	88.4010	0.082761
275	1.50082	0.016195	11.894	100.8836	0.084075
284	1.68549	0.015752	11.711	114.684	0.085391
293	1.88615	0.015327	11.533	129.800	0.086709
302	2.10349	0.014914	11.359	146.5923	0.088034
311	2.33819	0.014525	11.191	164.8744	0.089361
320	2.59090	0.014147	11.026	184.8282	0.090691
329	2.86218	0.013782	10.866	206.5394	0.092026
338	3.15265	0.013431	10.711	230.0969	0.093365
347	3.46282	0.013093	10.559	255.587	0.094709
356	3.79314	0 012878	10.410	283.0948	0.096059
365	4.14417	0.012453	10.265	312.7088	0.097413
374	4.51618	0.012147	10.124	344.4855	0.098774
383	4.90953	0.011854	9.986	378.5448	0.100140
392	5.32458	0.011570	9.851	414.9327	0.101514

SATURATED STEAM OF ETHER ($C_4H_{10}O$).

French Units.

For corresponding values of T and p, see Tables at end of work.

Temperature Centigrade t.	$\frac{dp}{dt}$ in millimeters of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
0	8.441	0.045778	12.497	73.90	0.08002
10	12.224	0.042619	12.061	110.94	0.08291
20	17.175	0.039685	11.628	161.38	0.08600
30	23.464	0.036963	11.200	228.00	0.08929
40	31.242	0.034444	10.781	313.60	0.09275
50	40.622	0.032117	10.374	420.78	0.09640
60	51.709	0.029976	9.982	552.20	0.10018
70	64.576	0.028017	9.610	710.32	0.10406
80	79.327	0.026243	9.264	898.02	0.10794
90	96.127	0.024659	8.951	1119.01	0.11172
100	115.313	0.023280	8.683	1379.34	0.11516
110	137.523	0.022139	8.475	1689.13	0.11799
120	163.979	0.021243	8.348	2066.66	0.11978

English Units.

Temperature Fahrenheit t.	$\frac{dp}{dt}$ in inches of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
32	0.18462	0.025432	12.497	8.3039	0.08002
50	0.26737	0.023677	12.061	12.4660	0.08291
68	0.37566	0.022047	11.628	18.1338	0.08600
86	0.51321	0.020515	11.200	25.6197	0.08929
104	0.68334	0.019135	10.781	35.2383	0.09275
122	0.88850	0.017843	10.374	47.2818	0.09640
140	1.13100	0.016653	9.982	61.8491	0.10018
158	1.41243	0.015565	9.610	79.8166	0.10406
176	1.73507	0.014579	9.264	100.9079	0.10794
194	2.10253	0.013699	8.951	125.7399	0.11172
212	2.52217	0.012933	8.683	154.9925	0.11516
230	3.00796	0.012299	8.475	189.8027	0.11799
248	3.58662	0.011801	8.348	232.2247	0.11978

SATURATED STEAM OF ALCOHOL (C_2H_6O).

French Units.

For corresponding values of T and p, see Tables at end of work.

Temperature Centigrade t.	$\frac{dp}{dt}$ in millimeters of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
0	0.8419	0.066295	18.099	7.37	0.05525
10	1.5185	0.062672	17.736	13.78	0.05638
20	2.6133	0.058779	17.222	24.55	0.05806
30	4.3193	0.055009	16.668	41.97	0.05999
40	6.8784	0.051450	16.104	69.04	0.06210
50	10.580	0.048114	15.541	109.59	0.06435
60	15.757	0.044992	14.982	168.26	0.06674
70	22.767	0.042072	14.431	250.43	0.06930
80	31.981	0.039342	13.887	362.04	0.07201
90	43.752	0.036788	13.354	509.32	0.07488
100	58.397	0.034401	12.831	698.53	0.07793
110	76.162	0.032168	12.320	935.46	0.08117
120	97.211	0.030080	11.821	1225.17	0.08459
130	121.597	0.028128	11.335	1571.58	0.08822
140	149.255	0.026302	10.863	1976.82	0.09206
150	179.998	0.024595	10.404	2441.72	0.09612

English Units.

Temperature Fahrenheit t.	$\frac{dp}{dt}$ in inches of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
32	0.01841	0.03683	18.099	0.8281	0.05525
50	0.03321	0.034817	17.736	1.5484	0.05638
68	0.05715	0.049321	17.222	2.7584	0.05806
86	0.09447	0.03056	16.668	4.1160	0.05999
104	0.15044	0 028583	16.104	7.7577	0.06210
122	0.23141	0.02673	15.541	12.3142	0.06435
140	0.34464	0.024995	14.982	18.9067	0.06674
158	0.49796	0.023373	14.431	28.1399	0.06930
176	0.6995	0.021856	13.887	40.6812	0.07201
194	0.9569	0.020437	13.354	57.2307	0.07488
212	1.2772	0.019112	12.831	78.4916	0.07793
230	1.6658	0.017871	12.320	105.1148	0.08117
248	2.1262	0.017831	11.821	137.6686	0.08459
266	2.6595	0.015626	11.335	176.588	0.08822
284	3.2645	0.014612	10.863	222.1294	0.09206
302	3.9369	0.013664	10.404	274.3691	0.09612

SATURATED STEAM FOR ACETONE (C_3H_6O).

French Units.

For corresponding values of T and p, see Tables at end of work.

Temperature Centigrade t.	$\frac{dp}{dt}$ in millimeters of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
0	3.769	0.059516	16.248	33.00	0.06155
10	5.726	0.051910	14.690	51.97	0.06807
20	8.349	0.046364	13.558	78.45	0.07378
30	11.797	0.042125	12.674	114.63	0.07834
40	16.242	0.038733	12.123	163.04	0.08248
50	21.858	0.035903	11.596	226.41	0.08623
60	28.808	0.033460	11.142	307.64	0.08975
70	37.237	0.031294	10.733	409.60	0.09316
80	47.266	0.029338	10.356	535.06	0.09656
90	58.971	0.027546	9.999	686.49	0.10001
100	72.391	0.025889	9.657	865.92	0.10355
110	87.515	0.024348	9.325	1074.90	0.10723
120	104.271	0.022907	9.002	1314.15	0.11108
130	122.555	0.021558	8.688	1583.89	0.11510
140	142.199	0.020293	8.380	1883.87	0.11932

English Units.

Temperature Fahrenheit t.	$\frac{dp}{dt}$ in inches of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
32	0.08243	0.033064	16.248	3.7081	0.06155
50	0.1252	0.028839	14.690	5.8397	0.06807
68	0.1607	0.025757	13.553	8.8152	0.07378
86	0.2580	0.023402	12.674	12.8804	0.07834
104	0.3552	0.021518	12.128	18.3203	0.08248
122	0.4780	0.019946	11.596	25.4409	0.08625
140	0.6300	0.018589	11.142	34.5686	0.08975
158	0 8144	0.017382	10.733	46.0256	0.09316
176	1.0338	0.016299	10.356	60.1232	0.09656
194	1.2898	0.015303	9.999	77.1389	0.10001
212	1.5833	0.014382	9.657	97.3009	0.10355
230	1.9141	0.013526	9.325	120.7834	0.10723
248	2.2806	0.012726	9.002	147.6673	0.11108
266	2.6803	0.011976	8.688	177.9772	0.11510
284	3.1099	0.011273	8.380	211.6839	0.11932

SATURATED STEAM OF CHLOROFORM ($CHCl_3$).

French Units.

For corresponding values of T and p, see Tables at end of work.

Temperature Centigrade t.	$\frac{dp}{dt}$ in millimeters of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
0	3.235	0.054178	14.790	28.32	0.06761
10	4.953	0.049297	13.951	44.95	0.07168
20	7.246	0.045156	13.231	68.08	0.07558
30	10.295	0.041593	12.603	100.04	0.07935
40	14.211	0.038485	12.046	142.64	0.08302
50	19.123	0.035741	11.544	198.08	0.08662
60	25.150	0.033292	11.086	268.58	0.09020
70	32.393	0.031084	10.662	356.31	0.09379
80	40.933	0.029079	10.265	463.38	0.09742
90	50.816	0.027244	9.889	591.55	0.10112
100	62.061	0.025555	9.532	742.36	0.10491
110	74.645	0.023994	9.190	916.83	0.10882
120	88.510	0.022546	8.860	1115.51	0.11286
130	103.549	0.021197	8.542	1338.26	0.11706
140	119.625	0.019937	8.234	1584.38	0.12145
150	136.584	0.018760	7.935	1852.80	0.12603
160	154.218	0.017658	7.646	2141.61	0.13079

English Units.

Temperature Fahrenheit t.	$\frac{dp}{dt}$ in inches of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
32	0.07075	0.030030	14.790	3.1822	0.06761
50	0.10586	0.027387	13.951	5.0509	0.07168
68	0.15848	0.025487	13.231	7.6499	0.07558
86	0.22518	0.023107	12.603	11.241	0.07935
104	0.31083	0.021380	12.046	16.028	0.08302
122	0.41827	0.019850	11.544	22.258	0.08662
140	0.55009	0.018496	11.086	30.180	0.09020
158	0.70851	0.017269	10.662	40.038	0.09379
176	0.89531	0.016155	10.265	52.069	0.09742
194	1.1114	0.015136	9.889	66.472	0.10112
212	1.3574	0.014198	9.532	83.417	0.10491
230	1.6326	0.013330	9.190	103.02	0.10882
248	1.9359	0.012526	8.860	125.35	0.11286
266	2.2649	0.011776	8.542	150.38	0.11706
284	2.6164	0.011076	8.234	178.03	0.12145
302	2.9874	0.010423	7.935	208.20	0.12603
320	3.3731	0.009810	7.646	240.65	0.13079

SATURATED STEAM OF CHLORIDE OF CARBON (CCl_4).

French Units.

For corresponding values of *T* and *p*, see Tables at end of work.

Temperature Centigrade *t*.	$\frac{dp}{dt}$ in millimeters of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
0	1.82	0.030743	15.107	15.96	0.06619
10	2.84	0.028071	14.350	25.76	0.06968
20	4.24	0.025874	13.646	39.82	0.07328
30	6.10	0.023823	12.993	59.28	0.07696
40	8.50	0.021991	12.390	85.35	0.08071
50	11.52	0.020354	11.834	119.30	0.08450
60	15.21	0.018890	11.323	162.46	0.08832
70	19.66	0.017582	10.855	216.23	0.09212
80	24.91	0.016411	10.427	281.99	0.09590
90	31.03	0.015363	10.038	361.27	0.09962
100	38.09	0.014424	9.685	455.64	0.10325
110	46.15	0.013584	9.365	566.83	0.10678
120	55.28	0.012830	9.076	696.70	0.11018
130	65.56	0.012153	8.816	847.28	0.11343
140	77.09	0.011547	8.584	1021.00	0.11650
150	89.96	0.011000	8.376	1220.31	0.11939
160	104.29	0.010509	8.191	1448.19	0.12208

English Units.

Temperature Fahrenheit *t*.	$\frac{dp}{dt}$ in inches of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
32	0.03980	0.030743	15.107	1.7934	0.06619
50	0.06211	0.028171	14.350	2.8946	0.06968
68	0.09274	0.025874	13.646	4.4745	0.07328
86	0.13342	0.023823	12.993	6.6611	0.07696
104	0.18591	0.021991	12.390	9.5906	0.08071
122	0.25197	0.020354	11.834	13.405	0.08450
140	0.33268	0.018890	11 323	18.255	0.08832
158	0.43001	0.017582	10.855	24.297	0.09212
176	0.54484	0.016411	10.427	31.687	0.09590
194	0.67870	0.015363	10.038	40.595	0.09962
212	0.83312	0.014424	9.685	51.199	0.10325
230	1.0094	0.013584	9.365	63.693	0.10678
248	1.2091	0.012830	9.076	78.286	0.11018
266	1.4339	0.012153	8.816	95.211	0.11343
284	1.6861	0.011547	8.584	114.73	0.11650
302	1.9676	0.011000	8.376	137.12	0.11939
320	2.2811	0.010509	8.191	162.73	0.12208

SATURATED STEAM OF BISULPHIDE OF CARBON (CS_2).

French Units.

For corresponding values of T and p, see Tables at end of work.

Temperature Centigrade t.	$\frac{dp}{dt}$ in millimeters of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
0	5.846	0.045704	12.477	51.18	0.08015
10	8.380	0.042226	11.950	76.05	0.08368
20	11.664	0.039137	11.467	109.60	0.08721
30	15.813	0.036383	11.024	153.65	0.09071
40	20.941	0.033911	10.614	210.20	0.09421
50	27.157	0.031682	10.233	281.30	0.09772
60	34.540	0.029661	9.877	368.85	0.10124
70	43.181	0.027821	9.548	474.98	0.10478
80	53.124	0.026137	9.226	601.38	0.10838
90	64.406	0.024591	8.926	749.76	0.11202
100	77.030	0.023166	8.641	921.42	0.11573
110	90.972	0.021847	8.367	1117.36	0.11951
120	106.173	0.020621	8.104	1338.12	0.12339
130	122.573	0.019482	7.851	1584.12	0.12737
140	140.057	0.018419	7.607	1855.00	0.13146
150	158.497	0.017425	7.371	2150.06	0.13567

English Units.

Temperature Fahrenheit t.	$\frac{dp}{dt}$ in inches of mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
32	0.12786	0.025390	12.477	5.7509	0.08015
50	0.18329	0.023459	11.950	7.8309	0.08368
68	0.25512	0.021743	11.467	11.241	0.08721
86	0.34587	0.020213	11.024	15.699	0.09071
104	0.45803	0.018839	10.614	21.395	0.00421
122	0.59399	0.017561	10.233	28.521	0.09772
140	0.75547	0.016478	9 877	37.251	0.10124
158	0.94447	0.015456	9.548	47.780	0.10478
176	1.1619	0.014520	9.226	60.251	0.10838
194	1.4087	0.013661	8.926	74.811	0.11202
212	1.6848	0.01287	8.641	91.556	0.11573
230	1.9898	0.012137	8.367	110.55	0.11951
248	2.3223	0.011456	8.104	131.81	0.12339
266	2.6810	0.010823	7.851	155.33	0.12737
284	3.0634	0.010233	7.607	181.04	0.13146
302	3.4667	0.0096806	7.371	208.82	0.13567

SATURATED STEAM OF MERCURY.—*French Units.*

Temperature Centigrade t.	Absolute Temperature, T	Pressure in millimeters mercury, p	$\frac{dp}{dt}$ in millimet's mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u}=AT\frac{dp}{dt}$	$\frac{Apu}{r}=\phi$
140	413	3.0592	0.1030	0.033684	13.911	1.36	0.07189
160	433	5.9002	0.1888	0.031999	13.855	2.62	0.07217
180	453	11.00	0.3336	0.030328	13.738	4.85	0.07279
200	473	19.90	0.5712	0.028703	13.576	8.66	0.07366
220	493	34.70	0.9418	0.027142	13.381	14.89	0.07478
240	513	58.82	1.5088	0.025652	13.159	24.82	0.07599
260	533	96.73	2.3443	0.024236	12.918	40.07	0.07741
280	553	155.17	3.5525	0.022894	12.660	63.00	0.07898
300	573	242.15	5.2362	0.021624	12.390	96.22	0.08071
320	593	368.73	7.5302	0.020422	12.110	143.20	0.08257
340	613	548.35	10.576	0.019287	11.823	207.90	0.08458
360	633	797.74	14.530	0.018214	11.529	294.96	0.08673
380	653	1139.65	19.602	0.017200	11.232	410.48	0.08903
400	673	1587.96	25.795	0.016244	10.932	556.73	0.09147
420	693	2177.53	33.405	0.015341	10.631	742.40	0.09406
440	713	2933.99	42.499	0.014485	10.328	971.73	0.09682
460	733	3888.14	53.186	0.013679	10.027	1250.24	0.09973
480	753	5072.43	65.520	0.012917	9.726	1582.19	0.10281
500	773	6520.25	79.534	0.012198	9.429	1971.57	0.10605
520	793	8264.96	95.204	0.011519	9.134	2421.18	0.10947

English Units.

Temperature Fahrenheit t.	Absolute Temperature, T	Pressure in inches mercury, p	$\frac{dp}{dt}$ in inches mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u}=AT\frac{dp}{dt}$	$\frac{Apu}{r}=\phi$
284	743.4	0.12044	0.002352	0.018713	13.911	0.15281	0.07189
320	779.4	0.23229	0.004129	0.017777	13.855	0.2944	0.07217
356	815.4	0.43306	0.007296	0.016849	13.738	0.54498	0.07279
392	851.4	0.78347	0.012493	0.015946	13.576	0.9731	0.07366
428	887.4	1.3661	0.020599	0.015079	13.381	1.6731	0.07473
464	923.4	2.3157	0.033001	0.014251	13.159	2.789	0.07599
500	959.4	3.8083	0.051276	0.013465	12.918	4.5025	0.07741
536	995.4	6.1091	0.077702	0.012719	12.660	7.0792	0.07898
572	1031.4	9.5335	0.11453	0.012013	12.390	10.812	0.08071
608	1067.4	14.517	0.16470	0.011345	12.110	16.091	0.08257
644	1103.4	21.588	0.23132	0.010691	11.823	23.361	0.08458
670	1139.4	31.407	0.31781	0.010119	11.529	33.144	0.08673
716	1175.4	44.868	0.42874	0.009555	11.232	46.125	0.08903
752	1211.4	62.518	0.56420	0.0090245	10.932	62.558	0.09147
788	1247.4	85.730	0.73065	0.0085228	10.631	83.421	0.09406
824	1283.4	115.51	0.92956	0.0080472	10.328	109.19	0.09682
860	1319.4	153.07	1.1633	0.0075995	10.027	140.49	0.09973
896	1355.4	199.70	1.4331	0.0071761	9.726	177.79	0.10281
932	1391.4	256.70	1.7396	0.0067765	9.429	221.54	0.10605
968	1427.4	325.40	2.0823	0.0063995	9.134	272.05	0.10947

SATURATED STEAM OF CARBONIC ACID.

French Units.

Temperature Centigrade t.	Absolute Temperature, T	Pressure in millimeters mercury, p	$\frac{dp}{dt}$ in millimet's mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
− 25	248	13007.02	399.64	0.030725	7.620	3178.42	0.13124
− 20	253	15142.44	456.63	0.030156	7.629	3704.91	0.13107
− 15	258	17582.48	518.66	0.029499	7.611	4291.33	0.13139
− 10	263	20340.20	585.19	0.028770	7.566	4935.58	0.13216
− 5	268	23441.34	656.01	0.027985	7.500	5638.04	0.13333
0	273	26906.60	730.70	0.027157	7.414	6397.22	0.13488
+ 5	278	30753.80	808.70	0.026296	7.310	7209.74	0.13679
+ 10	283	34998.65	889.42	0.025413	7.192	8072.03	0.13904
+ 15	288	39646.86	972.06	0.024518	7.061	8977.87	0.14163
+ 20	293	44716.58	1055.98	0.023615	6.919	9922.30	0.14453
+ 25	298	50207.32	1140.31	0.022712	6.768	10897.49	0.14775
+ 30	303	56119.05	1224.18	0.021814	6.610	11895.36	0.15129
+ 35	308	62447.30	1306.77	0.020926	6.445	12907.36	0.15515
+ 40	313	69184.45	1387.22	0.020051	6.276	13924.36	0.15934
+ 45	318	76314.60	1464.71	0.019193	6.108	14937.11	0.16384

English Units.

Temperature Fahrenheit t.	Absolute Temperature, T	Pressure in inches mercury, p	$\frac{dp}{dt}$ in inches mercury.	$\frac{dp}{p\,dt}$	$T\frac{dp}{p\,dt}$	$\frac{r}{u} = AT\frac{dp}{dt}$	$\frac{Apu}{r} = \phi$
− 13	446.4	512.09	8.7411	0.017069	7.620	357.15	0.13124
− 4	455.4	596.16	9.9876	0.016758	7.629	416.31	0.13107
+ 5	464.4	692.23	11.344	0.016388	7.611	482.21	0.13139
+ 14	473.4	800.8	12.799	0.015983	7.566	554.60	0.13216
+ 23	482.4	922.9	14.348	0.015547	7.500	633.53	0.13333
+ 32	491.4	1059.3	15.982	0.015087	7.414	718.84	0.13488
+ 41	500.4	1210.8	17.688	0.014609	7.310	810.13	0.13679
+ 50	509.4	1377.9	19.494	0.014118	7.192	907.04	0.13904
+ 59	518.4	1560.9	21.261	0.013621	7.061	1008.82	0.14163
+ 68	527.4	1760.5	23.097	0.013119	6.919	1114.94	0.14453
+ 77	536.4	1977.0	24.941	0.012618	6.768	1224.53	0.14775
+ 86	545.4	2209.4	26.776	0.012119	6.610	1336.61	0 15129
+ 95	554.4	2458.6	28.582	0.011625	6.445	1450.87	0.15515
+104	563.4	2723.8	30.342	0.011139	6.276	1564.64	0.15934
+113	572.4	3004.6	32.036	0.010663	6.108	1678.44	0.16384

CHAPTER XIX.

OTHER CHANGES OF CONDITION OF STEAM AND LIQUID MIXTURES, OF PRACTICAL IMPORTANCE.

I. *The Deportment of Steam and Liquid Mixtures, when Heat is Imparted or Abstracted under Constant Volume.*—Suppose 1 kilogram of steam and liquid inclosed in a vessel, of which x kilograms are steam and $1 - x$ liquid. The temperature of the mixture is t and the pressure p. The specific volume v of the mixture is then

$$v = xu + \sigma.$$

Let now heat be imparted or abstracted. The temperature is then t_1, the pressure p_1, and the specific steam quantity is x_1. For the final specific volume we have

$$v_1 = x_1u_1 + \sigma.$$

If now, the volume is kept constant or $v = v_1$,

$$xu + \sigma = x_1u_1 + \sigma,$$

hence the weight of steam at the end of the operation is

$$x_1 = \frac{u}{u_1}x \quad . \quad . \quad . \quad . \quad . \quad \text{(XXXIV.)}$$

EXAMPLE.

Suppose we have in a vessel a mixture of G kilograms of water and steam of which $0.8G$ are steam, and $0.2G$ water. The pressure is 1.5 atmospheres. What will be the specific steam weight x_1 when the steam has lost so much heat that the pressure is only $\frac{1}{10}$th of an atmosphere?

For 1½ atmospheres $u = 1.126$ (Table II.)

" $\frac{1}{10}$ " $u = 14.551$,

hence

$$x_1 = \frac{1.126}{14.551} \times 0.8 = 0.0774 \times 0.8 = 0.06192 \text{ kilograms.}$$

Hence the entire steam quantity at the end is $0.0619G$ kilograms, and the water quantity is $(1 - 0.0619)G = 0.9381G$ kilograms. There is then $0.8 - 0.0619 = 0.7381G$ kilograms of steam condensed. For English units the result is the same.

What is the amount of heat Q which must be imparted or abstracted from the mixture per kilogram?

Since, in this case, there is no outer work, all changes affect only the inner work. If then q is the heat of the liquid, ρ the inner vaporization heat at the beginning, and q_1 and ρ_1 at the end,

$$Q = q - q_1 + \rho x - \rho_1 x_1$$

or

$$Q = q - q_1 + \rho x - \rho_1 \frac{u}{u_1} x$$

$$= q - q_1 + xu\left(\frac{\rho}{u} - \frac{\rho_1}{u_1}\right). \quad . \quad \text{(XXXV.)}$$

EXAMPLE.

What is the amount of heat abstracted in the preceding example?

For 1½ atmospheres

$$q = 112.41 \quad \frac{\rho}{u} = 433.$$

For $\frac{1}{10}$ atmosphere $\quad q_1 = 46.28 \quad \frac{\rho_1}{u_1} = 37,$

Hence $Q = 112.41 - 46.28 + 0.8 \times 1.126\,(433 - 37) = 66.13 + 0.9 \times 396$
$= 422$ heat units per kilogram.

For English units we have $q = 202.33 \; \frac{\rho}{u} = 48.6$; $q_1 = 83.8 \; \frac{\rho_1}{u_1} = 4.16$;
$u = 18.03$; and hence $Q = 760$ heat units per pound.

The preceding finds application in the condenser of a steam engine. Let AB, Fig. 78, be the steam cylinder, C the piston, EF the condenser communicating with AB by the pipe and cock D. In steam engines, usually, communication of the cylinder with the condenser is opened before the end of the stroke. Thus, for example, when the piston is at C, and still going up, communication may be opened. Immediately the hot steam of high pressure in the cylinder mixes with the colder steam of lower pressure in the condenser, so that almost at once there is a medium pressure in both vessels. We shall see presently how to calculate this medium pressure. Since now the condenser is kept always cool either by water applied on the outside (surface condensation), or by water injected (jet condensation), this mean pressure quickly sinks, owing to the abstraction of heat, and we may assume that this fall of pressure takes place while the piston hovers at the end of its stroke. Heat is thus abstracted under constant volume. The steam in the entire space (cylinder and condenser) has now only the pressure corresponding to the temperature of the condenser, and the cylinder sides are kept hot. This was, as we know, the reason which led Watt to employ separate condensation, and it constitutes, indeed, the chief value of his discovery, regarded from an economical stand-point.

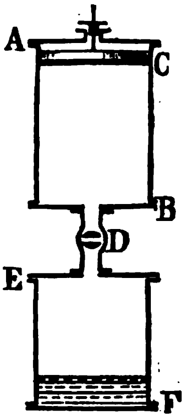

FIG. 78.

The question arises, how many kilograms of cooling water are necessary for each kilogram of mixture in our last example, if this water is heated, from say, $t = 15°$ to $t_1 = 35°$? We have from Equation II., page 390, for the heat of the liquid at 15°, $q' = 15.005$, and for that at 35°, $q'' = 35.037$. If G_1 kilograms of water are required for abstracting the $422\,G$ heat units, we have

$$G_1(35.037 - 15.005) = 422\,G,$$

and hence the ratio of the weight of cooling water to that of the steam is

$$\frac{G_1}{G} = \frac{422}{35.037 - 15.005} = \frac{422}{20.032} = 21.$$

Let us now investigate what amount of heat should be imparted to a mixture of steam and liquid in order that the pressure and temperature may rise to a certain point.

Let the initial steam weight be again xG kilograms, the water weight $(1 - x)G$ kilograms. At the end of the heat addition let there be x_1G kilograms of steam, and $(1 - x_1)G$ of liquid. In the beginning let the heat of the liquid be q and the inner latent heat ρ; at the end, q_1 and ρ_1. Then we have for the amount of heat imparted, for each kilogram

$$Q = q_1 - q + ux\left(\frac{\rho_1}{u_1} - \frac{\rho}{u}\right) \text{ heat units,}$$

or for G kilograms

$$Q = \left[q_1 - q + ux\left(\frac{\rho_1}{u_1} - \frac{\rho}{u}\right)\right] G \text{ heat units.}$$

By the aid of this formula we can solve a question of great practical interest, viz.:

In what time will the pressure of steam in a boiler rise by a certain amount (say, to double or treble), when from a given instant no more steam is drawn off?

Let us assume that there are Q_m heat units imparted every minute to the boiler, and that at the moment of closing the valve, there are xG kilograms of steam, and after Z minutes there are x_1G kilograms; the initial pressure being p, and that after Z minutes p_1. Since in Z minutes ZQ_m heat units are imparted, we have

$$ZQ_m = Q_1, \quad \text{or} \quad Z = \frac{Q_1}{Q_m}.$$

If in place of Q_1 we insert the value above, we have

$$Z = \frac{G}{Q_m}\left[q_1 - q + ux\left(\frac{\rho_1}{u} - \frac{\rho}{u}\right)\right].$$

This expression may be simplified. From page 490,

$$Z = \frac{G}{Q_m}(q_1 - q + \rho_1 x_1 - \rho x) = \frac{1}{Q_m}\left[(q_1 - q)\,G + (\rho_1 x_1 - \rho x)G\right] \qquad \text{(XXXVI.)}$$

Now, $(q_1 - q)\,G$ is the excess of the sensible heat over that at the beginning, and $(\rho_1 x_1 - \rho x)\,G$ is the excess of the inner latent heat. Since, however, ρ_1 and ρ differ by less than q_1 and q, and the steam weights x and x_1 are very small compared to the entire weight G of water and steam, especially when we remember that the boiler is about $\frac{2}{3}$ds full of water, we can neglect $(\rho_1 x_1 - \rho x)G$ in comparison with $(q_1 - q)G$, and then

$$Z = \frac{G}{Q_m}(q_1 - q) \quad . \quad . \quad . \quad \text{(XXXVII.)}$$

If c is the mean specific heat of water, and t_1 and t the initial and final temperatures, we have very nearly

$$Z = \frac{G}{Q_m}(t_1 - t)c. \quad . \quad . \quad \text{(XXXVIII.)}$$

This very simple expression shows, that the time in which the pressure or temperature of the steam in the boiler rises by a certain amount is proportional to the weight of the water, or capacity of the boiler, as well as to the difference of temperature, and inversely proportional to the heat (Q_m) imparted.

This is in fact evident, for it is clear that the time within which the temperature rises a certain amount must be greater the greater the weight of the entire mixture, and that for this rise of temperature less time will be required the more heat is imparted in a unit of time.

The heat imparted to a given mass of water in a unit of time depends, however, also upon the extent of heating surface, as well as upon the intensity of the heating. The first consists, in every boiler, of two parts, the direct and indirect heating surface. The direct heating surface is that directly in contact with the fire, the indirect is that in contact with the heated gases. Boilers which are required to generate steam quickly, and yet not hold much water, such as locomotive boilers, etc., possess a relatively great direct heating surface. The same weight of water thus receives in the same time a much greater amount of heat than in boilers where the direct heating surface is small in comparison to the indirect, and which are therefore larger and contain more water, as in stationary engines. In a locomotive boiler, therefore, the steam pressure rises much quicker than in that of a stationary engine.

We may illustrate the preceding by an example.

EXAMPLE.

The heating surface of a cylindrical steam boiler is 18 square meters (or 193.75 sq. feet; according to Zeuner, it is about 15 H. P.). The contents are 11 cubic meters (or 388.48 cubic feet), of which 0.6 are water and the rest steam. When the engine is in ordinary action, the boiler generates every hour 25 kilograms (or 55.115 pounds) of steam for every square meter of heating surface, of 5 atmospheres' pressure. In how many minutes will the pressure rise to 10 atmospheres ? First we have for the steam weight generated per minute,

$$\frac{18 \times 25}{60} = 7.5 \text{ kilograms.}$$

or

$$\frac{193.75 \times 55.115}{60} = 16.534 \text{ pounds.}$$

If this steam is generated from water at 15° (or 59° Fahrenheit), we can easily calculate the heat units necessary for its generation.

Since in the present case (when the machine is in motion) the heat has to perform outer work as well as inner, we have for the total heat which must be imparted in order to raise 1 kilogram of water at t degrees into 1 kilogram of steam at t_1 degrees,

$$W = q_1 - q + r = q_1 - q + \rho + Apu.$$

In our case $t_1 = 152.2$, $q_1 = 153.74$, $q = 15.005$, $\rho = 454.99$, and $Apu = 44.19$.

Hence $W = 153.74 - 15.005 + 454.99 + 44.19 = 637.92$ heat units.

or $W = 276.73 - 27 + 818.99 + 79.54 = 1148.28$ heat units English.

For 7.5 kilograms, therefore, we must have

$$7.5 \times 637.92 = 4784.5 \text{ heat units.}$$

or

$$16.534 \times 1148.26 = 18980.74 \text{ heat units English.}$$

This is then the heat imparted per minute to the water. It represents Q_m in Equation XXXVI. Since, further, the contents of the water space is $0.6 \times 11 = 6.6$ (or 233 cubic feet), the water weight is $= 6600$ kilograms (or 14562.5 pounds). The steam weight can be disregarded, and we have thus $G = 6600$ kilograms. If now we take $c = 1.022$, we have

$$Z = \frac{6600}{4784.5}(180.31 - 152.22) \times 1.022,$$

or

$$Z = \frac{14562.5}{18980.74}(356.56 - 305.99) \times 1.022,$$

where 180.31 is the temperature of the water at 10, and 152.22 that at 5 atmospheres. We have, therefore,

$$Z = 39.71 \text{ minutes.}$$

If we assume that a locomotive boiler furnishes in the same time 3.3 times as much steam, which is in general not far from correct, the time in raising the pressure from 5 to 10 atmospheres is only

$$\frac{39.71}{3.3} = 12 \text{ minutes.}$$

Fairbairn found the time required to raise the water of a locomotive boiler from t to t_1, to be given by the empirical formula

$$Z = 0.405\ (t_1 - t),$$

where Z is the number of minutes. If we take $t_1 = 180.31$ and $t = 152.22$, we have

$$Z = 0.405 \times 28.09 = 11.38 \text{ minutes},$$

which agrees well with our result.

We should bear in mind, that the steam pressure increases in a much more rapid ratio than the temperature, and hence in a boiler with high pressure less time is necessary for a certain increase than in a boiler with low pressure. Thus, for instance, let us suppose the normal pressure in the preceding example to be 9 instead of 5 atmospheres, and then see how many minutes (Z) are necessary for a rise of 5 atmospheres, from 9 to 14. We have now for t_1, 195.53, and $t = 178.02$; hence

$$Z = \frac{6600}{4784.5}\ (195.53 - 178.02) \times 1.022 = 24.7 \text{ minutes},$$

or much less time than in the first case. In order to conclude, therefore, whether the empirical formula of Fairbairn, applied to locomotives, gives results in accord with our formula, we must know for what mean pressures it is made out.

It is also easy to see from our tables that an increase of pressure of double, treble, etc., takes place in a shorter time for low pressures than for high.

The preceding calculations show that the time in which the pressure in the boiler of a stationary engine increases to double or treble when the steam pipe is closed, and the steam generation is as when in use, is tolerably great, even when heat is imparted as during ordinary action of the engine. If, when the engine is not in action, the fire is left to itself and only fed enough to keep it going, the time will be much greater. In stationary engines, then, there is less danger of explosion from this cause. In locomotives and such boilers, which have a large and direct heating surface, and the fire is kept up full, the danger is greater.

Mixture of Steam Quantities when in Different Conditions.—In the preceding we have considered the change of condition of a mixture of steam and liquid, under constant volume. Let us now consider two vessels containing a mixture of the same kind, but having in each a different temperature, pressure, etc.

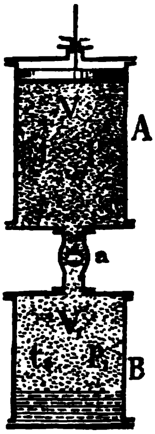

Fig. 79.

Let A have the volume V, and B the volume V_1. Both vessels are connected by the cock a. In the first we have xG kilograms of steam of the temperature t and pressure p; in the second, $x_1 G_1$ kilograms of steam of the temperature t_1 and pressure p_1. The water in A is then $(1-x)G$, and in B $(1-x_1)G_1$ kilograms. As soon as the cock a is opened the steam in the vessels mixes, and it is required to find the final condition after mixture, when heat is neither added nor taken away.

We have

$$V = (xu + \sigma)G \quad \text{and} \quad V_1 = (x_1 u_1 + \sigma)G_1.$$

After opening the cock, the total volume is

$$V_2 = V + V_1 = (x_2 u_2 + \sigma)(G + G_1),$$

where x_2 is the specific steam quantity after mixture. Hence

$$(xu + \sigma)G + (x_1 u_1 + \sigma)G_1 = (x_2 u_2 + \sigma)(G + G_1),$$

or, after reducing

$$(G + G_1)x_2 u_2 = Gxu + G_1 x_1 u_1 \; . \quad . \quad \text{(XXXIX.)}$$

In general, G_1 may be expressed in terms of G, so that $G_1 = nG$. We have then

$$(1 + n)x_2 u_2 = xu + nx_1 u_1 \quad . \quad . \quad . \quad . \quad \text{(XL.)}$$

We have in this equation two unknown quantities, x_2 and u_2. We must establish another equation between them.

Let the heat of the liquid in A be q, and in B, q_1, and the inner latent heats ρ and ρ_1. Then the amount of heat, measured in heat units, in the first vessel is

$$(q + x\rho)\, G,$$

and in the second

$$(q_1 + x_1\rho_1)\, G_1.$$

After mingling, let the heat of the liquid be q_2, and the inner latent heat be ρ_2. Then 1 kilogram of the mixture contains

$$q_2 + x_2\rho_2 \text{ heat units.}$$

Since, now, we had in A, G, and in B, G_1 kilograms of water and steam, we have still the same total amount, and hence the heat contained by the total mixture is

$$(G + G_1)(q_2 + x_2\rho_2) \text{ heat units.}$$

Since heat is neither added nor abstracted, we must have

$$(G + G_1)(q_2 + x_2\rho_2) = (q + x\rho)\, G + (q_1 + x_1\rho_1)\, G_1.$$

or, putting $G_1 = nG$,

$$(1 + n)(q_2 + x_2\rho_2) = q + x\rho + (q_1 + x_1\rho_1)\, n.$$

Hence

$$q_2 + x_2\rho_2 = \frac{q + x\rho + (q_1 + x_1\rho_1)n}{1 + n}.$$

Since all the quantities on the right are known, we can put the expression on the right, for the sake of brevity, equal to p, and thus have

$$q_2 + x_2\rho_2 = \pi.$$

Further, from Equation XL.,

$$x_2u_2 = \frac{xu + nx_1u_1}{1 + n}$$

or, denoting the expression on the right by π_1

$$x_2 = \frac{\pi_1}{u_2}, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(XLI.)}$$

hence

$$q_2 + \frac{\rho_2}{u_2}\pi_1 = \pi. \quad . \quad . \quad . \qquad \text{(XLII.)}$$

From Table II. we can find, by trial, the values of q_2 and $\frac{\rho_2}{u_2}$ in order that $q_2 + \frac{\rho_2}{u_2}p_1 = p$. As soon as q_2 and $\frac{\rho_2}{u_2}$ are known we can find at once the pressure p_2 and temperature t_2 of the specific steam weight x_2, that is, we can find the condition of the mixture after the cock is opened. An example will make the use of the formula clear.

EXAMPLE.

In the cylinder A we have G kilograms of pure saturated steam at 1.5 atmospheres, and in B, $G_1 = 24.380G$ kilograms of water and steam of $\frac{1}{10}$ atmosphere, of which 0.0095 kilograms are steam. What is the condition of the mixture after the cock is opened?

For the pressure 1.5 atmospheres, we have from Table II., $q = 112.41$, $\rho = 487.01$, and $u = 1.126$.

For the pressure 0.1 atmosphere we have

$$q_1 = 46.28, \quad \rho_1 = 538.85, \text{ and } u_1 = 14.55.$$

Also $\qquad n = 24.38$, and $x_1 = 0.0095$. Hence

$$x_2u_2 = \pi_1 = \frac{1.126 + 24.38 \times 0.0095 \times 14.55}{1 + 24.38} = \frac{1.126 + 3.376}{25.38} = 0.177.$$

Also

$$q_2 + x_2\rho_2 = \frac{112.41 + 487.01 + (41.28 + 0.0095 \times 538.85) \times 24.38}{1 + 24.38}$$
$$= 72.99 = \pi.$$

Accordingly,

$$q_2 + \frac{\rho_2}{u_2} \times 0.177 = 72.99.$$

Now from Table II., after a few trials, we hit upon $q_2 = 60.59$, for which $\frac{\rho_2}{u_2} = 70$, and these substituted give

$$60.59 + 70 \times 0.177 = 60.59 + 12.39 = 72.98,$$

a value almost exactly equal to 72.99. The heat of the liquid (q_2) after the mingling, is then 60.59 heat units. Hence, from the table, the temperature t_2 is 60.4°, and the pressure p_2 is 0.2 atmosphere. Since for this pressure $u_2 = 7.542$, we have from XLI., for the steam weight in each kilogram

$$x_2 = \frac{x_1}{u_2} = \frac{0.177}{7.542} = 0.0234 \text{ kilograms.}$$

Before the mingling, we had in A, G kilograms of steam and no water, and in B, $x_1 G_1 = 0.0095 \times 24.38G = 0.2316G$ kilograms steam and $(1 - 0.0095)$ $24.38G = 24.148G$ kilograms of water.

In both vessels, then, the total steam quantity was

$$(1 + 0.2316)G = 1.2316G \text{ kilograms.}$$

After mingling, the steam weight is

$$x_2 G + x_2 G_1 = x_2 G(1 + 24.38) = G \times 25.38 \times 0.0234 = 0.596G \text{ kilograms.}$$

Hence

$$(1.2316 - 0.596)G = 0.6356G \text{ kilograms}$$

of steam have been condensed.

We can also find the ratio of the volumes of the two vessels. If the volume of A is V and of B is V_1, we have

$$\begin{aligned} V : V_1 &= G(xu + \sigma) : G_1(x_1 u_1 + \sigma) \quad . \quad . \quad . \quad \text{(XLIII.)} \\ &= xu + \sigma : 24.38(x_1 u_1 + \sigma) \\ &= 1.127 : 3.388 \\ &= 1 \text{ to } 3 \text{ very nearly.} \end{aligned}$$

The preceding, together with what has been said on page 448 and the following, forms the basis of the theory of the condenser. We have already referred to the action of this apparatus. We noticed that the cylinder was put in communication with the condenser, when the piston is near the end of its stroke. We have, then, a sudden mixing of the hot and high pressure cylinder steam with the colder and lower pressure condenser steam. The above calculation includes this case. It shows how, from the pressure of the cylinder steam and the condenser steam, we can find the mean pressure of the steam in both vessels. This pressure then falls rapidly down to that of the condenser, by reason of the cooling produced by the jet or by the surrounding water. The calculations in the first part of this chapter, then, enable us to calculate the heat thus abstracted, under constant volume.

If the condenser is fitted with a gauge, we should see, at the moment of communication with the cylinder, a sudden rise of pressure, lasting but for an instant, and then a quick return to the condenser pressure.

It will now be easy to give a complete theory of the condenser. There are two kinds in use. The first presents a very great surface which is continually in contact with water, so that it is kept cool. The steam is condensed without coming into direct contact with the water. This is called the surface condenser. The other consists of a vessel, not only surrounded by cold water, but into which cold water is injected. The steam is thus condensed by direct contact and mixture with the water. It is called the jet condenser. In the first, we have only to cause a circulation of water by means of a pump, and by means of another pump to restore the condensed steam to the boiler, and if there is no loss by leakage of steam, we have a complete cycle process. In the jet condenser, we have not only to remove the condensed steam but also the water injected. Since this is in weight sometimes more than 20 times that of the steam, the pump must be much greater than for the surface condenser. The injection water also contains air, which must also be removed by the pump. For this reason it was called by Watt the "air pump."

QUESTIONS FOR EXAMINATION.

If heat is imparted to or abstracted from a mixture of steam and water, the volume of which remains constant, what is the new weight of steam? What is the heat imparted or abstracted? What amount of heat should be imparted to a mixture of steam and water in order that the pressure and temperature may rise to a certain point? In what time will the pressure of steam in a boiler rise by a certain amount, when no more steam is drawn off? If in two vessels communicating by a cock, we have mixtures of steam and water of given pressure, show how to find the condition of the mixture after the cock is opened.

CHAPTER XX.

THEORY OF THE CONDENSER.

A. THEORY OF THE SURFACE CONDENSER.

WHEN the piston *K* has reached the upper end of its stroke, the slide *S* has opened the port *o*, and the steam escapes into the condenser *CD*; at this moment, therefore, the cylinder steam mixes with the low steam in the condenser. The pressure in both spaces may thus rise to double the ordinary pressure in *CD*. We can calculate it as in the preceding chapter. But now, while the piston lingers at the end of its stroke, the pressure falls by reason of the cooling effect of the condenser. Heat is abstracted under constant volume, and we can find the final condition, as well as the heat abstracted, from Equation XXXV. Then the piston *K* descends, and drives the steam in the cylinder, now at the condenser pressure, under that pressure, into the condenser. Here, then, we have mechanical work under constant pressure. This work is transformed into heat, and this heat is also absorbed by the condenser.

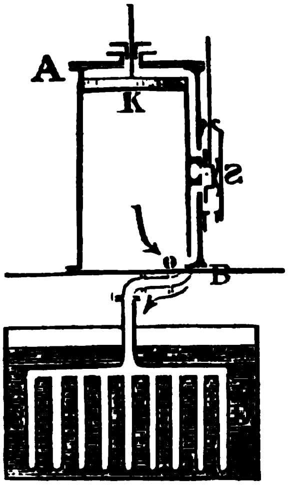

FIG. 80.

We see, therefore, that in every stroke there are three stages:

1st. Mixture of cylinder with condenser steam, and the attainment of a mean pressure.

2d. Lowering of this pressure by cooling under constant volume.

3d. Abstraction of heat under constant temperature, while the low cylinder steam is forced into the condenser.

If, therefore, we suppose in the cylinder AB, before opening of the port, G kilograms of steam and water, we have, in each kilogram of the mixture, x kilograms of steam and $1 - x$ kilograms of water. The steam quantity in AB is then xG kilograms, and the water quantity $(1 - x)\,G$ kilograms.

If we had only pure saturated steam, we should have $x = 1$, and the steam quantity in the cylinder would be G kilograms.

Suppose we have in the condenser CD, before the mingling, G_1 kilograms of steam and water, and in every kilogram of the mixture x_1 kilograms of steam and $1 - x_1$ of water. The steam weight in the condenser is then $x_1 G_1$, and the water weight $(1 - x_1)\,G_1$.

As soon, now, as we know the pressure of the steam in the cylinder and in the condenser, we can find, according to the preceding chapter, the condition of the steam in both spaces after the port is opened. We have

$$x_2 u_2 = \frac{Gxu + G_1 x_1 u_1}{G + G_1},$$

and

$$q_2 + x_2 \rho_2 = \frac{(q + x\rho)\,G + (q_1 + x_1 \rho_1)\,G_1}{G + G_1}.$$

If, as before, we denote the fraction on the right in the first equation by π_1, and in the second by π, we have

$$x_2 u_2 = \pi_1, \quad \text{or} \quad x_2 = \frac{\pi_1}{u_2},$$

and

$$q_2 + \frac{\rho_2}{u_2}\pi_1 = \pi,$$

and thus, by a few trials with the table, can find q_2 and the pressure p_2 and the temperature t_2 of the mixture directly the port is opened. This pressure p_2 is, of course, greater than p_1 and less than p.

Heat now is abstracted from the mixture, until the pressure sinks from p_2 to p_1. In order to find this heat we have Equation XXXV.

In order to impart to 1 kilogram of steam and water, of which x kilograms are steam, whose temperature is t and pressure p, a pressure p_1 and temperature t_1, we must add a quantity of heat (or subtract) equal to

$$Q = q - q_1 + \rho x - \rho_1 x_1,$$

or

$$Q = q - q_1 + xu\left(\frac{\rho}{u} - \frac{\rho_1}{u_1}\right).$$

If we have in the beginning $G + G_1$ kilograms, at the pressure p_2 and temperature t_2, the steam weight per kilogram being x_2, and, by abstracting heat, reduce the pressure to p_1, and the temperature to t_1, then we have

$$Q_1 = \left[q_2 - q_1 + x_2 u_2\left(\frac{\rho_2}{u_2} - \frac{\rho_1}{u_1}\right)\right](G + G_1),$$

or

$$Q_1 = (q_2 + x_2\rho_2)(G + G_1) - \left(q_1 + x_2 u_2\frac{\rho_1}{u_1}\right)(G + G_1).$$

If, as on the preceding page, we put

$$(q_2 + x_2\rho_2)(G + G_1) = (q + x\rho)\,G + (q_1 + x_1\rho_1)\,G_1,$$

and

$$x_2 u_2(G + G_1) = Gxu + G_1 x_1 u_1,$$

we have, after reducing

$$Q_1 = \left[q - q_1 + xu\left(\frac{\rho}{u} - \frac{\rho_1}{u_1}\right)\right]G.$$

The difference $\frac{\rho}{u} - \frac{\rho_1}{u_1}$ denotes the excess of inner latent heat of 1 cubic meter of steam in the cylinder AB before the port is opened, above that possessed by 1 cubic meter of steam in the condenser. The product $xu\left(\frac{\rho}{u} - \frac{\rho_1}{u_1}\right)G$ is, therefore, the excess of the inner latent heat possessed by the *entire* steam volume in the cylinder, before the port is opened, over that in the condenser. In like manner, $(q - q_1)\,G$ is the excess of sensible heat of the mixture in *AB* over that in the condenser. In other words, the entire expression on the right gives the excess of heat in the mixture in *AB* before the port is opened over that in *CD*. The heat quantity, therefore, which must be abstracted, *after* the mingling, from the entire contents of both vessels, in order that the pressure p_2 may sink to the condenser pressure p_1, is equal to that which would have to be withdrawn from the contents of *AB* before the mingling, in order to reduce the pressure from p to p_1. This is, in fact, evident, for if we first abstracted this heat from the mixture in *AB* and then opened communication with the condenser, the final condition would remain unchanged. The course which we have followed, however, corresponds perfectly to the actual changes which take place, and explains why we have, at each stroke of the piston, sudden variations in the condenser gauge.

We come now to the third part of the process, the abstraction of heat under constant temperature, while the now low pressure steam in the cylinder is driven out under constant pressure into the condenser.

The volume of the cylinder *AB* is

$$G\,(xu + \sigma) \text{ cubic meters,}$$

or if σ is very small in comparison to xu,

$$Gxu.$$

The inner latent heat in every cubic meter of steam after the mingling is $\frac{\rho_1}{u_1}$; in Gxu cubic meters we have then

$$Gxu\,\frac{\rho_1}{u_1}.$$

This heat must be abstracted. If we suppose the piston *K* to have an area of 1 square meter, the distance through which it must go is Gxu meters. In forcing the steam then under constant pressure p_1 into the condenser, the work performed is $Gxup_1$, and this corresponds to the heat $AGxup_1$.

Hence in this third period we must abstract

$$Q_3 = Gxu\,\frac{\rho_1}{u_1} + GxuAp_1 = Gxu\left(\frac{\rho_1}{u_1} + Ap_1\right) \text{ heat units.}$$

The total heat abstracted is then

$$Q = Q_1 + Q_3 = \left[q - q_1 + xu\left(\frac{\rho}{u} - \frac{\rho_1}{u_1}\right)\right] G + xu\left(\frac{\rho_1}{u_1} + Ap_1\right) G$$

$$= \left[q - q_1 + xu\left(\frac{\rho}{u} + Ap_1\right)\right] G. \quad . \quad . \quad \text{(XLIV.)}$$

This formula can be simplified. If we suppose at first only pure saturated steam without water, $x = 1$, also Ap_1 is very small and may be neglected, and then we have

$$Q = (q - q_1 + \rho)\,G.$$

Since $q + \rho$ is the "steam heat" J, we have also

$$Q = (J - q_1)\,G.$$

For low temperatures we can assume that q_1 is equal to the temperature. If, then, instead of q_1 we insert the temperature of the condensed steam, or generally the mean temperature of the condenser, t_1, we have

$$Q = (J - t_1)\,G \text{ heat units.}$$

If we assume that we require for condensing the steam, n times as much cold water as steam, and if the temperature of this water is raised from t_0 to t_1' degrees, where t_1' is always less than t_1, then the heat absorbed by the water is, provided that we again put the temperature in place of the heat of the liquid,

$$nG\,(t_1' - t_0).$$

We have, therefore,

$$nG\,(t_1' - t_0) = (J - t_1)\,G,$$

or

$$n = \frac{J - t_1}{t_1' - t_0} \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XLV.)}$$

As the steam heat J for those temperatures which occur in the steam engine varies but little (it varies hardly 23 heat units between 100° and 200°), we may take a mean value for it. Taking then, $J = 600$, we have the practical formula

$$n = \frac{600 - t_1}{t_1' - t_0} \quad . \quad . \quad . \quad . \quad . \quad \text{(XLVI.)}$$

or for Fahrenheit degrees, $n = \frac{1080 - (t_1 - 32)}{t_1' - t_0}$.

EXAMPLE.

How much more water than steam must be used in a surface condenser, when the water enters with a temperature $t_0 = 15°$ and departs with $t_1' = 35°$, the mean temperature of the condenser being $t_1 = 46.2°$, which corresponds to a pressure of $\frac{1}{10}$ atmosphere ?

We have

$$n = \frac{600 - 46.2}{35 - 15} = \frac{553.8}{20} = 27.7 \text{ times as much.}$$

If the steam used per stroke is 0.15 kilograms, the water quantity is

$$0.15 \times 27.7 = 4.155 \text{ kilograms}$$

per stroke, or since 1 kilogram of water occupies a space of $\frac{1}{1000}$ cubic meters,

$$4.155 \times \frac{1}{1000} = 0.00416 \text{ cubic meters.}$$

B. THEORY OF THE JET CONDENSER.

Let A be the steam cylinder with the piston KK. The mixture of steam and water in it weighs G kilograms. In every kilogram of this mixture there are x kilograms of steam and $1 - x$ of water. The temperature is t, and the pressure is p.

B is the condenser. In this we have G_1 kilograms, of the pressure p_1 and temperature t_1, and each kilogram contains x_1 kilograms of steam.

Finally, C is a vessel filled with cold water at the temperature t_0. It holds just as much water as is necessary to condense the steam used per stroke, viz., G_0 kilograms. Upon the surface of the water is a piston which is pressed by the atmosphere p_0.

Both A and C communicate with the condenser by cocks a

and *b*. Let both these be simultaneously open. The contents of *A* and *C* suddenly mingle, and the cold water injected reduces the pressure in both *A* and *B* to the mean pressure of the condenser p_1. The piston *KK*, as well as that in *C*, now descends. When *KK* reaches the end of its stroke, so has the piston in *C*, and all the water has entered the condenser, and all the steam has been condensed.

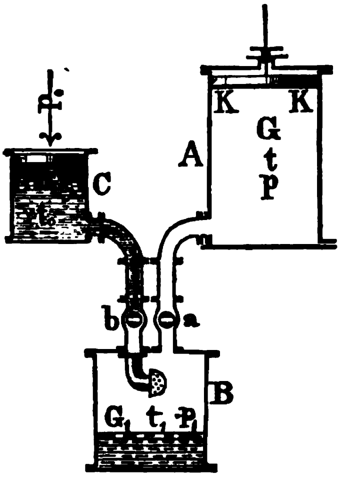

FIG. 81.

The total heat in *A* is

$$(q + x\rho)\,G,$$

and in *B*,

$$(q_1 + x_1\rho_1)\,G_1,$$

and in *C*,

$$q_0\,G_0.$$

Hence the heat in all three vessels is

$$(q + x\rho)\,G + (q_1 + x_1\rho_1)\,G_1 + q_0G_0.$$

After the mixture, all the steam and water is in the condenser, and we have there a mixture of steam and water weighing

$$G + G_1 + G_0 \text{ kilograms.}$$

In each kilogram of this mixture there is much more water, and hence much less steam, than before *a* was opened there was in *A* and *B*. Let the steam weight in each kilogram be x_1', then x_1' is less than x_1, and we have in the $G + G_1 + G_0$ kilograms

$$x_1'\,(G + G_1 + G_0) \text{ kilograms of steam,}$$

the pressure of which is p_1 and temperature t_1. Since the heat of the liquid is q_1, and the inner latent heat ρ_1, the heat in the condenser is

$$(q_1 + x_1'\rho_1)\,(G + G_1 + G_0),$$

or, denoting $G + G_1 + G_0$ by M,

$$(q_1 + x_1'\rho_1)\ M \text{ heat units.}$$

This is not all the heat in the condenser.

While the piston KK moves down, it overcomes the constant pressure p_1 through a certain distance, and therefore performs mechanical work in compressing the steam. This work generates heat. Since the cylinder volume is

$$(xu + \sigma)\ G \text{ cubic meters,}$$

the work performed is

$$(xu + \sigma)\ Gp_1,$$

or, neglecting σ,

$$xu\ Gp_1.$$

The heat equivalent to this work is

$$Ap_1xuG.$$

Mechanical work is also performed by the descent of the piston in C. If we denote, as always, the volume of 1 kilogram of water by σ, then since the atmospheric pressure is p_0, the work performed is

$$G_0\sigma p_0,$$

and the heat equivalent is

$$A\,G_0\sigma p_0 \text{ heat units.}$$

The increase of heat due to these two causes is therefore

$$Ap_1xuG + A\,G_0\sigma p_0.$$

If we add this heat to that which existed before the mingling, in the three vessels, we have the heat in the condenser. Hence,

$$(q + x\rho)G + (q_1 + x_1\rho_1)G_1 + G_0q_0 + A(Gp_1xu + G_0\sigma p_0) = (q_1 + x_1'\rho_1)M.$$

Now, as a matter of fact, the heat due to the work performed is very small, so that we can still have

$$(q + x\rho)\,G + (q_1 + x_1\rho_1)\,G_1 + q_0 G_0 = (q_1 + x_1'\rho_1)\,M.$$

Since now $(q_1 + x_1'\rho_1)M = q_1\,(G + G_1 + G_0) + x_1'\rho_1 M$, we have, after reduction,

$$(q - q_1 + x\rho)\,G + x_1\rho_1 G_1 - x_1'\rho_1 M = (q_1 - q_0)\,G_0.$$

Here x_1' is unknown, but it can be easily proved that $x_1\rho_1 G_1 = x_1'\rho_1 M$.

Before the mingling, the volume v of 1 kilogram of mixture in the condenser was

$$v = x_1 u_1 + \sigma,$$

and since there were G_1 kilograms, the volume (V_1) in the condenser was

$$V_1 = G_1\,(x_1 u_1 + \sigma).$$

After the mingling, the volume of 1 kilogram is

$$v = x_1' u_1 + \sigma.$$

Since the pressure and temperature are the same, u_1 is the same. But after the mingling, there are M kilograms of steam and water in the condenser, and hence the volume is

$$V_1 = M\,(x_1' u_1 + \sigma).$$

Hence,

$$G_1\,(x_1 u_1 + \sigma) = M\,(x_1' u_1 + \sigma) \text{ or } G_1 x_1 = M x_1'.$$

The product $G_1 x_1$ is the steam weight in the condenser at the beginning, and $M x_1'$ that at the end. As, then, this steam weight is constant, all the steam in the cylinder must be condensed. Since now

$$x_1 G_1 = x_1' M, \text{ we have also}$$

$$x_1\rho_1 G_1 = x_1'\rho_1 M,$$

and hence the equation above becomes

$$(q_1 - q_0)\,G_0 = (q - q_1 + x\rho)\,G.$$

We have, therefore, the water required for condensation

$$G_0 = \frac{(q + x\rho - q_1)\,G}{q_1 - q_0} \quad . \quad . \quad . \quad \text{(XLVII.)}$$

If we make here $x = 1$, and put for $q + \rho$ the steam heat J, and for q_1 and q_0 the temperatures t_1 and t_0, we have

$$G_0 = \frac{J - t_1}{t_1 - t_0}\,G,$$

or if $G_0 = nG$

$$n = \frac{J - t_1}{t_1 - t_0} \quad . \quad . \quad . \quad . \quad \text{(XLVIII.)}$$

This equation differs from XLV. in that t_1 in the denominator is the temperature of the condenser water, while in XLV. t_1' is the temperature of the heated condensing water, which is always less than that of the water of condensation. Since here also $J = 600$ about, we have

$$n = \frac{600 - t_1}{t_1 - t_0} \quad . \quad . \quad . \quad . \quad . \quad \text{(XLIX.)}$$

or for Fahrenheit degrees,

$$n = \frac{1080 - (t_1 - 32)}{t_1 - t_0}.$$

EXAMPLE.

A high pressure steam engine using steam of 5 atmospheres has a condenser in which the average pressure is 0.1 atmosphere. The cooling water has a temperature of 18°. How much more water than steam must be used ?

For $\frac{1}{10}$ atmosphere, $t_1 = 46°$, hence

$$n = \frac{600 - 46}{46 - 18} = \frac{554}{28} = 20 \text{ times as much.}$$

If then the steam used per stroke is 0.12 kilograms, we have per stroke $20 \times 0.12 = 2.40$ kilograms of water necessary, or $\frac{2.40}{1000} = 0.0024$ cubic meters.

We have already noticed the fact that the air contained in the injection water is set free. This air increases the pressure in the condenser, on the average about 0.05 atmosphere. The amount of injection water is not thereby increased, but the air pump must have greater dimensions in order that it may remove both air and water from the condenser.

CHAPTER XXI.

THE FLOW OF STEAM AND HOT WATER THROUGH ORIFICES.

A. Flow of Steam through Orifices.

As in Chapter X., Part I., let *ABCD* be a large vessel with a narrow discharge pipe *EF*. In the first is a piston *HJ*, of *F* sq. meters cross-section, and in the pipe a smaller one *G*, of *f* sq. meters. Upon *HJ* we have the pressure of *p* kilograms per square meter. The pressure upon *G* is p_1 kilograms per square meter, and $p_1 < p$. Suppose the space between the pistons filled with some liquid, as water. Let the piston *HJ* move through the distance *s* per second, and *G* through s_1. Then the work of the first piston is *Fsp*, and of the second fs_1p_1.

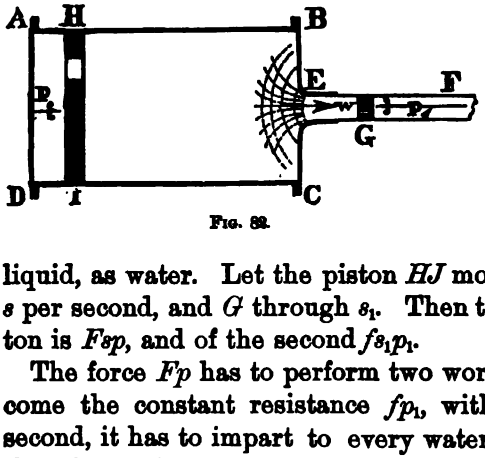

Fig. 82.

The force *Fp* has to perform two works. First, it must overcome the constant resistance fp_1, with uniform velocity, and second, it has to impart to every water particle which enters the pipe an increased velocity, so that the velocity *s* becomes s_1. Since the volume fs_1 issues through *E* per second, the weight of this volume is, if 1 cubic meter weighs γ kilograms, $fs_1\gamma$. To increase the velocity of this from *s* to s_1, requires the work

$$\frac{s_1^2 - s^2}{2g} fs_1\gamma.$$

The force *Fp*, which drives the piston *HJ*, has then to perform not only the work fs_1p_1, but also the work $\frac{s_1^2 - s^2}{2g}fs_1\gamma$. Hence

$$Fsp = fs_1p_1 + \frac{s_1^2 - s^2}{2g}fs_1\gamma,$$

or

$$Fsp - fs_1p_1 = \frac{s_1^2 - s^2}{2g}fs_1\gamma \quad . \quad . \quad . \quad . \quad . \quad \text{(L.)}$$

Since, also, $Fs = fs_1$, we have

$$p - p_1 = \frac{s_1^2 - s^2}{2g}\gamma \quad . \quad . \quad . \quad . \quad . \quad \text{(LI.)}$$

This is a well-known formula of hydraulics of frequent application. If we assume the vessel *ABCD* very great, with reference to the pipe *EF*, or the diameter very great in comparison to that of the orifice *E*, the distance s is very small in comparison to s_1, and we have

$$p - p_1 = \frac{s_1^2}{2g}\gamma,$$

or putting w^2 in place of s_1^2,

$$p - p_1 = \frac{w^2}{2g}\gamma \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(LII.)}$$

We omit, of course, here the fact that the pressure of the water particles above the orifice, due to their own weight, assists the force Fp.

Now let us suppose that instead of water between the piston *HJ* and *G*, we have saturated steam. Let the pressure of this steam in *ABCD* be p. The piston *HJ* is pressed, as before, from left toward the right by the force p. If here also the pressure p_1 upon the piston *G*, in the pipe *EF*, is less than p, the efflux of the steam is essentially different from that of water, and is completely similar to that of a gas. The steam expands when it is subjected, on one side, to a less pressure than on the other. For this reason, one kilogram of steam in the pipe *EF* occupies a greater space than a kilogram in *ABCD*, that is, the specific volume v in *EF* is different from that in

ABCD. The distance between the molecules when they pass the orifice E is greater, and they have then been moved apart. This motion can be caused by heat received from without, or by the heat of the steam itself. In the latter case, a part of the molecular motion of the steam is converted into outer motion. In the case of permanent gases, the pressure p_1 and the specific volume v_1 in the pipe can be different from that in the vessel, while still the temperature is unchanged (Chap. X., Part I.) With steam this cannot be, so long at least as it is in the saturated condition. For this condition, so long as the temperature remains constant, the pressure is unchanged.

Let us conceive that the expansion of the steam while passing through E extends to some distance, as shown by the dotted lines. As the steam molecules reach the first arc, their expansion, or the increase of their mutual distances, is still small. As they approach the innermost arc, it is greater, until in the pipe EF the volume is v_1. Since, then, the mutual distance of the molecules increases gradually, the pressure exerted by any molecule at any instant upon the next one which lies nearer the orifice E is greater than the counter-pressure of this last molecule only by an infinitely small amount. If, then, the expansion follows any given law, we can calculate the work necessary for this expansion.

If, for example, we suppose the expansion to follow the law of constant steam weight, heat must be imparted. We have seen (page 419) that the heat required by 1 kilogram of saturated steam, expanding according to this curve, varies with the temperature; for high temperatures it is less and for low temperatures greater. If the steam pressure, for example, in *ABCD* is 5 atmospheres, or $p = 5 \times 10334$ kilograms, and the pressure on G per square meter is 1×10334 kilograms, we have from Table III. for the heat imparted per kilogram during expansion,

$$200.46 - 148.47 = 51.99 \text{ heat units.}$$

(Compare the example, page 426.) The temperature falls from 152.2°, corresponding to 5 atmospheres, to 100°.

If we denote generally the heat which must be imparted, when the expansion of the steam at efflux follows the curve of constant steam weight, by P, then the work corresponding is $\frac{P}{A}$. It is this work, which in the efflux of steam (just as for gas) is to be added to the work Fsp (Equation L.), and which in combination with this work generates the velocity w and overcomes the constant pressure p_1 in the pipe EF with this velocity. If we assume that 1 kilogram of steam issues per second, and denote the volume of this kilogram when in the vessel $ABCD$, by v, and when in the pipe EF, by v_1, we have in Equation L., $Fs = v$ and $fs_1 = v_1$, and have then for the work necessary to impart the velocity w to each kilogram

$$pv + \frac{P}{A} - p_1v_1 = \frac{w^2}{2g} . 1,$$

since in Equation L., $fs_1\gamma$ can be put $= 1$.

The specific volume v or v_1 is given by

$$v = (xu + \sigma)$$
$$v_1 = (xu_1 + \sigma),$$

because the steam weight x is constant. For $x = 1$

$$v = u + \sigma$$
$$v_1 = u_1 + \sigma.$$

It is easy to see how, from the pressures p and p_1, by the aid of the above equations, the velocity w can be found. In practice, the case here considered occurs but seldom if at all, and we shall not, therefore, pursue it further.

Velocity of Efflux when Heat is neither Added nor Abstracted.—The efflux of steam from the safety-valve of a boiler, or from the cylinder of an engine, takes place without the addition of heat from without. Let us find then the velocity w under the assumption that heat is neither added nor abstracted by outer bodies. The expansion then follows the law of the adiabatic curve, and by the gradual change of pressure from p to p_1, the work of the molecules is (Equation XXVII.)

$$L_1 = \frac{q - q_1 + x\rho - x_1\rho_1}{A}.$$

Hence we have

$$pv - p_1v_1 + \frac{q - q_1 + x\rho - x_1\rho_1}{A} = \frac{w^2}{2g} \quad . \quad \text{(LIII.)}$$

Multiplying both sides by A, and putting $v = xu + \sigma$, and $v_1 = x_1u_1 + \sigma$,

$$q + x\rho + Apux - (q_1 + x_1\rho_1 + Ap_1x_1u_1) + A\sigma(p - p_1)$$
$$= A\frac{w^2}{2g} \quad . \quad . \quad \text{(LIV.)}$$

Here we can disregard the last member on the left. Also remembering that $x(\rho + Apu) = xr$, and $x_1(\rho_1 + Ap_1u_1) = x_1r_1$, we have

$$A\frac{w^2}{2g} = q - q_1 + xr - x_1r_1 \quad . \quad . \quad . \quad \text{(LV.)}$$

Since p, p_1 and x are known, q, q_1, r and r_1 are also known, and x_1 can be found from Equation XXV.,

$$\tau + \frac{xr}{T} = \tau_1 + \frac{x_1r_1}{T_1}$$

whence

$$x_1 = \left(\tau - \tau_1 + \frac{xr}{T}\right) \div \frac{r_1}{T_1}.$$

The values of τ and $\frac{r}{T}$ are given in Table III. If then x_1 is thus determined, we have

$$w = \sqrt{\frac{2g}{A}(q - q_1 + xr - x_1r_1)}.$$

Since $2g = 2 \times 9.81$ meters, and $\frac{1}{A} = 424$,

$$w = 91.2\sqrt{q - q_1 + xr - x_1r_1}, \quad . \quad . \quad \text{(LVI.)}$$

or for English units, $w = 222.8\sqrt{q - q_1 + xr - x_1r_1} =$ ft. per second.

Let us illustrate the above by an example.

EXAMPLE.

Steam issues from the safety-valve of a steam boiler, in which the pressure is 5 atmospheres, into the air. What is the velocity of efflux w, when we assume the efflux to take place without friction, and that only pure saturated steam, without admixture of water, approaches the orifice? How much steam is condensed during efflux?

We have evidently to answer the last question first. From Table III.,

$$\frac{r}{T} = 1.174, \quad \frac{r_1}{T_1} = 1.438, \quad \tau = 0.447, \quad \tau_1 = 0.314,$$

hence $x_1 = (0.447 - 0.314 + 1.174) \div 1.438 = 0.908$ kilogram.

For English units, $x_1 = (0.516 - 0.383 + 1.174) \div 1.438 = 0.908$ lb.

Hence during efflux, $1 - 0.908 = 0.092$ kilograms of steam are condensed. Further,

$$q = 153.74, \quad q_1 = 100.5, \quad r = 499.19, \quad r_1 = 536.5,$$

hence $q - q_1 + xr - x_1 r_1 = 153.74 - 100.5 + 499.19 - 0.908 \times 536.5 = 65.29$,

or for English units, $276.73 - 180.9 + 898.5 - 0.908 \times 965.7 = 117.48$;

and $w = 91.2\sqrt{65.29} = 91.2 \times 8.08 = 736.9$ meters per second,

or $w = 222.9\sqrt{117.48} = 2417.08$ ft. per second.

Transformation of the Preceding Equations.—The preceding formulæ can only be used when we have our tables at hand. Let us see if we can find w without them.

First, we can put for q and q_1 the temperatures t and t_1. Thus if we assume the specific heat of water as constant, and take it at 1.0224, which corresponds to about 145°, we have

$$q - q_1 = 1.0224\,(t - t_1) = 1.0224\,(T - T_1).$$

Also,

$$\tau = c \log \text{nat} \frac{T}{a} = 1.022 \log \text{nat} \frac{T}{a},$$

and

$$\tau_1 = c \log \text{nat} \frac{T_1}{a} = 1.022 \log \text{nat} \frac{T_1}{a},$$

hence

$$\tau - \tau_1 = 1\ 022 \log \text{nat} \frac{T}{T_1} = 2.353 \log \frac{T}{T_1}.$$

Further, according to Clausius (page 393),

$$r = 607 - 0.708t, \qquad r_1 = 607 - 0.708t_1.$$

Hence

$$x_1 = \left[2.353 \log \frac{T}{T_1} + \frac{x(607 - 0.708t)}{T}\right] \div \frac{607 - 0.708t_1}{T_1}.$$

Equation LVI. becomes

$$w = 91.2 \sqrt{1.022\,(t - t_1) + x\,(608 - 0.708t) - x_1\,(607 - 0.708t_1)} \qquad \text{(LVII.)}$$

For English units, put 1092.6 − 0.708 ($t - 32$) in place of 607 − 0.708t and 222.8 for 91.2. The result will give w in feet per second.

Let us calculate from these last two formulas x_1 and w for the last example. We have $t_1 = 100$, and $T_1 = 273 + 100 = 373$; hence

$$\frac{607 - 0.708t_1}{T_1} = \frac{607 - 70.8}{373} = 1.438.$$

Further,

$$2.353 \log \frac{T}{T_1} = 2.353 \log \frac{273 + 152.2}{373} = 2.353 \times 0.0569 = 0.1339.$$

Hence

$$\frac{x\,(607 - 0.708t)}{T} = \frac{607 - 0.708 \times 152.2}{273 + 152.2} = 1.174.$$

We have then

$$x_1 \doteq (0.1339 + 1.174) \div 1.438 = 0.909 \text{ kilograms.}$$

By means of the tables we found before $x_1 = 0.908$.
We have now

$$\begin{aligned} w &= 91.2 \sqrt{1.022 \times 52.2 + 499.24 - 0.909\,(607 - 70.8)} \\ &= 91.2 \sqrt{53.35 + 499.24 - 487.41} \\ &= 91.2 \sqrt{65.18} = 91.2 \times 8.07 = 735.98 \text{ meters.} \end{aligned}$$

This value agrees closely with that found before.

If the pressures p and p_1, or the corresponding temperatures, differ but little, we can put

$$2.353 \log \frac{T}{T_1} = \frac{T - T_1}{T_1}.$$

We should have then

$$x_1 = \left(\frac{T - T_1}{T_1} + \frac{xr}{T}\right) \times \frac{T_1}{r_1},$$

or

$$x_1 r_1 = T - T_1 + xr\frac{T_1}{T}.$$

If we insert this in LVI., and remember that $T - T_1 = t - t_1$, and that we have without great error $c\,(t - t_1) = t - t_1$, we have

$$w = 91.2\sqrt{\frac{xr}{T}(T - T_1)} \quad . \quad . \quad . \quad \text{(LVIII.)}$$

or finally,

$$w = 91.2\sqrt{\frac{x\,(607 - 0.708t)}{273 + t}(t - t_1)} \quad . \quad . \quad \text{(LIX.)}$$

For English units, put 222.8 in place of 91.2, 1092.6 – 0.708 $(t - 32)$ in place of $607 - 0.708t$ and 459.4 in place of 273. The result will give w in feet per second.

EXAMPLE.

With what velocity does the steam issue from the boiler of a low-pressure engine, the pressure being only 1.2 atmospheres, when $x = 1$?

In this case $t = 105.2°$ and $t_1 = 100$, hence

$$w = 91.2\sqrt{\frac{607 - 0.708 \times 102.2}{273 + 105.2} \times 5.2} = 246.24 \text{ meters per sec.}$$

In English units we have

$$w = 222.8\sqrt{\frac{1092.6 - 0.708 \times 189.806}{459.4 + 221.806} \times 9.806} = 806.54 \text{ feet per sec.}$$

Another Expression for the Velocity of Efflux.—We have made use above, page 515, of the expression

$$L_1 = \frac{q - q_1 + x\rho - x_1\rho_1}{A}$$

for the work of 1 kilogram of steam in expanding from the pressure p to p_1. On page 440, we have seen that this work is given by the expression

$$L_1 = \frac{pv}{m - 1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]$$

where the value of m depends not only upon p and p_1 but also upon the proportion of water in the mixture. If we have only pure saturated steam without water, m has the mean value 1.135.

We have then

$$\frac{w^2}{2g} = pv - p_1v_1 + \frac{pv}{m-1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]. \quad . \quad \text{(LX.)}$$

The law of the adiabatic curve, that is, the law of expansion of the steam during efflux, is

$$pv^m = p_1v_1^m,$$

hence

$$v_1 = \left(\frac{p}{p_1}\right)^{\frac{1}{m}} v \quad \text{and} \quad p_1v_1 = p_1^{1-\frac{1}{m}} p^{\frac{1}{m}} v.$$

Inserting this value of p_1v_1 and reducing,

$$\frac{w^2}{2g} = \frac{mpv}{m-1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]. \quad . \quad . \quad \text{(LXI.)}$$

or

$$w = 4.43\sqrt{\frac{mpv}{m-1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]}. \quad \text{(LXII.)}$$

For w in feet per second, put 8.022 in place of 4.43.

We can now find by this formula the velocity w for the example on page 517.

First, $\frac{p_1}{p} = \frac{1}{5} = 0.2$. Put $m = 1.135$, then $\left(\frac{p_1}{p}\right)^{\frac{m-1}{m}} = (0.2)^{0.119}$.

By the aid of logarithms we find this equal to 0.8257. Hence

$$\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right] = 1 - 0.8257 = 0.1743.$$

Further, for 5 atmospheres $v = xu + \sigma = 0.364$. Since $p = 5 \times 10334$,

$$w = 4.43\sqrt{\frac{1.135 \times 5 \times 10334 \times 0.364}{0.135} \times 0.1743} = 735.38 \text{ meters.}$$

This result agrees closely with that already found.

For practical use, Eq. LXII. can be simplified. Thus, making $m = 1.135$, and taking the sq. root of $\frac{m}{m-1}10334$, we have

$$w = 1305.83\sqrt{pv\left[1-\left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]}. \quad \text{(LXIII.)}$$

For w in feet per second put 1070 in place of 1305.83. Here p is expressed in atmospheres.

If the steam contains 10 per cent. of water, $m = 1.125$, and

$$w = 1351\sqrt{pv\left[1-\left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]}. \quad \text{(LXIV.)}$$

For w in feet per second put 1107 in place of 1351.

Table V. gives for different pressure ratios $\left(\frac{p_1}{p}\right)$ the values of $1-\left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}$ and of $\left(\frac{p_1}{p}\right)^{\frac{1}{m}}$, which last serves to determine the steam volume and weight which issues per second.

Steam Volume and Weight per Second.—We have thus far assumed, for the sake of simplicity, that one kilogram per second passes through E, and have found w under this assumption. The velocity evidently will not change when 2, 3, 4, or G kilograms issue per second. Apart from the pressures p and p_1, this quantity depends only upon the area of the orifice. If this area is F, the volume issuing per second is

$$V_1 = Fw,$$

provided that there is no contraction of the stream, as is the case with water and other liquids. If G is the weight of steam,

$$V_1 = v_1G,$$

or since

$$v_1 = \left(\frac{p}{p_1}\right)^{\frac{1}{m}}v$$

$$V_1 = \left(\frac{p}{p_1}\right)^{\frac{1}{m}}vG \text{ cubic meters.}$$

Hence

$$Fw = \left(\frac{p}{p_1}\right)^{\frac{1}{m}} vG,$$

and

$$G = Fw\left(\frac{p_1}{p}\right)^{\frac{1}{m}} \cdot \frac{1}{v} \text{ kilograms.} \quad . \quad . \quad \text{(LXV.)}$$

In order to show the use of Table V., we take an example.

EXAMPLE.

Let $p = 4$, and $p_1 = 1$ atmosphere. The steam at 4 atmospheres is pure saturated, without water. What is u and G?

We have $\frac{p_1}{p} = 0.25$. We find in the column for $\frac{p_1}{p}$ the number 0.25. In the same horizontal line we find for the column giving $1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}$, the number 0.1521. Hence

$$w = 1805.83 \sqrt{4 \times v \times 0.1521}.$$

Since $v = xu + \sigma$, and u, for 4 atmospheres, is from Table II., 0.447, we have $v = 0.447 + 0.001 = 0.448$. Hence

$$w = 1805.83 \sqrt{4 \times 0.448 \times 0.1521} = 681.9 \text{ meters per second.}$$

For English units, $v = 7.1668 + 0.016 = 7.1828$. Hence

$$w = 1070 \sqrt{4 \times 7.1828 \times 0.1521} = 2236.8 \text{ feet per second.}$$

Again, for 0.25, we find in the column for $\left(\frac{p_1}{p}\right)^{\frac{1}{m}}$ (m being 1.135) 0.2948.

Hence $$G = 681.9 \times 0.2948 \times \frac{F}{0.448} = 448.8F \text{ kilograms,}$$

or $$G = 2236.8 \times 0.2948 \times \frac{F}{7.1828} = 91.78F \text{ pounds.}$$

Calculation of the Size of Safety-Valve.—In general, we make the area F of the valve so great that it will discharge 10 to 20 times as much steam in a given time as the boiler can generate. This latter quantity depends, however, upon the heating surface, hence the valve and heating area must stand in a certain relation. According to Prussian regulations (Weisbach, Vol. II., Art. 434), the valve area should be $\frac{1}{1000}$ of the heating area. This gives indeed, for high pressure boilers, a greater safety than for low pressure, since for the latter the steam issues with less velocity.

We shall now show by an example how the diameter of the valve (d) may be found under the assumption that the steam issues with the velocity given by our formula, and that the loss of velocity due to friction, etc., is little or nothing.

EXAMPLE.

What diameter (d) should the safety-valve of a steam boiler have, which generates per hour 250 kilograms (or 551.15 lbs.) of steam of 4 atmospheres, for 20-fold security?

We have already found, in the last example, the weight of steam issuing per second through the area F under a pressure of 4 atmospheres. It was

$$G = 448.3F \text{ kilograms,}$$

or

$$G = 91.783F \text{ pounds.}$$

If the diameter of the valve is d, we have

$$G = 448.3\frac{\pi d^2}{4} = 448.3 \times 0.785d^2 \text{ kilograms,}$$

or

$$G = 91.783 \times 0.785d^2 \text{ pounds.}$$

The boiler generates 250 kilograms of steam per hour, or $\frac{250}{3600} = 0.0695$ kilograms (or 0.1531 pounds) per second. Since we wish 20-fold security, we have for G, $20 \times 0.0695 = 1.39$ kilograms (or 3.062 pounds). Hence

$$1.39 = 448.3 \times 0.785d^2, \quad \text{or} \quad d = 0.067 \text{ meters} = 6.7 \text{ centimeters.}$$

or

$$3.062 = 91.783 \times 0.785d^2, \quad \text{or} \quad d = 0.206 \text{ feet} = 2.5 \text{ inches.}$$

B. EFFLUX OF HOT WATER.

In the chapter upon the adiabatic curve of steam and liquid mixtures, we have seen that steam is generated when a mixture of water and steam containing a preponderance of the first expands adiabatically. When, then, hot water flows from the vessel $ABCD$, the particles expand in approaching the orifice F, and steam is formed. If, then, we have simply hot water and no steam, the Equation XXV., page 437,

$$\tau + \frac{xr}{T} = \tau_1 + \frac{x_1 r_1}{T_1}$$

becomes, since $x = 0$,

$$\frac{x_1 r_1}{T_1} = \tau - \tau_1.$$

Since r is always greater than r_1 so long as the temperature of the water is greater than that in the pipe EF, $\frac{x_1 r_1}{T_1}$ is positive, and since x_1 is the steam quantity at the orifice E, we see from the equation, that by the efflux of hot water steam is generated, and that, therefore, both steam and water issue. The weight of this steam in every kilogram of the mixture is

$$x_1 = (r - r_1) \times \frac{T_1}{r_1}.$$

If we put this value of x_1 in Equation LV., page 516, we have

$$A\frac{w^2}{2g} = q - q_1 - (r - r_1)\,T_1. \quad . \quad . \quad \text{(LXVI.)}$$

or

$$w = 91.2\sqrt{q - q_1 - (r - r_1)\,T_1}\,. \quad \text{(LXVII.)}$$

If instead of 1 kilogram, G kilograms issue per second, and if F is the area of the orifice, we have for the steam weight per second

$$D = x_1 G \text{ kilograms,}$$

and the water weight

$$W = (1 - x_1)\,G \text{ kilograms.}$$

The volume of 1 kilogram of the mixture at the orifice is

$$x_1 u_1 + \sigma,$$

hence of G kilograms it is

$$(x_1 u_1 + \sigma)\,G \text{ cubic meters.}$$

We have therefore

$$(x_1 u_1 + \sigma)\,G = Fw,$$

and

$$G = \frac{Fw}{x_1 u_1 + \sigma}. \quad . \quad . \quad . \quad \text{(LXVIII.)}$$

EXAMPLE.

Hot water flows from the test cock of a boiler under the pressure of 5 atmospheres. What is the specific steam weight at the orifice? With what velocity w does the mixture issue? What is the discharge G per second? How much steam and water are contained in the mixture?

For 5 atmospheres $\tau = 0.447$, for 1 atmosphere $\tau_1 = 0.314$, and $\frac{r_1}{T_1} = 1.438$. Hence $x_1 = (0.447 - 0.314) + 1.438 = 0.092$ kilograms. We have, then, as much steam as in the efflux of saturated steam alone is condensed (page 517).

Since, further, $q = 153.74$, $q_1 = 100.5$, we have

$$w = 91.2 \sqrt{153.74 - 100.5 - (0.447 - 0.314)\,373}$$
$$= 91.2 \times 1.9 = 173.28 \text{ meters.}$$

or $$w = 223.8 \sqrt{276.734 - 180.9 - (0.517 - 0.333)\,671.4} = 560.78 \text{ feet.}$$

For 1 atmosphere $u_1 = 1.650$, hence $x_1 u_1 + \sigma = 0.092 \times 1.650 + 0.001 = 0.154$, and

$$G = \frac{173.28}{0.154} F = 1125\,F \text{ kilograms.}$$

For English units $u_1 = 26.4216$, $x_1 u_1 + \sigma = 0.092 \times 26.4216 + 0.016 = 2.44$. The steam weight issuing per second is

$$G = \frac{560.78}{2.44} F = 229.9 F \text{ pounds.}$$

$$D = x_1 G = 0.092 \times 1125 F = 103.5 F \text{ kilograms.}$$

The water weight is therefore $(1125 - 103.5) F = 1021.5 F$. If then F is given, we can find G, D and W. If, for example, F is 1 square centimeter $= \frac{1}{10000} = 0.0001$ square meter, we have $G = 0.1125$ kilogram. $D = 0.01035$ and $W = 0.10215$ kilogram per second. The volume of water is, since 1 kilogram $= 0.001$ cubic meter, $0.1125 \times 0.001 = 0.0001125$ cubic meter.

We see from this calculation, how to find for any time the quantity of hot water and steam which flows through a given orifice under a certain pressure. Zeuner was the first to endeavor to determine how far the theoretical result agrees with the fact. "I allowed," he says in his Wärme Theorie, "water to flow from a locomotive boiler under 6 atmospheres pressure, and endeavored to collect and measure the issuing water. In spite of variations in the method of experiment, the measurement did not succeed. The steam carried off with it the greatest part of the water." There remains only in further experiments of this kind, to measure therefore the efflux in the boiler itself.

The following table gives for different pressures the corresponding velocity of efflux, the specific steam quantity at the orifice, and the discharge in kilograms per second, according to Zeuner.

Pressure in atmosp.	Velocity of efflux. w.	Specific steam weight at orifice. x_1.	Efflux G in kilogrs. per second for cross-section F. G.
2	69.52	0.038	1095 F
3	112.85	0.062	1102 "
4	145.07	0.079	1106 "
5	171.02	0.093	1109 "
6	192.98	0.105	1113 "
7	212.00	0.115	1116 "
8	228.87	0.124	1118 "
9	244.14	0.132	1120 "
10	258.02	0.139	1123 "
11	270.78	0.145	1124 "
12	282.64	0.152	1127 "
13	293.71	0.157	1128 "
14	304.09	0.163	1130 "

This table shows the remarkable result that the discharge is nearly constant whether the pressure be small or great. The specific steam weight x_1, on the other hand, increases with the pressure, a proof that the total steam weight issuing increases with the pressure. Since, then, the entire discharge is nearly constant, the water weight issuing per second must decrease with increasing pressure. It follows, that for the same contents and area of orifice, a low pressure boiler can be emptied in about the same time as a high, provided that in both the initial pressure is constant during the efflux. If, however, during efflux, the pressure gradually sinks, the weight of the issuing mixture indeed remains about the same, but the steam weight decreases while the water weight increases. We have then almost the inverse phenomena from the efflux of cold water.

In order to make apparent the difference between the efflux of cold and hot water, we give the following small table. The velocity of efflux is found from page 513,

$$p - p_1 = \frac{w^2}{2g} \gamma$$

where γ, the weight of one cubic meter of water, is 1000 kilograms. Since σ is the volume of 1 kilogram, $\gamma\sigma$ is the volume of 1000, or of 1 cubic meter. Hence

$$\gamma = \frac{1}{\sigma}$$

$$\text{and} \quad w^2 = \sigma (p - p_1) 2g \quad \text{or}$$

$$w = 4.43 \sqrt{\sigma (p - p_1)} \quad . \quad . \quad . \quad \text{(LXIX.)}$$

For English units put 8.022 in place of 4.43.

If, for example, water flows under the constant pressure of 4 atmospheres from a vessel into the air, the velocity of efflux w is, neglecting all resistances,

$$w = 4.43 \sqrt{10334 \times \tfrac{1}{1000} (4-1)} = 4.43 \sqrt{10334 \times 3} = 24.66 \text{ meters,}$$

$$\text{or} \qquad w = 8.022 \sqrt{2116.21 \times 0.016 (4 - 1)} = 80.85 \text{ feet.}$$

The water volume per second is then Fw, and the water weight

$$G = 1000wF = 24660F \text{ kilograms} \quad . \quad \text{(LXX.)}$$

Hence we have

Pressure. p.	Velocity. w.	Efflux. G.
4	24.66	$24660F$
8	37.67	$37673F$
12	47.23	$47226F$

While, therefore, for the same pressure the velocity of the issuing water is much less than for a mixture of steam and water, the discharge is much greater. The explanation is as follows: The steam weight, in spite of its great velocity, is very small, but even this small weight occupies a relatively large space, and fills, in part, the orifice so that the water quantity is small.

It is much to be desired that these theoretical investigations may be tested by thorough and exact experiments.

Cases in which the Hot Water Issues with the same Velocity as *the Cold.*—The complete equation for the velocity of the issuing steam was, from Equation LIV., page 516,

$$q + x\rho + Apux - (q_1 + x_1\rho_1 + Ap_1x_1u_1) + A\sigma(p - p_1) = A\frac{w^2}{2g},$$

or

$$q + xr - q_1 - x_1r_1 + A\sigma(p - p_1) = A\frac{w^2}{2g}.$$

If, now, instead of steam we have only hot water, $x = 0$, and we have

$$q - q_1 - x_1r_1 + A\sigma(p - p_1) = A\frac{w^2}{2g}.$$

Now, from page 516,

$$x_1 = \left(\tau - \tau_1 + \frac{xr}{T}\right)\frac{T_1}{r_1},$$

and since here also $x = 0$,

$$x_1 = (\tau - \tau_1)\frac{T_1}{r_1}, \quad \text{or} \quad x_1r_1 = (\tau - \tau_1)\,T_1.$$

Instead of $q - q_1$ we can put $c(t - t_1) = c(T - T_1)$ and have

$$c(T - T_1) - (\tau - \tau_1)\,T_1 + A\sigma(p - p_1) = A\frac{w^2}{2g}. \quad \text{(L.)}$$

If the pressures p and p_1, or the temperatures T and T_1 are but little different from each other, we have

$$2.3026 \log\frac{T}{T_1} = \frac{T - T_1}{T_1}.$$

From page 517

$$\tau - \tau_1 = 2.3026c \log\frac{T}{T_1} = c\,\frac{T - T_1}{T_1}.$$

or

$$(\tau - \tau_1)\,T_1 = c\,(T - T_1).$$

In equation (1) the two first members then are equal, and we have

$$A\sigma(p - p_1) = A\frac{w^2}{2g},$$

or

$$\frac{w^2}{2g} = \sigma(p - p_1) = \frac{p - p_1}{\gamma}. \quad . \quad . \quad \text{(LXXI.)}$$

This is the same equation which we have found, page 513, for the efflux of water under ordinary circumstances. We have, however, called attention there to the fact that the pressure due to the head of water over the orifice should be taken into account in finding the velocity of efflux. The same is the case for hot water. Since γ is the weight of one cubic meter and p the pressure in kilograms per square meter, we have, when p is expressed in atmospheres, for the height of a column of water which would exert this pressure,

$$\frac{10334p}{\gamma} = \frac{10334}{1000}p = 10.334p.$$

If the head of water is h meters above the center of the orifice, we have instead of $\frac{p}{\gamma}$, $\frac{p}{\gamma} + h$. Hence the theoretical velocity of the cold water is

$$w = 4.43\sqrt{h + \frac{p - p_1}{\gamma}}. \quad . \quad . \quad \text{(LXXII.)}$$

For English units put 8.022 in place of 4.43.

And this will be the velocity also of the hot water when p and p_1 are nearly equal.

The efflux of hot and cold water must then be the same, if steam formation during efflux is prevented. This can only be the case when we abstract from the water as much heat as the steam requires for its formation. If the heat of the liquid in the vessel is q, and outside q_1, we must abstract for each kilogram of water at the orifice the heat $q - q_1$. When this is the case, the formula for efflux of cold water will apply also to hot water, because the diminution of volume of the water in passing from the higher to the lower temperature, which in the efflux of gas and steam is so great, is for water so small as to have no influence upon the flow. The velocity *w*, and the discharge in kilograms per second or minute, must then be the same, in this case, both for cold and hot water.

CHAPTER XXII.

CONSTRUCTIONS WHICH DEPEND UPON SIMILAR PRINCIPLES.—THE INJECTOR.

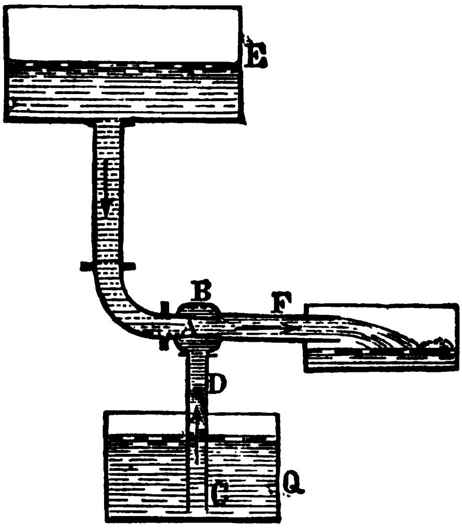

Fig. 83.

The principle of the ordinary suction pump consists in a pipe with one end in the water and the other connected with a pump, by means of which a partial vacuum is created in the pipe, so that the pressure of the air upon the outside water forces water up the pipe. For the production of the vacuum different methods have been recently adopted. One of the simplest and most ingenious is that of Professor James Thompson. It consists in causing a stream of water, of considerable velocity, to carry away, in part, the air with it. We give a sketch of the apparatus, Fig. 83. From the tank *E* a vertical pipe descends and ends in a conical mouth-piece at *A*, inclosed by the spherical vessel *B*. From this we have the diverging pipe *F* and the suction pipe *D*. While now the water flows through *A* with great velocity, it drives out the air in *F* as well as in *B*. Air is thus sucked out of the pipe *CD*, a partial vacuum is created, and the water rises in *CD*. This water enters *B* with a certain velocity, and is there carried out along with the stream in *F*.

While here, we make use of the velocity of a stream of water for the formation of a vacuum, we might make use of a current of air or steam. The latter is applied thus in the blast-pipe of the locomotive and in Giffard's injector. In the case of the blast-pipe, the steam passes from the cylinder at about 1.25 atmospheres pressure, through a nozzle in the lower part of the stack. The velocity is by reason of this pressure great, and the steam drives out with it the gases of combustion, and imparts to them a greater upward velocity, so that a partial vacuum is created in the smoke-box and the outer air enters rapidly through the grate, thus causing more rapid combustion. The operation of the blast-pipe consists then in sucking in the outer air by means of a current of steam. Giffard's injector uses steam for the purpose of sucking up water, as in Thompson's water-jet pump. It is remarkable that here we can force the water thus sucked up, into the boiler from which we obtain the steam. This apparatus is much used instead of ordinary force pumps for furnishing feed-water to boilers.

Description of the Injector.—Our sketch, Fig. 84, shows a section of the apparatus. The pipe *A* connects with the steam space of the boiler, and when the cock *H* is opened the steam passes through a number of holes in the pipe *BC* into this pipe, which ends in a conical mouthpiece *C*. This mouthpiece empties into a chamber *D*, which communicates by the pipe *FF* with the feed-water tank *QQ*. The feed water and the in large part condensed steam, pass through *E* and enter a second conical mouthpiece *G*. It then passes through the pipe *K*, valve *V* and pipe *L*, which communicates with the water-space in the boiler. The flow of steam is regulated by a conical spindle *N*, which can be raised or lowered by the crank *M*. The flow of feed water can also be regulated by means not shown. By the pipe *S* the surplus water which collects in the chamber *R* is removed.

The action of the apparatus is easily understood. *H* is opened and the spindle *C* raised; steam flows with great velocity into the chamber *D*, carries out the air with it, and thus causes in the chamber and suction pipe *F* a partial vacuum. The feed water rises through *FF* into the chamber *D*. The spindle is then raised further, and more steam enters, a great part of which is condensed by contact with the cold feed water. The steam still coming from the boiler drives, by reason of its living force, the feed water and condensed steam through *E*, into the mouthpiece *G*, and it then passes by *K*, *V*, and *L* into the boiler. If the ratio of the entering steam and feed water is just right, the stream of water where it passes from *E* to *G*, and which can be observed by a small window at *B*, should be perfectly transparent, so that neither steam nor water departs by *S*.

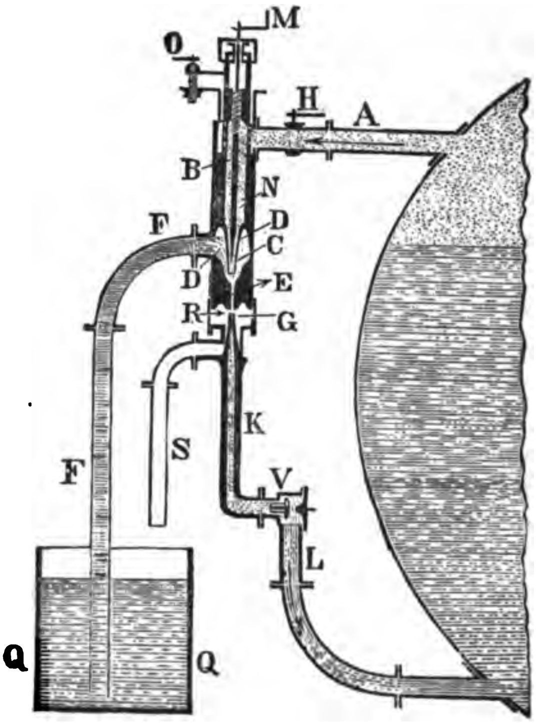

FIG. 84.

Theory of the Apparatus.—Dimensions.—As has been remarked in Part I., this apparatus testifies to the correctness of the mechanical theory of heat. For if the valve *V* is open, one would say that the boiler water ought to escape just as much as the steam, since the pressure upon the water is the same as the steam tension. If, however, we consider that the work inherent in the steam—in other words, the total heat of the steam—is much greater than that of the boiler water, we can understand how the excess of inner work can not only suck up water and force it into the boiler, but also heat this water to the temperature of the boiler. That which in Thompson's pump is performed by the *outer* living force of the water, is here performed by the inner living force of the steam, by that which we call the vibration work of the molecules.

Let the velocity with which the steam passes the orifice at C be w; the steam pressure in the boiler be p kilograms per square meter; t be the temperature and q the heat of the liquid. Every kilogram of steam which passes, contains x kilograms of pure saturated steam and $1 - x$ of water. If G kilograms pass C per second, we have in this time xG kilograms of steam and $(1 - x)G$ of water. We assume further, the pressure in the condensing chamber $D_1 = p_1$.

According to page 516, the heat which imparts to the steam the velocity w is

$$q + x\rho + Apxu - (q_1 + x_1\rho_1 + Ap_1x_1u_1) + A\sigma(p - p_1) = A\frac{w^2}{2g},$$

where x is the specific steam weight in the boiler, and x_1 that in the condensing chamber. If we assume that here the steam is completely condensed, $x_1 = 0$, and we have

$$q + x\rho + Apxu - q_1 + A\sigma(p - p_1) = A\frac{w^2}{2g}. \quad \text{(LXXIV.)}$$

Here, q_1 is the heat of the liquid in the condensing chamber, hence $q + x\rho + Apxu - q_1$, is the heat which the steam has given up. We have then, in this condensed steam, q_1 heat units per kilogram. If the flow is G kilograms per second, we have

$$GA\frac{w^2}{2g} = G\left[q + x\rho + Apxu - q_1 + A\sigma(p - p_1)\right]. \quad \text{(LXXV.)}$$

From page 520, Equation LXI., we have also

$$\frac{w^2}{2g}G = \frac{mpv}{m - 1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]G.$$

In the following investigations and calculations, we shall make use of one or the other of these formulæ.

Let us first apply the former.

The entering steam generates at first in the condensation chamber a vacuum. Cold water is then forced up by the outer pressure of the air into this chamber. If now water flows from a reservoir, upon whose surface there is a pressure p_0 per square meter, into another in which the pressure is p_1, and if h is the head, we have for the living force in the issuing water, G_0,

$$\frac{u^2}{2g} G_0 = \left[\frac{p_0 - p_1}{\gamma} + h\right] G_0,$$

where u is the velocity and $\frac{1}{\gamma} = \sigma$; hence

$$\frac{u^2}{2g} G_0 = [(p_0 - p_1)\sigma + h]\, G_0.$$

If h is negative, or the water is raised, which can only be when $p_1 < p_0$, we have

$$\frac{u^2}{2g} G_0 = [(p_0 - p_1)\sigma - h]\, G_0 \quad . \quad . \quad \text{(LXXVI.)}$$

or

$$u = \sqrt{2g\,[(p_0 - p_1)\sigma - h]} = 4.43\sqrt{(p_0 - p_1)\sigma - h}.$$

For English units put 8.022 in place of 4.43.

The heat equivalent to the above work is

$$A\frac{u^2}{2g} G_0 = A\,[(p_0 - p_1)\sigma - h]\, G_0 \text{ heat units} \quad . \quad \text{(LXXVII.)}$$

If the heat of the liquid for the cold water is q_0, the heat in it is $q_0 G_0$ heat units. At the beginning of the entire process we have the heat

$$G(q + rx - q_1) + GA\sigma(p - p_1) + A\,[(p_0 - p_1)\sigma - h]\, G_0 + q_0 G_0. \quad \text{(LXXVIII.)}$$

This heat performs the following works: 1. It has to heat the cold feed water. 2. It has to impart to it a certain velocity, so that the water mass G_0 and condensed steam mass G may pass with equal velocity through E. 3. The combined mass $G_0 + G$ must overcome on entering the chamber R the outer air pressure, since R communicates with the atmosphere by S.

Let now the velocity of the mass $G + G_0$ be u_1, the mechanical work inherent in it is

$$\frac{u_1^2}{2g}(G + G_0),$$

and this represents the heat

$$A\frac{u_1^2}{2g}(G + G_0) \quad . \quad . \quad . \quad . \quad \text{(LXXIX.)}$$

Since, further, the G_0 kilograms of feed water must have the same heat of liquid as the condensed steam, it must contain

$q_1 G_0$ heat units.

Finally, the work required for overcoming the air pressure in R, p_0, is

$$(p_0 - p_1)\,\sigma\,(G + G_0).$$

This corresponds to the heat

$$A\,(p_0 - p_1)\,\sigma\,(G + G_0) \text{ heat units} \quad . \quad \text{(LXXX.)}$$

These three last quantities of heat must equal that given by Equation LXXVIII. Hence we have

$$G\,(q + rx - q_1) + GA\sigma\,(p - p_1) + G_0A\,[(p_0 - p_1)\,\sigma - h] + q_0G_0$$
$$= A\,\frac{u_1^2}{2g}\,(G + G_0) + q_1G_0 + A\sigma\,(p_0 - p_1)\,(G + G_0).$$

Or, reducing and canceling equal quantities on both sides,

$$G\,(q + rx - q_1) + GA\sigma\,(p - p_0) - G_0Ah + q_0G_0 =$$
$$A\frac{u_1^2}{2g}\,(G + G_0) + q_1G_0. \quad . \quad . \quad . \quad \text{(LXXXI.)}$$

This equation contains two unknown quantities, the heat q_1 and the velocity u_1. If the apparatus feeds the same boiler from which the steam is taken, the living force $\frac{u_1^2}{2g}\,(G + G_0)$ of the water must be so great as to be able to force the mass $(G + G_0)$ into the boiler.

If we assume that the condensation chamber D lies at the level of the boiler water, we have, since the pressure in the boiler is p and that in R is p_0,

$$\frac{u_1^2}{2g}(G + G_0) = \sigma(p - p_0)(G + G_0).$$

The right side represents the work necessary for overcoming the pressure $p - p_0$, neglecting all resistances. Hence

$$u_1 = \sqrt{2g\sigma(p - p_0)} = 4.43\sqrt{\sigma(p - p_0)}. \quad \text{(LXXXII.)}$$

where p and p_0 are given in kilograms per square meter. If we express p and p_0 in atmospheres, then, since, $\sigma = 0.001$ (or 0.016),

$$u_1 = 4.43\sqrt{10.334(p - p_0)} = 14.24\sqrt{p - p_0}.$$

For English units put 46.70 in place of 14.24.

By reason of resistances, we may put $u_1 = 14.67\sqrt{p - p_0}$, and, since p_0 is always 1,

$$u_1 = 14.67\sqrt{p - 1} \text{ meters.} \quad . \quad \text{(LXXXIII.)}$$

For English units put 48 in place of 14.67.

If in LXXXI. we put $\frac{u_1^2}{2g}(G + G_0) = \sigma(p - p_0)(G + G_0)$, we have

$$G(q + rx - q_1) + GA\sigma(p - p_0) - G_0Ah + q_0G_0$$
$$= A\sigma(p - p_0)(G + G_0) + q_1G_0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XC.)}$$

This equation leads to interesting considerations. We may consider it as composed of two equations, viz.:

$$G(q + rx - q_1) = (q_1 - q_0)G_0 \quad . \quad . \quad . \quad . \quad \text{(XCI.)}$$

and

$$GA\sigma(p - p_0) - G_0Ah = A\sigma(p - p_0)(G + G_0),$$

or

$$0 = G_0A\sigma(p - p_0) + G_0Ah = G_0[\sigma(p - p_0) + h].\ A \quad \text{(XCII.)}$$

The first of these equations would indicate that the heat given up by G kilograms of steam and water of the temperature t, when cooled under constant pressure p into water at t_1, is sufficient to heat G_0 kilograms from t_0 to t_1 degrees, or to impart $G_0(q_1 - q_0)$ heat units. But if this is so, then, as is seen from the second equation, there is no heat or mechanical work remaining, in order to raise the mass G_0 and force it into the boiler. If, therefore, the ratio $\frac{G}{G_0}$ and the values of q, q_0, and x are known, we should find q_1 from the first equation, too large, and the excess is that heat, or work, necessary for raising G_0 and forcing it into the boiler. Thus it follows, unmistakably, that in our apparatus, during the entire process, a part of the molecular work of the steam is transformed into outer work.

As has been remarked, we can only give a satisfactory account of this apparatus when we assume this principle as correct. That, however, a very small amount of heat suffices for the raising of the water, can be easily shown by a practical example. This, also, follows from the fact that one heat unit corresponds to a mechanical work of 424 meter-kilograms.

From an experiment by the French engineer Villiers, the height h to which the injector raised the water was 4 meters, the pressure p in the boiler was $4\frac{1}{4}$ atmospheres corresponding to a temperature t_1 of 146.19°, and heat of liquid $q = 147.55$ heat units, and total latent heat of $r = 503.54$ heat units. The steam contained about 7 per cent. of water. Hence x was $1 - 0.07 = 0.93$, and $rx = 503.54 \times 0.93 = 468.29$. The temperature t_0 of the feed water was 23.5°, hence $q_0 = 23.51$. Finally, the temperature of the issuing water (mixture of steam and water) was $t_1 = 60.5°$, and hence $q_1 = 60.64$. If we substitute these values in the first of the above equations, we have for the ratio $\frac{G_0}{G}$ of the feed water to the steam used,

$$\frac{G_0}{G} = \frac{147.55 + 468.29 - 60.64}{60.64 - 23.51} = 14.95.$$

If, then, no mechanical work had been necessary for raising the water and forcing it into the boiler, we should have for the given data of the apparatus, 14.95 times as much cold feed water as steam. But now, for this raising and forcing, work is required, and hence this ratio should be less. And inversely, if this ratio were correct, the heat of the liquid q_1 should be less. The second equation tells us by how much, viz., by

$$A\,[h + \sigma\,(p - p_0)] \text{ heat units.}$$

Since now $h = 4$, $\sigma = 0.001$, $p = 10334 \times 4\frac{1}{4}$, $p_0 = 10334$, we have

$$\tfrac{1}{424}\,[4 + 10334\,(4.25 - 1)] = 0.089 \text{ heat units per kilogram.}$$

Of this, $\frac{1}{424} \times 4 = 0.01$ per kilogram are used for raising the water and 0.079 for forcing it into the boiler. We see that this heat is so insignificant that it can only be observed by specially constructed thermometers. This is explained, as already remarked, by the fact that 1 heat unit represents the considerable work of 424 meter-kilograms, and hence a very small loss of molecular work can cause a considerable amount of outer work. For these reasons we may use, in all practical cases, the first equation for determining $\frac{G_0}{G}$, when t, t_1 and t_0 are known, or for determining t_1 from t, t_0 and $\frac{G_0}{G}$.

From the equation

$$G\,(q + rx - q_1) = G_0\,(q_1 - q_0)$$

we have

$$G\,(q + rx) + G_0q_0 = G_0q_1 + Gq_1, \text{ hence}$$

$$q_1 = \frac{G\,(q + rx) + G_0q_0}{G_0 + G}.$$

Instead of $q + rx$, we can put the total heat (w) in 1 kilogram of steam of the temperature t. Since, further, q_0 and q_1 cannot be large (q_0 is, for example, on an average 15°, and q_1 at most 60°), we can put the temperatures in place of the liquid heats. We have then

$$t_1 = \frac{GW + G_0t_0}{G_0 + G} \quad . \quad . \quad . \quad . \quad \text{(XCIII.)}$$

where $W = 606.5 + 0.305t$ (or for English units $1091.7 + 0.305(t-32)$). We have also

$$\frac{G_0}{G} = \frac{W - t_1}{t_1 - t_0} \quad . \quad . \quad . \quad . \quad . \qquad \text{(XCIV.)}$$

Hence we have the temperature of the mixture of steam and liquid. This formula could have been found directly, if we had assumed the views as to the action of the apparatus referred to as correct.

Before proceeding to further calculations of the dimensions of the apparatus, let us consider here a question of practical interest, viz., which is the most economical, an ordinary feed-pump or the injector?

We have assumed above that the injector requires G kilograms of steam per second of the temperature t. The generation of this steam out of water at $t°$, if in each kilogram there are x kilograms of steam, requires Grx heat units. Now the injector forces every second $G + G_0$ kilograms of water at t_1 degrees into the boiler, in which it is heated up to $t°$. This requires the heat $(G_0 + G)\,(q - q_1)$ heat units. The total heat required per second is then

$$Q = Grx + (G_0 + G)\,q - q_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(XCV.)}$$

Now we have from the general equation, page 537,

$$G_0\,(q_1 - q_0) + G_0A\,[\sigma\,(p - p_0) + h] = G\,(q + rx - q_1),$$

or

$$G_0A\,[\sigma\,(p - p_0) + h] - G_0q_0 = Grx + G\,(q - q_1) - G_0q_1,$$

and, adding G_0q to both sides,

$$G_0\,(q - q_0) + G_0A\,[\sigma\,(p - p_1) + h] = Grx + (G + G_0)\,(q - q_1).$$

From this equation and Eq. XCV., we have

$$Q = G_0\,(q - q_0) + G_0A\,[\sigma\,(p - p_0) + h]. \quad . \qquad \text{(XCVI.)}$$

The first member on the right gives the heat which G_0 kilograms of water require in order to become heated from t_0 to t, that is, to the boiler temperature. The second member is the heat required to raise G_0 kilograms of water to the height h and then to force it into the boiler. *Hence the heat required to run the injector depends only on these quantities, and is independent of the steam quantity G used by the apparatus.* This is evident when we consider that the quantity of heat required by the injector is always returned to the boiler. If it uses more steam, or steam of higher tension, it can furnish more water, but in all cases the heat required for the generation of this greater tension is returned to the boiler. Of course, in this, we disregard all losses of heat due to radiation, conduction, and loss of steam. If we remember still, that the second member on the right is almost vanishingly small with respect to the first, we have

$$Q = G_0(q - q_0) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad \text{(XCVIII.)}$$

Hence the heat required by the injector is greater the more water it furnishes to the boiler, or the more water or steam the engine uses per second; the greater the temperature of the boiler, and the less that of the feed water. Precisely the same is true of the ordinary feed pump.

Suppose that the pump has first to raise the cold water to the height h. The work required is G_0h.

This water is now to be forced into the boiler, where the pressure is p. This requires the work

$$G_0p\sigma - G_0p_0\sigma = G_0\sigma(p - p_0).$$

If, therefore, the pump makes one revolution per second, it furnishes per second G_0 kilograms of water at the temperature $t_0°$, and the work required is

$$G_0h + G_0\sigma(p - p_0),$$

or in heat units

$$AG_0[\sigma(p - p_0) + h],$$

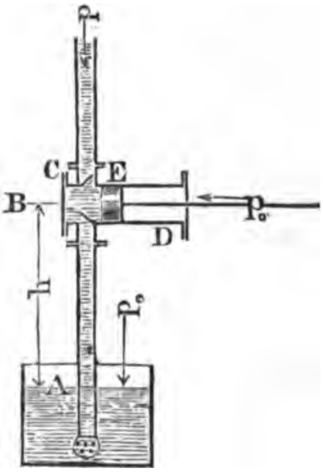

FIG. 85.

or the same as in equation XCVI. was found for the injector.

In the boiler, the G kilograms are heated from t_0 to t degrees, which requires

$$G_0(q - q_0) \text{ heat units.}$$

The total heat for the working of an ordinary pump is therefore precisely the same as for the injector.

Theoretically, then, the one apparatus has no advantage over the other. If we consider, however, that the frictional resistances in the pump are much greater, the injector is the best. This is not the case, however, when the apparatus is used simply for the raising of water only, as we shall soon point out.

As to the height of suction h, this, as has been shown by experiment, is much less when the injector first begins to work than when in full action. The reason may be as follows: When the apparatus is set in action, we have in the condensing chamber DD air of atmospheric tension. The steam rushing through C carries with it especially those particles near the orifice, and causes a partial vacuum. But in consequence of this there is a quick vaporization of the particles of water in the steam, or of those remaining in the condensing chamber, which diminishes the vacuum. When once the apparatus is in full activity, the steam is at once condensed by the cold feed water. We have indeed still steam mixing with the rarefied air, but steam whose pressure depends only upon the temperature of the resulting mixture of steam and water.

According to the experiments of Villiers, at St. Etienne, we have the following results for the height of suction and steam pressure in boiler at beginning of action:

For

$p =$ 2.	2.5	3.	3.5	4.	4.5	atmos.
$h =$ 1.4	2.	2.47	2.8	3.	3.1	meters.

When the apparatus was in full action

$h =$ 3.14	4.	4.49	4.74	4.99	4.99	meters.

We see that both in beginning as in normal action, beyond a certain steam pressure, there is no further increase in height of suction.

The results obtained by Beuther from a steam jet pump coincide well with the preceding. According to these for

$p = 2.$ $\quad 2.5 \quad 3.$ atmos.
$h = 1.32 \quad 1.88 \quad 2.35$ meters,

and when in full action

$h = 3.45 \quad 3.69 \quad 4.4$ meters.

The first results for beginning of action, are given closely by the empirical formula

$$h = -1.124 + 1.46x - 0.1x^2 \quad . \quad . \quad . \quad \text{(XCIX.)}$$

where x is given in atmospheres.

For 3 atmospheres we have

$$h = -1.124 + 1.46 \times 3 - 0.1 \times 9 = 2.356 \text{ meters.}$$

These values should not be exceeded for good feeding, otherwise the water drawn up enters the condensing chamber with small velocity, and less may be furnished than is required for feed. In general, we allow the water to enter with about 10 to 20th part of the velocity which it possesses at E. If we denote then the velocity in the suction pipe by u, we have

$$u = \tfrac{1}{10} \text{ to } \tfrac{1}{20} u_1.$$

Hence, the cross-section F_2 of the suction pipe should be 10 or 20 times as great as that of E. If this is F_1, we have $F_2 = 10F_1$ to $20F_1$. If we denote the diameter by d_2 and that of E by d_1, then

$$\frac{\pi d_2^2}{4} = 10\frac{\pi d_1^2}{4} \text{ to } 20\frac{\pi d_1^2}{4},$$

or

$$\left(\frac{d_2}{d_1}\right)^2 = 10 \text{ to } 20.$$

If, therefore, we know the velocity u with which the water enters the condensing chamber, we can find the pressure p_1 as soon as we know the height of suction h. Thus we have, Equation LXXVII,

$$\frac{u^2}{2g} = p_0\sigma - p_1\sigma - h \quad \text{or}$$

$$p_1 = p_0 - \frac{h}{\sigma} - \frac{u^2}{2g\sigma} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (\text{C.})$$

where p_1 and p_0 are given in kilograms per sq. meter. If p_1 and p_0 are given in atmospheres, and $\sigma = 0.001$, we have

$$p_1 = \frac{10334 - 1000h - 51u^2}{10334}$$

or

$$p_1 = 1 - 0.097h - 0.005u^2. \quad . \quad . \quad . \quad . \quad (\text{CI.})$$

In English units, $p_1 = 1 - 0.0295h - 0.00046u^2$.

If, for example, $h = 1.3$ meters, and $u = 3$ meters,

$$p_1 = 1 - 0.097 \times 1.3 - 0.005 \times 9 = 1 - 0.1711 = 0.83 \text{ atmos.}$$

is the pressure in the condensing chamber.

As soon as p_1 is known, we have the velocity of efflux w of steam.

$$w = 4.43\sqrt{\frac{mpv}{m-1}\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]} \quad . \quad . \quad (\text{CII.})$$

For English units put 8.022 in place of 4.43.

If we have dry saturated steam, $m = 1.135$, and

$$w = 1305.83\sqrt{pv\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]} \quad . \quad . \quad (\text{CIII.})$$

For English units put 1070 in place of 1305.83.

If the steam contains 10 per cent. water, $m = 1.125$, and

$$w = 1351\sqrt{pv\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]} \quad . \quad . \quad (\text{CIV.})$$

For English units put 1107 in place of 1351.

On account of resistances, we may put

$$w = 1324.5\sqrt{pv\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]}. \quad . \quad (CV.)$$

For English units put 1085 in place of 1324.5.

where $1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}$ is given in Table V.

Since now we can either calculate, or find by experiment, the steam weight used per second by a projected or existing steam engine of given horse power, we can determine by Equation XCIV. the steam weight required by the injector, if we fix upon the temperature of the mixture of cold water and steam furnished by the boiler, and know the temperature of the injection water. If we denote the steam quantity in kilograms required by the injector by G, and if the specific steam volume in the boiler is v, we have for the area F_1 of the orifice C, from Equation LXV.,

$$F_1 = \frac{Gv}{w}\left(\frac{p}{p_1}\right)^{\frac{1}{m}} = \frac{Gv}{w} \div \left(\frac{p_1}{p}\right)^{\frac{1}{m}}.$$

The diameter d_1 of this orifice is then

$$d_1 = \sqrt{\frac{4F_1}{\pi}} = 1.129\sqrt{F_1}.$$

EXAMPLE.

The steam pressure p in the boiler is 5 atmospheres, the height of suction $h = 1.75$ meters (or 5.74 feet). The condensing chamber is on a level with the boiler-water level. The engine uses 7.5 kilograms (or 16.5 lbs.) of steam per minute (page 453). What should be the area of the mouthpiece C in a Giffard injector? what of the suction pipe F and the pipe E, when the feed water has a temperature of 15° (59° Fah.), and the mixture of water and condensed steam 40° (104° Fah.)?

The velocity u_1 of the water in E is, Equation LXXXIII.,

$$u_1 = 14.67\sqrt{p-1} = 14.67\sqrt{4} = 29.34 \text{ meters per sec.}$$

$$u_1 = 48\sqrt{4} = 96 \text{ feet per sec.}$$

Since the engine requires per minute 7.5, or per second $\frac{7.5}{60} = 0.125$ kilograms, (0.275 lbs.) this is the feed. Since the mixture of steam and water has a temperature of 40°, we have from Equation XCIV.,

$$\frac{G_0}{G} = \frac{0.125}{G} = \frac{W-40}{40-15}, \quad \text{or} \quad \frac{0.275}{G} = \frac{W-104}{104-59}.$$

Now $W = 606.5 + 0.305t$, and t for 5 atmospheres is 152.22 (or 305.996 Fah.), hence $W = 650.33$ (or 1175.2) and

$$\frac{0.125}{G} = \frac{650.93-40}{25} = 24.44,$$

$$\text{or } \frac{0.275}{G} = \frac{1175.2-104}{45} = 24.$$

Hence the steam weight used per second by the injector is

$$G = \frac{0.125}{24.44} = 0.00512 \text{ kilograms.}$$

$$G = \frac{0.275}{24} = 0.01126 \text{ pounds.}$$

We have then, flowing through E, $0.125 + 0.00512 = 0.130$ kilograms (or 0.286 lbs.) of water at 40° (or 140° Fah.). Since 1 kilogram occupies the space of 0.001 cubic meter (or 1 lb. 0.016 cub. ft.), 0.130 occupies 0.000130 (or 0.00457 cub. ft.). If the area of E is F, then

$$Fu_1 = 0.000130,$$
$$\text{or } Fu_1 = 0.00457.$$

Since $u_1 = 29.34$ (or 96),

$$F = 0.00000443 \text{ sq. meter.}$$
$$F = 0.000047 \text{ sq. ft.}$$

If d is the diameter,

$$\frac{\pi d^2}{4} = 0.785d^2 = 0.00000443, \text{ or } d = 2.4 \text{ millimeters, or } 0.09 \text{ inch.}$$

If we take the diameter of the suction pipe 5 times as large,

$$d_2 = 12 \text{ millimeters.}$$

If d_2 is $5d$, the water flows $\frac{1}{25}$ as fast in the suction pipe as in E, or has a velocity of $\frac{29.34}{25} = 1.174$ meters (or 3.84 ft.). Hence the pressure p_1, in the condensing chamber, is from Equation CI.,

$$p_1 = 1 - 0.097 \times 1.75 - 0.005\,(1.174)^2,$$
$$\text{or } p_1 = 1 - 0.0296 \times 5.74 - 0.00046\,(3.84)^2,$$

or taking into account resistances in the suction pipe,

$$p_1 = 1 - 0.097 \times 1.75 - 0.01\,(1.174)^2 = 0.82 \text{ atmos.},$$
$$\text{or } p_1 = 1 - 0.0296 \times 5.74 - 0.00092\,(3.84)^2 = 0.82 \text{ atmos.}$$

Now we can find the velocity w with which the steam passes C. If it contains 10 per cent. water, we have from CV.,

$$w = 1824.5\sqrt{pv\left[1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}\right]}.$$

For English units we put 1085 in place of 1824.5.

Since $v = xu + \sigma = 0.90 \times 0.363 + 0.001 = 0.828$ (or 5.288), and $\frac{p_1}{p} = \frac{0.82}{5} = 0.164$, we have from Table V., $1 - \left(\frac{p_1}{p}\right)^{\frac{m-1}{m}} = 0.1728$, and hence

$$w = 1324.5\sqrt{5 \times 0.828 \times 0.1728} = 705.96 \text{ meters},$$

or

$$w = 1085\sqrt{5 \times 5.288 \times 0.1728} = 2314.8 \text{ feet}.$$

The cross-section F_1 of C is now

$$F_1 = \frac{0.00512 \times 0.828}{705.96} + 0.1821 = 0.0000131 \text{ sq. meters},$$

or

$$F_1 = \frac{0.01126 \times 5.288}{2314} + 0.1821 = 0.0001413 \text{ sq. ft.}$$

$$d_1 = 1.129\sqrt{F_1} = 1.129 \times 0.00362 = 4.09 \text{ millimeters}.$$

$$d_1 = 1.129\sqrt{F_1} = 0.012 \text{ ft.} = 0.14 \text{ inches}.$$

Remark.—Although, as has been remarked, the injector is a good feed apparatus for boilers, and is to be preferred to ordinary pumps, this is by no means the case when the apparatus is simply used for raising water. The steam passing C has a much greater velocity than the water in E. The condensed steam particles experience then a sudden change of velocity. There is thus impact, by which a large part of the living force of the steam is lost and does not contribute to useful effect. By this impact there is indeed heat, but as in the present case we have to do only with outer work, this is of no account. The application of steam in such a case is no more advantageous than would be its application in the case of an impact or reaction wheel.

CHAPTER XXIII.

SUPERHEATED STEAM.

WHAT we understand by superheated steam has been already specified in Chapter XIV. As to saturated steam, we know that it obeys different laws from those which govern permanent gases. The question arises, whether the same holds true for superheated steam? This is in part true, at least in the vicinity of the point of saturation, that is, where it passes into saturated steam, superheated steam differs in its deportment from the permanent gases. Only when it is far removed from this point are its properties the same as air or other of the so-called permanent gases. In the case of air, for example, we have learned that it expands $\frac{1}{273} = 0.00366$ of its volume for each degree rise of temperature. Saturated steam, on the other hand, when heated apart from water, under constant pressure from 100 to 110°, expands 5 times as much as air, and for further heating from 110 to 115.6, 126.5 and 186.1°, it expands respectively, 4, 3, and 2 times as much. It follows that only at a considerable distance from the point of saturation is the deportment of superheated steam that of a gas. When it has arrived at this state, we have the same uniformity in expansion which we have for gases, and the formulæ for gases apply.

This deportment of saturated steam, and of superheated steam near the point of saturation, can only be explained on the supposition that the molecules are more strongly attracted than those of permanent gases, but that this attraction decreases the more the steam is superheated, or the farther it is from the point of saturation. If, then, superheated steam expands under constant temperature, the heat imparted is not equal to the outer work, as is the case with air, but rather more heat is necessary in order to force the molecules apart or to decrease their mutual attraction. If this heat is not added, the temperature cannot remain constant, but the heat required for this inner work must be furnished by the steam itself.

Illustration by Diagram of Saturated and Superheated Steam.—Let OA, Fig. 86, be the volume (s) of one kilogram of saturated steam at 100°, and AB the corresponding pressure of 1 atmosphere. Then B is a point in a curve of constant steam weight. If D_1D_1 is this curve, we have for the volume

$$OA = s = u + \sigma \text{ cub. m.}$$

If OC is the volume of the steam for a less temperature and CD the pressure, D is also a point in the curve, and

$$OC = s_1 = u_1 + \sigma, \text{ etc.}$$

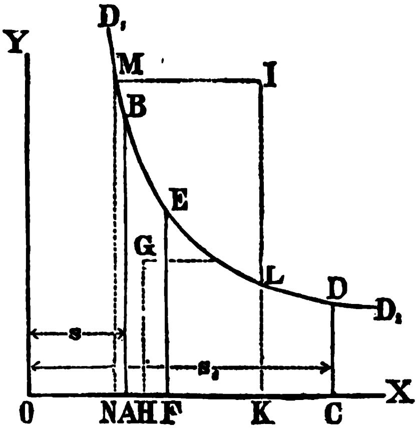

Fig. 86.

If, therefore, from any point E of the curve D_1D_1 we let fall EF, we have the pressure for any temperature. This last can be found directly from Table II., or calculated from the formula of Magnus (page 383). In like manner OF gives to the scale of abscissas the volume corresponding to this pressure and temperature. If, now, we suppose a point G, between the curve D_1D_1 and the axis, this corresponds to a mixture of steam and liquid. The perpendicular GH gives the pressure, but OH is not the specific *steam* volume (volume of one kilogram), but the specific volume of the mixture, which we denote by

$$v = xu + \sigma.$$

Since we can determine GH in the same way as EF, and since for each pressure the value of u is known, and since OH gives v, x is given by

$$x = \frac{v - \sigma}{u}.$$

If we wish to find the specific steam volume for the pressure GH, it is only necessary to draw a parallel to OX through G till it intersects D_1D_1, and a perpendicular from this point cuts off upon OX the required volume. All points, therefore, between D_1D_1 and the axes are points which relate to a mixture of steam and water. For each of these points there is a curve of constant steam weight.

If we take a point I on the other side of D_1D_1, this relates to superheated steam. The perpendicular IK gives the pressure and OK the corresponding steam volume. Since IK is greater than LK, the pressure for the same volume of saturated steam, the temperature must be higher. If we draw IM, M is that point for which saturated steam of the same pressure possesses the less volume ON. The further I is from the curve D_1D_1, the more is the steam superheated, and the more perfectly the formulæ for perfect gases apply.

The Law of Hirn.—We suppose in a space v, 1 kilogram of saturated steam confined. Let it flow into a vacuum, or conceive the vessel enlarged. Then the steam will occupy a greater volume, and be no longer saturated. By this operation no outer work is performed. In the case of gases, as we know, the temperature would be constant, since there is no disgregation work. Their inner work is then unchanged. Now Hirn asserts (Zeuner, Mech. Wärmetheorie, page 435), *that for steams also, from the point of saturation up to that where they have the deportment of gases, the* INNER WORK *must be constant, when they expand in a vacuum, or adiabatically without performing outer work.*

There is indeed no reason, from the standpoint of the mechanical theory of heat, for calling in question the truth of this principle. Now, we know that the so-called isodynamic curve is that which gives the change of condition when the inner work is constant. This curve then must give the deportment of steam, when it expands in a vacuum, from its point of saturation up to that where it coincides with a gas in its properties.

Hirn concludes further that the law of this curve is given by

$$pv = p_1v_1 = p_2v_2, \text{ etc.},$$

that is, by the same equation which, in the case of gases, gives the isothermal curve. If, starting with this principle, we calculate the specific volume of superheated steam for various temperatures and pressures, we find certainly a very satisfactory agreement with experimental results. That the principle cannot be perfectly valid will appear from the following: We have already seen (page 434) that, as shown by Zeuner, the law for the isodynamic curve of saturated steam is given very exactly by

$$pv^n = p_1v_1^n = \text{etc.},$$

where $n = 1.0456$. Although this value of n is indeed not far from 1, it follows that saturated steam, when it expands but a very little, or is but little superheated, cannot suddenly pass into a condition where the equation

$$pv = p_1v_1 = p_2v_2$$

holds good. Near the point of condensation, therefore, the isodynamic curve of steam must follow a somewhat different law from Hirn's. It thus seems justified when we assume that in the adiabatic expansion of saturated steam in an empty space, the law of change of condition is given by

$$pv^n = p_1v_1^n$$

where the value of n changes from $n = 1.0456$ to $n = 1$, which corresponds to a perfect gas.

Calculation of the Specific Volume of Superheated Steam by Hirn's Law.—Let BF be the curve given by

$$pv = p_1v_1 = p_2v_2,$$

or the isodynamic curve of superheated steam according to Hirn. We have then

$$p : p_1 : p_2 = v_2 : v_1 : v.$$

Through B pass the curve GG of constant steam weight.

Suppose one unit weight of steam at say 5 atmospheres tension (p) to expand in a vacuum, so that the pressure p_2 at the end is only 1 atmosphere. Then, according to Hirn, the entire inner work for the pressure p_2 is equal to that at p. Denote this inner work by U_1. The line $U_1 U_2$, parallel to OX, intersects GG at U_2. This line, then, gives the law according to which saturated steam of 1 kilogram and volume v_4 expands under constant pressure to v_2, and thus passes into the superheated condition. If the specific heat is c_p, and if we assume that c_p is constant during the expansion (which, according to Regnault's experiments, is nearly true), and if the absolute temperature at U_2 is T_2, and at U_1, T_1, we have for the heat imparted

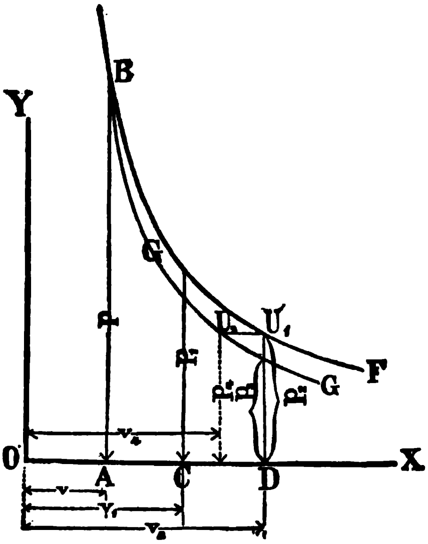

Fig. 87.

$$c_p (T_1 - T_2).$$

The change in inner work caused by this heat, measured in heat units, is

$$A(U_1 - U_2),$$

and the outer work performed is

$$p_2 (v_2 - v_4),$$

or in heat units

$$Ap_2 (v_2 - v_1).$$

Now, according to Hirn's law, the inner work U_1 in the state $p_2 v_2$ is equal to that in the state pv, or equal to that at B. As soon as we know p, then U_1 or $q + \rho$ is known. We also know then the inner work in the state $p_4 v_4$, U_2, or $q_2 + \rho_2$.

We have thus

$$A \ (U_1 - U_2) = q + \rho - (q_2 + \rho_2).$$

Hence

$$c_p (T_1 - T_2) = q + \rho - (q_2 + \rho_2) + Ap_2 (v_2 - v_4).$$

But now $p_2v_2 = pv$, or $v_2 = \frac{pv}{v_2}$.

Hence

$$c_p(T_1 - T_2) = q + \rho - (q_2 + \rho_2) + Ap_2\left(\frac{pv}{p_2} - \frac{p_2v_4}{p_2}\right)$$

$$= q + \rho - (q_2 + \rho_2) + A(pv - p_2v_4)$$

$$= q + \rho - (q_2 + \rho_2) + A(pv - p_4v_4).$$

But $v = u + \sigma$ and $v_4 = u_4 + \sigma$.

Hence

$$c_p(T_1 - T_2) = q + \rho - q_2 - \rho_2 + Ap(u + \sigma) - Ap_4(u_4 + \sigma)$$

$$= q + \rho + Apu - (q_2 + \rho_2 + Ap_4u_4) + Ap\sigma - Ap_4\sigma.$$

Now $q + \rho + Apu$ is the total heat W of 1 kilogram of steam in the state pv, and $q_2 + \rho_2 + Ap_4u_4$ is the total heat W_2 in the condition p_4v_4. We have then

$$c_p(T_1 - T_2) = W - W_2 + A\sigma(p - p_4). \quad . \quad (\text{CVI.})$$

Instead of $T_1 - T_2$, we can put $t_1 - t_2$. Further

$W = 606.5 + 0.305t$, and $W_2 = 606.5 + 0.305t_2$, hence

$$W - W_2 = 0.305\,(t - t_2).$$

We have therefore

$$c_p(t_1 - t_2) = 0.305\,(t - t_2) + A\sigma(p - p_2).$$

If, then, the specific heat c_p is known, we have for the superheating of the steam alone U_2U_1,

$$t_1 - t_2 = \frac{0.305\,(t - t_2) + A\sigma(p - p_2)}{c_p}. \quad . \quad (\text{CVII.})$$

and for the temperature of the superheated steam in the state p_2v_2

$$t_1 = t_2 + \frac{0.305\,(t - t_2) + A\sigma(p - p_2)}{c_p}. \quad (\text{CVIII.})$$

Our formula then gives us the temperature t_1 of superheated steam, when saturated steam of any pressure p and temperature t expands in a vacuum, down to the pressure p_2. We can find from

$$pv = p_2v_2$$

the volume v_2 of the superheated steam of the pressure p_2. If we subtract the volume v_4 of saturated steam of the same pressure $p_4 = p_2$, we can find by how much the volume v_4 of this steam must be heated under constant pressure, apart from water, in order that it may have the volume v_2. An example will make this clearer.

EXAMPLE.

We have 1 kilogram of dry saturated steam of $p = 3$ atmospheres. What temperature will it have when it expands in vacuo down to 1 atmosphere? How many degrees must saturated steam of 1 atmosphere be heated under constant pressure, in order that for the same temperature it may have the same volume?

For $p = 3$ we have $t = 133.91°$; for $p_2 = 1$, $t_2 = 100°$. According to Regnault, $c_p = 0.4805$, hence

$$t_1 - 100 = \frac{0.305\,(133.91 - 100) + \frac{1}{424} \times 0.001 \times 2}{0.4805},$$

For Fahrenheit degrees, $$t_1 - 212 = \frac{0.305\,(273.038 - 212) + \frac{1}{772} \times 0.061 \times 2}{0.4805}$$

$$t_1 = 100 + 21.63 = 121.68° \text{ Centigrade.}$$
$$t_1 = 212 + 70.815 = 282.815 \text{ Fah.}$$

The temperature then falls from 133.91 to 121.63, or 12.28°. Since no outer work is performed in the expansion, this loss of vibration work must be ascribed entirely to the disgregation work.

The specific volume v for $p = 3$ is

$$v = u + \sigma = 0.587,$$

hence we can find v_2 from $pv = p_2v_2$, or

$$v_2 = \frac{pv}{p_2} = \frac{3 \times 0.587}{1} = 1.761 \text{ cubic meters.}$$

The volume is then three times as great. The specific volume of saturated steam of the pressure $p_2 = p_4 = 1$, is 1.650.

If, then, saturated steam at 1 atmosphere is heated under constant pressure, until its temperature is 121.63°, that is until it is raised 21.63, its volume is $\frac{1.761}{1.650} = 1.067$ times increased.

If we disregard in our formula the member $A\sigma(p - p_2)$, we have the very simple equation

$$t_1 = t_2 + \frac{0.305\,(t - t_2)}{c_p} \quad . \quad . \quad . \quad . \quad . \quad (\text{CX.})$$

If, for example, we know from experiment what volume v_2 one kilogram of saturated steam of, say, 1 atmosphere tension assumes, when heated from $t_2 = 100°$ to $t_1°$, we can find the temperature t which 1 kilogram of saturated steam must possess when, in expanding in vacuo and cooling to t_1, it has the same volume. Thus

$$t = \frac{(t_1 - t_2)\,c_p + 0.305t_2}{0.305} \quad . \quad . \quad . \quad . \quad (\text{CXI.})$$

Thus Hirn found that saturated steam of 1 atmosphere, when heated under this pressure to 148.5°, occupies a space of 1.87 ($= v_2$) cubic meters. How great must be the temperature t of that saturated steam which, after expanding in vacuo down to 1 atmosphere, shall have the same temperature and volume?

We have $$t = \frac{(148.5 - 100)\,0.4805 + 30.5}{0.305} = 176.4°.$$

This temperature corresponds to a pressure of 9.20 atmospheres. The specific volume of this steam is 0.203 cubic meters. Since at the end of expansion the pressure must be 1 atmosphere, we have from $pv = p_2v_2$

$$9.20 \times 0.203 = 1 \times v_2, \quad \text{or} \quad v_2 = 1.87 \text{ cubic meters},$$

or exactly as found by Hirn.

By means of our formulæ we can also find what volume 1 kilogram of saturated steam, of given pressure, has when it is superheated to any degree under constant pressure. Suppose we have 1 kilogram of saturated steam at 3 atmospheres, whose temperature is 133.9°, and that we heat it 30°. What volume will it then have?

We have

$$t = \frac{(163.9 - 133.9)\,0.4805 + 133.9 \times 0.305}{0.305} = 181.2.$$

This temperature of 181.2° corresponds to a pressure of about 10.25 atmospheres, and a specific volume of 0.184 cubic meters. We have then for the volume v_2 required $(pv = p_2v_2)$,

$$10.25 \times 0.184 = 3v_2, \quad \text{or} \quad v_2 = 0.629.$$

If then 1 kilogram of saturated steam at 3 atmospheres is heated 30°, it expands from 0.586 to 0.629, or 0.043 cubic meter. It follows that saturated steam of a high pressure expands less for a given superheating than that of lower pressure. We see, also, that we cannot use our formula for great degree of superheating and high pressures, because t soon becomes so great as to exceed the limits of our Table II.

Let us turn once more to our figure. Suppose 1 kilogram of saturated steam in the condition v_2p_3, and assume that it is heated under constant volume v_2. If the temperature is t_3 and becomes by heating t_1, we can find the specific heat for constant volume. Thus the heat imparted is

$$c_v(t_1 - t_3).$$

Since no outer work is performed, this heat increases the inner work. The inner work of 1 kilogram of saturated steam in the condition v_2p_3 is $q_3 + \rho_3 = J_3$, and in the condition p_2v_2, $q_2 + \rho_2 = J_2$.

Hence

$$c_v(t_1 - t_3) = J_2 - J_3.$$

Now the inner heat J_2 is the inner heat at B, according to Hirn's law, which we denote by J. Hence

$$c_v(t_1 - t_3) = J - J_3,$$

or

$$c_v = \frac{J - J_3}{t_1 - t_3} \quad . \quad . \quad . \quad . \quad . \quad \text{(CXII.)}$$

We have found above for the specific volume v_2 of 1 kilogram heated under 1 atmosphere 21.63°, $v_2 = 1.76$. Saturated steam, which by expanding in vacuo has this temperature, must have a pressure of 3 atmospheres, or a temperature of 133.91°. For this temperature we have the inner work $J = 604.47$. This inner work is possessed by the 1 kilogram of steam after expansion, when its pressure is $p_2 = 1$. We find from Table II. what temperature saturated steam possesses whose specific volume is 1.761. We find by interpolation about 98.9°. For this temperature t_3 we have $J_3 = q_3 + \rho_3 = 596.67$. Hence

$$c_v = \frac{604.47 - 596.67}{121.63 - 98.9} = 0.343.$$

More exact calculation gives 0.347.

Just as for gases, then, the specific heat for constant volume is less than for constant pressure. The ratio k is

$$k = \frac{c_p}{c_v} = \frac{0.4805}{0.347} = 1.385.$$

Hirn has found for various degrees of superheating, the specific volume of steam expanding under constant pressure, the following experimental results:

100°	$v_2 = 1.65$ (saturated).	162°	$v_2 = 1.93$
118.5	1.74	200	2.08
141	1.85	205	2.14
148.5	1.87	246	2.29

With these data Zeuner has computed by Equation CVIII. a table, in order to see how far calculation agrees with experiment, and thus to test the validity of Hirn's law.

SPECIFIC VOLUME AND TEMPERATURE OF SUPERHEATED STEAM ACCORDING TO ZEUNER.

Atmos.	$v_2 = 1.65$	$t = 100$
$p = 2$	1.72	113.1
3	1.76	121.6
4	1.79	128.1
5	1.82	133.4
6	1.84	137.8
7	1.86	141.8
8	1.87	145.3
9	1.89	148.5
10	1.90	151.4
11	1.91	154.1
12	1.92	156.7
13	1.93	159.1
14	1.94	161.3

If we compare the numbers in this table with the experimental results of Hirn, we find a good agreement.

APPENDIX TO CHAPTER XXIII.

THEORY OF SUPERHEATED STEAM.

Zeuner has deduced a formula for superheated steam* which holds good equally well for saturated steam also, which enables us to find easily the volume from the pressure and temperature, or inversely, and which agrees very closely with experimental results. We give here an abstract of his article.

The formula to which the discussion conducts, is

$$pv = BT - Cp^{\frac{k-1}{k}},$$

where B and C and k are constants, whose values are

$$B = \frac{c_p\,(k-1)}{Ak}, \qquad c_p = 0.4805, \qquad k = 1.333, \qquad C = 192.50,$$

Hence $pv = 50.933\ T - 192.5\sqrt[4]{p}$ where the temperature is Centigrade and p is given in kilograms per square meter, and v in cubic meters. We see that this equation differs from that for permanent gases

$$pv = RT$$

only in term $Cp^{\frac{k-1}{k}}$, or $C\sqrt[4]{p}.$

If p is given in atmospheres, v in cubic meters and temperature Centigrade, we have $pv = 0.0049287\ T - 0.187815\sqrt[4]{p}.$

If p is in atmospheres, v in cubic feet and temperature Fahrenheit, we have $pv = 0.043862\ T - 3.00859\sqrt[4]{p}.$

If p is in pounds per square inch and v in cubic feet and temperature Fahrenheit, we have $pv = 0.644592\ T - 22.581875\sqrt[4]{p}.$

If we use this formula for *saturated steam*, since for a given pressure there is but one corresponding temperature, we have only to insert the given p and corresponding t, and we can calculate v, the specific volume. The specific volumes thus calculated agree perfectly with those calculated from the mechanical theory of heat, within ordinary limits of pressure, from 1 to 14 atmospheres, as we shall see in the following discussion.

To use the formula for *superheated steam* for a given pressure, we can find v for any desired temperature greater than the corresponding temperature for saturated steam. Volumes thus calculated agree very closely with those given by Hirn's experiments, as will be seen hereafter.

The formula is thus perfectly reliable, and enables us to solve many practical problems which otherwise are incapable at present of solution. The importance of such a formula cannot be overestimated, and we therefore devote a few pages to a presentation of Zeuner's method of deduction. We shall have occasion to use the Calculus, but that need not deter any reader from satisfying himself, at least as to the reliability of the formula, and familiarizing himself with its use.

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* Zeitschrift des Vereins deutscher Ingenieure, Bd. XI., p. 1, 1866.

It has been often observed that the use of superheated steam instead of saturated in the steam engine possesses special advantages. Observations and experiments, among which those of Hirn hold a high place, point decisively to this conclusion, and indicate that engines working with superheated steam are more economical in fuel, and must come more and more into favor.

For more than a decade experiments to determine the advantages of superheated steam have been made, especially in America ; and when we consider the extremely favorable results of Wethered with mixtures of saturated and superheated steam, it seems remarkable that the use of superheated steam has not had a more extended application.

Apart from certain practical difficulties in the application of highly superheated steam, we may find an explanation of this in the fact, that although all experiments thus far have proved the advantages of the use of superheated steam, yet the *degree* of advantage is uncertain. The results, in this respect, are very contradictory, and some are very properly received not without incredulity.

In such a state no amount of experiments can avail, because the question can only be decided by a thorough knowledge of the physical properties of steam in general. A theory of superheated steam is also of great scientific importance. We know at present only the deportment of such steam at its two limiting states, viz., at its point of condensation when it passes into the saturated condition, and in the highly superheated state in which its properties coincide with those of a permanent gas.

The formulæ of the mechanical heat theory for these two conditions are entirely different both in construction and method of deduction, and thus far it has not been possible to deduce from the equations for saturated steam, or for steam and liquid mixtures, those for permanent gases or reversely, or to represent the deportment of steam in transition from one to the other of the limiting conditions.

Theoretical investigations upon the deportment of superheated steam have, thus far, been made by Hirn only. In what follows we shall present the results of our own investigations, together with applications to the most important technical problems. We shall confine our discussion to steam of water, but there will be no difficulty in extending the method to other steams.

Preliminary Investigation.—We denote by v the specific volume, *i. e.*, the volume of the unit of weight (one kilogram) of steam, by p the specific pressure (pressure in kilograms per square meter), and by t the temperature Centigrade. If pressure and volume are given, we can easily determine whether in any given case we have to do with pure saturated, or superheated steam, or with a mixture of steam and liquid. For saturated steam, whose volume we shall denote by v_1, pressure and volume stand in a known determinate relation, and from the principles of the mechanical heat theory we can calculate the volume v' corresponding to the pressure p. If we lay off for pure saturated steam (without admixture of water) the volumes as abscissas and the corresponding pressures as ordinates, we obtain a curve DD of constant steam weight, which I call "the limiting curve," the course of which we shall investigate hereafter. For every point of this curve, or for every pressure there is a corresponding determinate absolute temperature T_1, known by experiment.

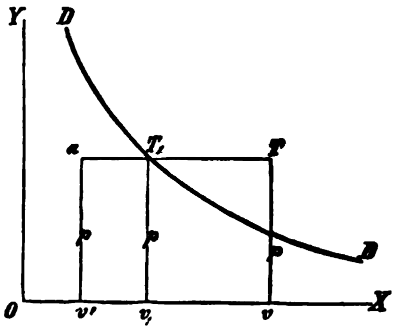

FIG. 88.

If now in any given case we lay off the volume of the unit of weight as abscissa, and the pressure as ordinate, and the point thus found falls on the limiting curve, we know that we have to do with pure saturated steam. If, however, the point a falls in the space between the limiting curve and the co-ordinate axes, we have for the same pressure, the same temperature T_1, but a less volume. We have, therefore, a mixture of steam and water. The steam is of the same character as before. If x is the specific steam quantity, *i. e.*, the weight of steam in the unit weight of mixture, then $1 - x$ is the weight of the water, and if σ is the specific volume of the water the volume v' of the mixture is

$$v' = xv_1 + (1 - x)\sigma,$$

and from this we can easily calculate the mixture ratio x for the given volume v'.

If, again, the point falls in the space outside of the limiting curve, as T, we have to do with superheated steam. In this case the temperature $T > T_1$, and is not given by the pressure p alone, but depends also upon the volume v. This relation

$$T = F(p, v)$$

is that which thus far is unknown for superheated steam, and it is this which we shall first deduce. We shall call this relation the equation of condition, or "condition equation." Thus far it has only been assumed that the condition equation takes the form

$$pv = RT \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

which holds good for permanent gases, in which R is a constant depending upon the kind of gas, when the point T, in Fig. 88, lies very far from the limiting curve, that is, when the steam is highly superheated.

In the deduction of the condition equation for superheated steam, we make use of the following principle of the mechanical heat theory. (See Appendix to Chap. V.)

If the pressure p_2 and the volume v_2 are given for the unit weight, and the body expands or is compressed adiabatically, the end of the ordinate describes a curve A_2A_2, Fig. 89, called the adiabatic curve. If the body is in the condition a_1, given by the pressure p_1 and the volume v_1, the point a_1 lies upon a second adiabatic curve A_1A_1. If the body passes from the condition a_2 to the condition a_1, and if heat dQ is the heat imparted or abstracted during an infinitely small change of condition, then $\int \frac{dQ}{AT}$ is constant, no matter how the pressure p changes with the volume v during the transfer, that is, no matter what the curve a_2a_1 may be, wherever the point a_2 may be on the first adiabatic, or the point a_1 on the second. This quantity we have called the "heat weight," and for the sake of simplicity we denote it by P. Let us now determine the heat weight for a mixture of water and steam.

In the unit of weight of mixture let there be x kilograms of steam at the pressure p and tension t. Let c be the specific heat of water, which, according to Regnault, is

$$c = 1 + 0.00004t + 0.0000009t^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (2)$$

and let r be the total latent heat, which, according to Regnault, is

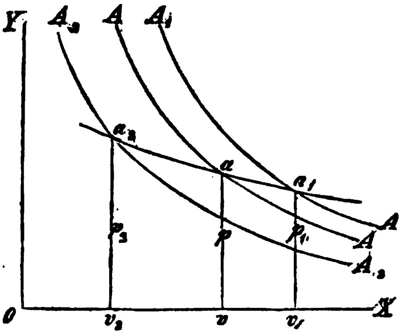

Fig. 89.

$$r = 606.5 + 0.305t - \int_0^t cdt. \quad (3)$$

then we have, according to Clausius,

$$dQ = cdt + Td\left(\frac{xr}{T}\right).$$

If we divide both sides by AT, and put, for the sake of simplicity,

$$\int \frac{cdt}{T} = \tau \quad . \quad . \quad (4)$$

we have, using the index 2 for the condition a_2 and 1 for a_1,

$$P = \int \frac{dQ}{AT} = \frac{1}{A}\left[\left(\tau_1 + \frac{x_1r_1}{T_1}\right) - \left(\tau_2 - \frac{x_2r_2}{T_2}\right)\right] \quad . \quad . \quad . \quad (5)$$

If we assume that the transfer takes place along the limiting curve DD, as shown in Fig. 88, we have for this curve

$$x_1 = x_2 = 1,$$

and

$$AP = \left(\tau_1 + \frac{r_1}{T_1}\right) - \left(\tau_2 + \frac{r_2}{T_2}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

and this can be easily calculated for given initial and final temperatures.

For the sake of simplicity we put $\varphi = \tau + \frac{r}{T}$ . . . . (7)

and then

$$AP = \varphi_1 - \varphi_2. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8)$$

Using Equations (2), (3) and (4), we have calculated φ as given by (7) for a number of values of the pressure, and placed the results in column 8 of the following Table I.

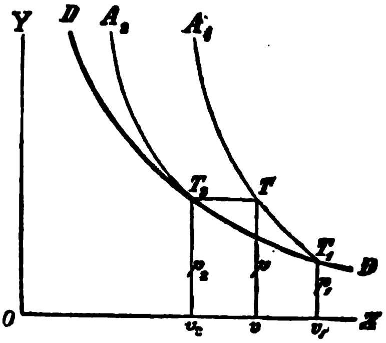

FIG. 90.

We pass now through the two points T_2 and T_1 of the limiting curve *DD*, Fig. 90, the two adiabatics A_2 and A_1. The course of both curves is unknown, since they extend into the space corresponding to superheated steam. If now we heat the saturated steam at the temperature T_2, *under constant pressure* $p_2 = p$, until we reach the second adiabatic at the point *T*, we have for the heat weight, when the specific heat c_p is taken as *constant*,

$$P = \int \frac{dQ}{AT} = \int_{T_2}^{T} \frac{c_p\,dT}{AT} = \frac{c_p}{A} \log \frac{T}{T_2}.$$

Since the heat weight from T_2 to T_1 is the same, we have

$$c_p \log \frac{T}{T_2} = \varphi_1 - \varphi_2 \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (9)$$

This formula holds good, to be sure, only under the express assumption that c_p, the specific heat of steam for constant pressure, is constant. That this assumption is allowable is shown by the experiments of Regnault. He finds by four experiments

$$c_p = 0.46881,\ 0.48111,\ 0.48080,\ 0.47963,$$

and considers only the first of these values not entirely reliable. We have then for the mean

$$c_p = 0.4805,$$

and this value of c_p for steam we shall assume in all further discussions. We shall see further on that this hypothesis of the constancy of the value of the specific heat for constant pressure, and the assumption of the correctness of Regnault's experimental values, are justified.

By the aid of Equation (9) we can find easily the temperature *T* of superheated steam, when the temperatures T_2 and T_1 are given, or by taking different values for T_2 can calculate for a number of points of the adiabatic through T_1, the temperature *T* of the superheated steam. Such a proceeding would lead to little, and the actual course of the curve A_1 would be in no way known. We must rather transform Equation (9). If we add and subtract $c_p \log T_1$ on the left side, we have

$$c_p \log \frac{T}{T_1} = \varphi_1 - \varphi_2 - c_p \log \frac{T_1}{T_2} \quad . \; . \; . \; . \; . \quad (10)$$

From this we can find the ratio $\frac{T}{T_1}$ for two points upon the *same* adiabatic A_1. If this curve answered to a permanent gas, we should have between pressure and temperature of both points the relation

$$\frac{T}{T_1} = \left(\frac{p}{p_1}\right)^{\frac{k-1}{k}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (11)$$

where k is constant and denotes the ratio of the specific heat of the gas for constant pressure to that for constant volume.

I make now the hypothesis *that for superheated steam, or at least for water-steam, Equation* (11) *also holds good,* and that here also k is constant, but the previous signification of k no longer holds for superheated steam. The following mathematical discussion must fix the more general significance of the value of k.

Substituting Equation (11) in (10), we have, since in the entire discussion, as shown in Fig. 90, the pressure p is identical with p_2,

$$\varphi_1 + c_p \log \frac{p_1^{\frac{k-1}{k}}}{T_1} = \varphi_2 + c_p \log \frac{p_2^{\frac{k-1}{k}}}{T_2},$$

and hence it follows, in case of the correctness of our hypothesis, generally the value

$$\varphi + c_p \log \frac{p^{\frac{k-1}{k}}}{T}$$

for *saturated* steam must be a constant. If we denote this constant by φ_0, we have instead of (7)

$$\varphi = c_p \log \frac{T}{p^{\frac{k-1}{k}}} + \varphi_0 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12)$$

That this formula, with a proper choice of the constants k and φ_0, actually gives the values of φ with great exactness, may be shown by the following calculations.

We take $k = \frac{4}{3} = 1.3333$, hence $\frac{k-1}{k} = \frac{1}{4} = 0.25$, and $\varphi_0 = 1.0933$. Then in common logarithms

$$\varphi = 0.2766 \log_{10} \frac{T^4}{p} - 1.0933 \quad . \quad . \quad . \quad . \quad . \quad . \quad (13)$$

where p is in atmospheres, and the temperature to be taken from Regnault's Tables.

The agreement of the values of φ, as given by this formula and by (7), is shown in the following table. The agreement might be made still greater by taking k a little different from $\frac{4}{3}$, but for the sake of simplicity in numerical calculations, we retain this round number.

TABLE I.

1 Pressure in Atmos. p.	2 Temperature Cent. according to Regnault, t.	3 $\phi = \tau + \frac{r}{T}$.	4 ϕ According to Eq. (13).
0.1	46.21	1.9548	1.9538
0.2	60.45	1.8929	1.8912
0.5	81.71	1.8116	1.8111
1	100.00	1.7519	1.7520
2	120.60	1.6940	1.6946
3	133.91	1.6612	1.6619
4	144.00	1.6384	1.6391
5	152.22	1.6209	1.6217
6	159.22	1.6071	1.6076
7	165.34	1.5955	1.5958
8	170.81	1.5856	1.5858
9	175.77	1.5769	1.5769
10	180.31	1.5691	1.5691
11	184.50	1.5625	1.5621
12	188.41	1.5563	1.5557
13	192.08	1.5506	1.5508
14	195.53	1.5454	1.5445

The great probability of the correctness of our hypothesis may also be shown in the following manner:

If we differentiate Equation (13), we have

$$d\varphi = c_p \frac{dt}{T} - c_p \frac{k-1}{k} \frac{dp}{p} \quad . \; . \; . \; . \; . \; . \; . \quad (14)$$

If we differentiate Equation (7), we have, with reference to the value of τ as given by Equation (4)

$$d\varphi = \frac{1}{T}\left(c + \frac{dr}{dt} - \frac{r}{T}\right) dt.$$

The expression in the parenthesis is the same as that which we have called (Chap. XVII., page 420) the specific heat of saturated steam for constant steam quantity, and denoted by h. We can, therefore, easily calculate it. We have then

$$d\varphi = \frac{h}{T} dt \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (15)$$

The union of (14) and (15) gives then

$$h = c_p - c_p \frac{k-1}{k} \frac{Tdp}{pdt} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (16)$$

We can use this formula as proof of the correctness of our hypothesis. Thus we have for steam at the temperature 0°, 100°, 200°,

$$h = -1.9166, \quad -1.1333, \quad -0.6766.$$

We have from Regnault's formula, which gives the relation between pressure and temperature,

$$\frac{Tdp}{pdt} = 19.520, \quad 13.344, \quad 9.851.$$

Inserting these values in (16), and assuming c_p as constant, and according to Regnault 0.4805, we find for k, for the above temperatures,

$$k = 1.3434, \quad 1.3362, \quad 1.3234.$$

If, on the other hand, we take, as we have done, $k = \frac{4}{3} = 1.3333$, we have from Equation (16),

$$c_p = 0.49397, \quad 0.48514, \quad 0.46255,$$

and the mean of these is 0.4805, or, strangely enough, exactly Regnault's mean value.

From the preceding, then, we conclude that the quantities c_p and k, even if future exact investigations may show that one or the other, or both, are variable, vary very slowly, and may at present be assumed as constant, with the values as given above. The results of the following investigations will further justify these assumptions.

Deduction of the Equation of Condition for Superheated Steam.—The "condition equation" must give the relation between p, v, and t, or $T = 273 + t$. If we consider the absolute temperature as a function of the pressure and volume, we have

$$dT = \left(\frac{dT}{dp}\right) dp + \left(\frac{dT}{dv}\right) dv, \quad \text{or,}$$

since we can replace dT by dt,

$$dT = \left(\frac{dt}{dp}\right) dp + \left(\frac{dt}{dv}\right) dv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (17)$$

The fundamental equations of the mechanical heat theory, as given by Zeuner, are

$$A = \frac{d}{dp}\left[c_p\left(\frac{dt}{dv}\right)\right] - \frac{d}{dv}\left[c_v\left(\frac{dt}{dp}\right)\right] \quad \ldots\ldots \quad \text{(I.)}$$

$$AT = (c_p - c_v)\left(\frac{dt}{dv}\right)\left(\frac{dt}{dp}\right) \quad \ldots\ldots\ldots \quad \text{(II.)}$$

$$\left.\begin{aligned} dQ &= c_v\left(\frac{dt}{dp}\right)dp + c_p\left(\frac{dt}{dv}\right)dv \\ dQ &= c_v\,dt + AT\left(\frac{dp}{dt}\right)dv \\ dQ &= c_p\,dt - AT\left(\frac{dv}{dt}\right)dp \end{aligned}\right\} \quad \ldots\ldots \quad \text{(III.)}$$

If we divide the last equation by T, we have

$$\frac{dQ}{T} = c_p\frac{dt}{T} - A\left(\frac{dv}{dt}\right)dp.$$

But according to the notation of Equations (5) and (8), $\frac{dQ}{T}$ is identical with $d\varphi$, and for this latter we have found according to (14)

$$d\varphi = c_p\frac{dt}{T} - c_p\frac{k-1}{k}\frac{dp}{p}.$$

Comparing these two expressions, we have for the first new relation for superheated steam

$$c_p\left(\frac{dt}{dv}\right) = \frac{Akp}{k-1} \quad \ldots\ldots\ldots\ldots \quad (18)$$

Inserting this in Equation I., we have, since c_p and k are constant,

$$\frac{d}{dv}\left[c_v\left(\frac{dt}{dp}\right)\right] = \frac{A}{k-1},$$

and hence by integration

$$c_v\left(\frac{dt}{dp}\right) = \frac{Av}{k-1} \quad \ldots\ldots\ldots\ldots \quad (19)$$

wherein we assume indeed that the constant of integration, which in general is a function of p, is zero. This assumption will be justified by the correspondence of calculated results with those of observation hereafter.

If we make use of (18) and (19) in Equation II., we have, after easy transformation,

$$\frac{c_p}{c_v} = 1 + c_p \frac{(k-1)^2}{k} \frac{T}{Apv} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (20)$$

If we determine c_v from this and insert in (19), we have

$$\left(\frac{dt}{dp}\right) = \frac{Av}{c_p(k-1)} + \frac{k-1}{k} \frac{T}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

while Equation (18) gives

$$\frac{dt}{dv} = \frac{Akp}{c_p(k-1)} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

The substitution of these two in Equation (17) gives then

$$dT = \frac{A}{c_p(k-1)}(vdp + kpdv) + \frac{k-1}{k} \frac{T}{p} dp \quad . \quad . \quad . \quad . \quad (23)$$

and *this is the differential equation of the equation of condition for superheated steam.*

This can be integrated easily. We have then

$$pv = BT - Cp^{\frac{k-1}{k}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

where B and C are constants, and

$$B = \frac{c_p(k-1)}{Ak} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (25)$$

Now, for superheated steam, we have from the preceding $c_p = 0.4805$ and $k = 1.333$. Hence $B = 50.933$. The other constant can be easily determined, as we shall soon see.

Equation (24) is the condition equation for superheated steam. By means of it, when any two of the quantities p, v, and T are given, the third can be found. It differs from the equation for permanent gases only in the term $Cp^{\frac{k-1}{k}}$, which becomes, for superheated steam, $C\sqrt[4]{p}$.

Test of the New Equation.—If the equation for superheated steam is correct, it must hold for the limiting condition also, that is, when the steam is in the saturated condition. It should therefore give the specific volume of saturated steam when we substitute for a given pressure p the corresponding temperature t. This leads us at once to the value of the constant C.

Thus the mechanical theory of heat gives for saturated steam of one atmosphere tension ($p = 10334$), and $t = 100°$, or $T = 373°$, the volume v of one kilogram, $v = 1.6506$ cubic meters. Using these values in Equation (24), we find $C = 192.50$.

If now the equation is correct, it should give the specific volume of saturated steam for all other pressures. How far this is so is shown by Table II.

The second column gives the specific volumes for various pressures, according to the mechanical theory of heat. The third column is calculated from Equation (24). It is to be remarked that the values given for B and C hold good when the pressure p is given in kilograms per sq. meter. If we wish p in atmospheres, we have

$$\left.\begin{array}{l} pv = BT - C\sqrt[4]{p} \\ B = 0.0049287, \quad \text{and} \quad C = 0.187815 \end{array}\right\} \quad . \quad . \quad . \quad . \quad (26)$$

If p is in atmospheres, v in cubic feet and temperature Fahrenheit, we have $B = 0.043862$ and $C = 3.00859$.

The table contains also, in the last column, the values of $\frac{C}{B}\sqrt[4]{p}$, for French and English units, of which use will be made in what follows.

We see that the agreement is very satisfactory, and that we can use the above equation for pure saturated steam. Only for pressures of less than one atmosphere is the deviation on any account. For such pressures, however, the values in the second column are not entirely reliable. By the slightest change in the constants used, we might also produce for such steam still better agreement. It would only be necessary to do as Regnault has done in the construction of his formulæ for the relation between pressure and temperature, and to distinguish steam of more than one atmosphere pressure from that of less. For our purposes it will be sufficient to retain the constants as given already, since we have

TABLE II.

Pressure in atmos.	Specific Volume of Saturated Steam		Values of $\frac{C}{B}\sqrt[4]{p}$ French units.	Values of $\frac{C}{B}\sqrt[4]{p}$ English Units.
	by Mech. Theory of Heat.	by Eq. (24).		
0.1	14.552	14.677	21.429	38.5722
0.2	7.543	7.583	25.483	45.8694
0.5	3.171	3.181	32.043	57.6774
1	1.6504	1.6506	38.106	68.5908
2	0.8598	0.8588	45.316	81.5688
3	0.5874	0.5861	50.151	90.2713
4	0.4484	0.4474	53.891	97.0038
5	0.3686	0.3630	56.982	102.5676
6	0.3064	0.3060	59.640	107.352
7	0.2652	0.2650	61.983	111.5694
8	0.2339	0.2339	64.087	115.3566
9	0.2095	0.2096	66.002	118.8036
10	0.1897	0.1900	67.764	121.9752
11	0.1735	0.1739	69.398	124.9164
12	0.1599	0.1601	70.924	127.6632
13	0.1483	0.1489	72.357	130.2426
14	0.1383	0.1388	73.711	132.6798

in view more especially the needs of practice, and as in the steam engine higher steam pressures are coming into favor.

From Equation (24) or (26) we now can easily calculate the volume of superheated steam for any given pressure and temperature. If we take $t = 100, 110, 120,$ etc., and $p = 1$ in Equation (26), we have, for example, for superheated steam of one atmosphere tension the following values for the specific volume:

$t = 100°$	$v = 1.6506$ cubic meters.	$t = 160°$	$v = 1.9463$ cubic meters.
110	1.6999	170	1.9956
120	1.7492	180	2.0449
130	1.7984	190	2.0942
140	1.8477	200	2.1435
150	1.8970	210	2.1927

Hirn has observed the specific volume for different pressures and temperatures. The following table shows how excellently the results of our formula agree with his experimental results.

Pressure in Atmos.	Temperature. C.	Specific Volume in Cubic Meters.	
		Hirn.	Eq. (26).
1	118.5	1.74	1.7417
1	141	1.85	1.8526
3	200	0.697	0.6947
4	165	0.4822	0.4733
4	200	0.522	0.5164
4	246	0.5752	0.5781
5	162.5	0.3758	0.3781
5	205	0.414	0.4150

If we calculate for the same pressures and temperatures the specific volume v' of air, the ratio of v to v' gives the specific weight of the steam with reference to air. We obtain thus for one atmosphere and the temperatures 100°, 150°, 200°, the specific weights 0.6401, 0.6316, 0.6250, that is, decreasing with increasing superheating.

As another test of the reliability of our equation, we may determine the coefficient of expansion for superheated steam. If α is the coefficient of expansion, we have for gases, according to the law of Mariotte and Gay-Lussac,

$$\frac{pv}{p_1 v_1} = \frac{1 + \alpha t}{1 + \alpha t_1}.$$

For constant pressure $p = p_1$, and hence

$$\alpha = \frac{v_1 - v}{v t_1 - v_1 t}.$$

For constant volume, $v_1 = v$, and

$$\alpha = \frac{p_1 - p}{p t_1 - p_1 t}.$$

The first formula gives the coefficient of expansion for change of volume, the second for change of pressure. If we pass to the differentials, we have for change of volume

$$\alpha = \frac{1}{v\frac{dt}{dv} - t} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (27)$$

and for change of pressure

$$\alpha = \frac{1}{p\frac{dt}{dp} - t} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (28)$$

For a perfect gas both formulæ give the same value, $\alpha = 0.003665$, but not so for actual gas or steam.

If in Equation (24) we put for T, $a + t$, where $a = 273$, and differentiate, regarding first v and then p as constant, we have

$$v\frac{dt}{dv} = \frac{pv}{B}$$

$$p\frac{dt}{dp} = \frac{pv}{B} + \frac{C}{B}\frac{k-1}{k}p^{\frac{k-1}{k}}$$

These values substituted in Equations (27) and (28), and replacing pv by its equivalent in Equation (24), give us for superheated steam the coefficient of expansion α for change of volume

$$\alpha = \frac{1}{a - \frac{C}{B}p^{\frac{k-1}{k}}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (29)$$

and for change of pressure

$$\alpha = \frac{1}{a - \frac{C}{Bk}p^{\frac{k-1}{k}}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (30)$$

These two values are therefore different, and since $k > 1$, the second is always somewhat less than the first. This agrees perfectly with Regnault's observations. Also α is always greater than $\frac{1}{a} = 0.003665$, which is also confirmed by experiment. Further, α is greater the greater the pressure p, which is also confirmed by experiment. Regnault has observed even for hydrogen, which approaches nearest to a perfect gas, for different pressures, somewhat different values for the coefficient of expansion.

From Equations (29) and (30) the following values are computed for different values of p, for superheated steam. As p is given in atmospheres, the values of the constants as given in Equation (26) are used, also $\frac{k-1}{k} = \frac{1}{4}$.

	Coefficient of Expansion.	
	Change of Volume.	Change of Pressure.
$p = 0.1$	$\alpha = 0.003975$	0.003892
0.5	0.004150	0.004017
1	0.004257	0.004090
5	0.004629	0.004343
10	0.004872	0.004501

The formulæ above confirm all the facts with reference to the coefficients of expansion of gases and steams thus far made known by experiment. We may consider this, therefore, as a further proof of the reliability of our new formula. We can deduce, however, a new result from (29) and (30), viz., that the coefficient of expansion α depends only upon the pressure, and not upon the degree of superheating or the volume. There are no observations which contradict this principle. If we do not accept it in its full generality, we must at least admit that it is exact enough for superheated steam in the neighborhood of the point of condensation.

We may now finally determine more closely the specific heat of superheated steam for constant volume. For perfect gases the value of k is identical with the ratio $\frac{c_p}{c_v}$. This is not so for steam. Here we have Equation (20)

$$\frac{c_p}{c_v} = 1 + c_p \frac{(k-1)^2}{k} \frac{T}{Apv}.$$

Making use of Equations (24) and (25) we have, after reduction,

$$\frac{c_p}{c_v} = 1 + \frac{k-1}{1 - \frac{C}{B} \frac{p^{\frac{k-1}{k}}}{T}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

By means of this formula we can find for any given condition of superheated steam $\frac{c_p}{c_v}$, and then, since c_p is constant, the value of c_v. We see at once from the formula, that with increasing superheating the value of $\frac{c_p}{c_v}$ approaches k. For small pressure and very great superheating, we should have for steam $\frac{c_p}{c_v} = \frac{4}{3}$, and hence $c_v = 0.3604$, and this latter value I regard as the specific heat of steam for constant volume, when by high superheating and low pressure the steam has passed into the condition of a permanent gas. The equation also shows that the ratio $\frac{c_p}{c_v}$ increases and c_v diminishes the more the steam approaches the saturated condition. Thus, for example, Equation (31) gives for saturated steam of 0.1, 0.5, 1 and 5 atmospheres, the following values :

$p = 0.1$	0.5	1	5
$\frac{c_p}{c_v} = 1.358$	1.3664	1.3718	1.3849
$c_v = 0.3538$	0.3516	0.3504	0.3470

It appears striking that the value of the specific heat c_v for constant volume *increases* the more the steam departs from the saturated condition. According to the usual conceptions of gases and steam, we should rather expect the contrary. Further investigations must confirm our result and explain it.

The Fundamental Equations of the Mechanical Theory of Heat applied to Superheated Steam.—If the unit of weight of superheated steam is given with certain pressure, volume, and temperature, the imparting of the heat dQ will cause a change in these quantities. The Equations III. give the relations generally which subsist between these quantities. For superheated steam we have, when we make use of Equations (18) to (23), from these general equations

$$\left.\begin{aligned} dQ &= \frac{A}{k-1}\left(vdp + kpdv\right) \\ dQ &= c_p\left(dt - \frac{k-1}{k}\,\frac{T}{p}\,dp\right) \\ dQ &= c_v\left(dt + (k-1)\frac{T}{v}\,dv\right) \end{aligned}\right\} \quad \ldots\ldots \quad (32)$$

These equations do not differ from those of permanent gases in form, but c_v is here variable and to be determined by Equation (20), while for gases it is constant and $= \frac{c_p}{k}$.

If the change of state is reversible, *i. e.*, if during the change of volume the steam tension p is in equilibrium with the outer pressure, the work of the expansion dv is pdv, and the corresponding heat is $Apdv$. This portion of the heat dQ goes then to perform outer work, and the rest goes to perform vibration work (rise of temperature), and disgregation work, both of which constitute the inner work. If we denote the change of inner work by dU, we have

$$AdU = dQ - Apdv,$$

or replacing dQ by the first of Equations (32),

$$AdU = \frac{A}{k-1}d(pv). \quad \ldots\ldots\ldots \quad (33)$$

Integrating this from a given initial condition, we have

$$A(U - U_1) = \frac{A}{k-1}(pv - p_1v_1).$$

The difference $U - U_1$ gives the increase of inner work, and $A(U - U_1)$ is this difference measured in units of heat. If we assume the initial condition to be water at 0°, and let J stand for $A(U - U_1)$, we have

$$J = J_0 + \frac{A}{k-1}pv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (34).$$

where J_0 is a constant to be determined, and J shows how much more heat is contained in superheated or saturated steam of the pressure p and volume v, than in water at 0°. We have called J the "steam heat."

The formula must hold good both for superheated and saturated steam. For the latter we know already how to determine J, and thus can find not only the value of J_0, but can test whether Equation (34) holds good for saturated steam of any pressure.

The heat of the liquid, or the heat necessary to raise one kilogram of water, under the pressure p, from 0° to t° is

$$q = \int_0^t c dt,$$

where c is from Equation (2)

$$q = t + 0.00002t^2 + 0.0000003t^3.$$

If now the water under pressure p is completely changed into saturated steam, the inner latent heat ρ must be added.

For steam we have given

$$\rho = 575.40 - 0791t.$$

Now for saturated steam

$$J = q + \rho. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (35).$$

Thus, for example, for saturated steam of one atmosphere, $p = 10334$, $t = 100°$, $J = 596.80$. Inserting this value in Equation (34) and taking $v = 1.6506$, from Table II. we find

$$J_0 = 476.11.$$

For English units we have $J_0 = 857$.

Using now the values of v in Table II., we can calculate from Equation (34) the values in the last column of the following Table III., and compare with those obtained by Equation (35).

TABLE III.

Pressure in Atmos.	Steam Heat J for Saturated Steam.	
	Equation (35).	Equation (34).
1	596.80	596.80
2	601.42	601.62
3	604.47	604.68
4	606.81	606.97
5	608.73	608.81
6	610.39	610.38
7	611.86	611.74
8	613.18	612.96
9	614.38	614.05
10	615.49	615.05

The agreement is very satisfactory, and we see in this a new proof of the reliability of our formula for the deportment of superheated steam.

We obtain very remarkable results, which afford further confirmation of the correctness of our formula, when we insert Equation (24) in (34).

We have then

$$J = J_0 + \frac{AB}{k-1}\left(T - \frac{C}{B}p^{\frac{k-1}{k}}\right),$$

or using the value of B in Equation (25),

$$J = J_0 + \frac{c_p}{k}\left(T - \frac{C}{B}p^{\frac{k-1}{k}}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; (36).$$

For small tension p and high superheating we can neglect the square of the quantity

$$\frac{C}{B}\frac{p^{\frac{k-1}{k}}}{T},$$

and thus obtain from Equation (31)

$$\frac{c_p}{c_v} = k + (k-1)\frac{C}{B}\frac{p^{\frac{k-1}{k}}}{T}.$$

If we determine from this $\frac{c_p}{k}$, and insert in (36), we have for the same assumptions, after reduction,

$$J = J_0 + \left(1 - \frac{C}{B}\frac{p^{\frac{k-1}{k}}}{T}\right)c_v T \quad . \; . \; . \; . \; . \; . \; (37).$$

For very low tension and very high superheating, in which case the steam passes almost completely into the condition of a permanent gas, the second member in the parenthesis may be neglected, and we have

$$J = J_0 + c_v T,$$

and this equation is the same as we actually have for permanent gases. If we put as above $J = AU$, where U is the inner work, then

$$AdU = c_v dT,$$

from which we see that the increase of inner work is directly proportional to the rise of temperature, a principle laid down by Clausius in his Theory of Gases.

Finally, we can determine for superheated steam that which for saturated steam we have called the "total heat" and denoted by W, for which Regnault gives the empirical formula

$$W = 606.5 + 0.305t.$$

This is the heat which is necessary in order to convert the unit of weight of water, under the constant pressure corresponding to the steam temperature t, completely into saturated steam.

If we suppose superheated steam of the volume v generated under the same circumstances, the work done during its formation is $p(v - \sigma)$, and the equivalent heat is $Ap(v - \sigma)$, if σ is the volume of the unit of weight of water. We can neglect σ with respect to v, and have therefore for the total heat

$$W = J + Apv,$$

or from Equation (34),

$$W = J_0 + \frac{Ak}{k-1} pv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (38).$$

Referring to Equations (24) and (25)

$$W = J_0 + c_p \left(T - \frac{C}{B} p^{\frac{k-1}{k}} \right) \quad . \quad . \quad . \quad . \quad . \quad (39).$$

where $J_0 = 476.11$.

Since this equation must hold good also for saturated steam, we should have, in case this behaved like a permanent gas, $C = 0$, and then

$$W = J_0 + c_p T,$$

and the comparison with Regnault's formula would give for the specific heat of steam for constant pressure $c_p = 0.305$, which has been obtained by Rankine in a different manner. Equation (39) shows clearly the reason of the deviation from the correct value $c_p = 0.4805$.

Values of W calculated from Equation (39) for saturated steam, agree very satisfactorily with those found by Regnault's formula. Thus we have for

	$p =$ 0.5	1	5 atmos.
by Equation (39),	$W = 681.15$	687.02	653.05
Regnault,	$W = 681.42$	687.00	652.93

Our Equation (39) has the advantage that it holds generally good for superheated steam also.

Recapitulation of Formulæ Deduced for Superheated Steam.—[Before passing on to applications, we shall group together here, for convenience of reference and for the benefit of those who wish results presented in compact shape, the formulæ deduced in the preceding for superheated steam.

We have for both saturated and superheated steam the equation

$$\left.\begin{aligned} pv &= BT - Cp^{\frac{k-1}{k}} \\ \text{where} \quad B &= \frac{c_p(k-1)}{Ak}, \quad c_p = 0.4805, \quad k = 1.333 \end{aligned}\right\} \quad . \quad . \quad . \quad (40).$$

and hence $B = 50.933$, $C = 192.50$, when *p is in kilograms per sq. meter*, v in cubic meters and temperature Centigrade.

If p is in pounds per square inch and v in cubic feet, we have for temperature Fahrenheit $B = 0.644592$, $C = 22.581875$.

If *p is in atmospheres*, we have

$$\left.\begin{aligned} pv &= BT - C\sqrt[4]{p} \\ B &= 0.0049287, \quad C = 0.187815 \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (41).$$

For p in atmospheres, v in cubic feet, and temperature Fahrenheit, we have $B = 0.048362$, $C = 3.00859$.

We have for the ratio of c_p to c_v,

$$\frac{c_p}{c_v} = 1 + \frac{k-1}{1 - \frac{C}{B}\frac{p^{\frac{k-1}{k}}}{T}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (42).$$

Here we can take p either in atmospheres or kilograms per sq. meter or pounds per square inch according to the values taken of C and B, as given above. For saturated steam we must insert, for any given pressure, the *corresponding temperature*. For superheated steam we can insert any desired temperature greater than this. The formulæ are quite correct within practical limits (1 to 14 atmospheres). For the "steam heat" we have

$$\left.\begin{aligned} J &= J_0 + \frac{A}{k-1}pv \\ J &= J_0 + \frac{c_p}{k}\left(T - \frac{C}{B}p^{\frac{k-1}{k}}\right) \end{aligned}\right\} \quad . \quad . \quad . \quad . \quad . \quad (43).$$

where $J_0 = 476.11$ (or 857), and p is to be taken in *kilograms per sq. meter* or pounds per square inch. k and c_p are the same as always, viz., $k = \frac{4}{3} = 1.333$, $c_p = 0.4805$.

For the "total heat,"

$$W = J_0 + c_p\left(T - \frac{C}{B}p^{\frac{k-1}{k}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (44).$$

where p is in kilograms per sq. meter, or pounds per square inch, and J_0, c_p, and k as above.

The "heat of liquid" is found, as always when the temperature is known, by

$$q = t + 0.00002t^2 + 0.0000003t^3 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45).$$

or for English units, $q = (t - 32) + 0.0000111\,(t - 32)^2 + 0.0000000926\,(t - 32)^3$ or by our steam tables.

The inner latent heat is found by

$$\rho = J - q \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (46).$$

The outer latent heat by

$$Apv = W - J \, . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (47).$$

By the aid of these formulæ we may solve problems concerning superheated steam of great practical importance, and which heretofore have been impossible of solution.]

APPLICATIONS.

If we regard our equation for the deportment of superheated steam as correct (and from what precedes, the great probability of its correctness, at least for those pressures usual in practice, seems without doubt), we shall be able to solve many questions otherwise impossible of solution. Especially easy of solution are those problems of practical importance, and a theory of engines working with superheated steam presents no longer any difficulty. We shall investigate here a few of the most important cases, many of which, by reason of known experiments in relation to them, will serve as further confirmation of the practical correctness of our formulæ.

Adiabatic Curve.—If the unit of weight of superheated steam expands performing work, without heat being added or abstracted, the adiabatic curve gives the law of variation of the pressure with the volume. This curve gives also the law of the curve of expansion of the indicator diagram of a steam engine working with superheated steam. If the initial condition is given by p_1, v_1, T_1 (Fig. 91), and the final condition by p_2, v_2, and T_2, we can find by Equation (32) the relation between these quantities. If in these equations we make $dQ = 0$, we have by integration

$$p_1 v_1{}^k = p_2 v_2{}^k, \quad \frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{k-1}{k}}, \quad \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{k-1} \quad . \quad . \quad . \quad (48).$$

These are the same equations as for permanent gases, only there, for air $k = 1.41$, while in the present case, for superheated steam, $k = 1.333$.

The work L during expansion, that is, the work of each unit of weight of steam, is

$$L = \int_{v_1}^{v_2} p dv,$$

or, replacing p by the relation $pv^k = p_1 v_1{}^k$,

$$L = \frac{p_1 v_1}{k-1}\left[1 - \left(\frac{v_1}{v_2}\right)^{k-1}\right] \quad . \; . \; . \; . \; . \; . \; . \quad (49).$$

where p is in kilograms per square meter.

The application of this formula to the steam engine, supposes, indeed, that the steam remains superheated during expansion. By great expansion it may happen that at a certain moment the steam becomes saturated, and from there on condenses, so that the expansion curve is different. At the moment of the change, the adiabatic curve cuts the limiting curve DD at the point T_0 (Fig. 91). The pressure p_0 and volume v_0 for this point can be determined. The curve DD is given by Equation XVI., Chapter XVII., viz. :

$$pv^n = D,$$

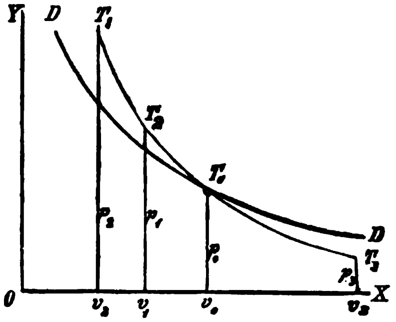

FIG. 91.

where $n = 1.0646$, and $D = 1.704$, and p is in atmospheres. For p in atmospheres and v in cubic feet, $D = 32.658$.

Since now the point $T_0 p_0 v_0$ is in both curves, we have

$$p_0 v_0{}^k = p_1 v_1{}^k \quad \text{and} \quad p_0 v_0{}^n = D.$$

Hence

$$\frac{v_0}{v_1} = \left(\frac{p_1 v_1{}^n}{D}\right)^{\frac{1}{k-n}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (50).$$

where p is in atmospheres.

This equation gives us the expansion ratio $\frac{v_0}{v_1}$, for which the superheated steam just reaches by expansion the point of saturation. If the actual expansion ratio $\frac{v_2}{v_1}$ is less than this, the work is given by the simple Equation (49). If it is greater, for instance, $\frac{v_3}{v_1}$, we can find the work up to T_0, by putting v_0 in place of v_2 in Equation (49). From T_0 to T_3 we must insert in Equation (49) $p_0 v_0$ in place of $p_1 v_1$, and $\frac{v_3}{v_0}$ in place of $\frac{v_1}{v_2}$, and $k = 1.135$ in place of $k =$

1.333, as has been shown for saturated steam originally without admixture of water (Chapter XVIII., page 429, and Equation XXXIII.).

EXAMPLE.

An engine works with superheated steam of $p_1 = 5$ atmos. and temperature $t_1 = 180°$. What is the expansion ratio when the steam at the end of expansion is just in the saturated condition ?

The specific volume of this steam is by Equation (41)

$$v_1 = 0.39037.$$

Equation (50) gives then for the required degree of expansion

$$\frac{v_0}{v_1} = 1.322.$$

Isodynamic Curve.—The isodynamic curve gives the law of variation of pressure with volume where the inner work, that is for steam, the steam heat, is constant. If the initial pressure and volume are p_1 and v_1, we have from Equation (34)

$$J = J_0 + \frac{A}{k-1} p_1 v_1 = J_0 + \frac{A}{k-1} pv,$$

whence

$$p_1 v_1 = pv \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51).$$

We see then, that for superheated steam, the isodynamic curve is an equilateral hyperbola, precisely as is the case for permanent gases.

From Equation (36), we obtain in similar manner

$$T_1 - \frac{C}{B} p_1^{\frac{k-1}{k}} = T - \frac{C}{B} p^{\frac{k-1}{k}},$$

from which we can find the temperature T for any pressure p. During expansion and *fall* of pressure there is then an *increase* of temperature, while for permanent gases the temperature remains constant. For superheated steam the change of temperature is

$$t_1 - t = \frac{C}{B}\left(p_1^{\frac{k-1}{k}} - p^{\frac{k-1}{k}}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad (52).$$

This last formula solves an interesting problem. If a vessel filled with steam communicates with a vacuum, the steam expands, and after it has come to rest will be superheated, provided it was originally dry and saturated. The Equations (51) and (52) give the final condition p, v, t, for the steam heat is evidently here constant. There is. therefore, a fall of temperature during expansion into a vacuum, while for a perfect gas there is no change of temperature.

EXAMPLE.

Let a vessel contain one unit of weight of pure saturated steam at a pressure $p_1 = 5$ atmospheres. Then the volume is, by formula or tables, $v_1 = 0.3630$, and temperature $t_1 = 152.22°$. Let the steam in this vessel expand into another in which is a vacuum, whose volume is 4 times as large. Then the final volume is $v_2 = 5v_1$, and hence by Equation (51), $p_2 = 1$ atmosphere. From Equation (52), we have for the fall of temperature (using Table II.),

$$t_1 - t_2 = 18.876°$$

Hence the final temperature $t_2 = 133.34°$. As this is greater than 100° which is the temperature of saturated steam of 1 atmosphere, the steam is superheated.

Isothermal Curve.—If the steam expands under constant temperature, the law of change of volume with pressure is given by the isothermal curve. For perfect gases this curve is identical with the isodynamic, *i. e.*, it is an equilateral hyperbola. This is not so for steam. Here we have T constant in our equation

$$pv = BT - Cp^{\frac{k-1}{k}},$$

and hence the relation between p and v is given by it.

For the initial condition

$$p_1v_1 = BT - Cp^{\frac{k-1}{k}},$$

and by subtraction

$$pv = p_1v_1 + C\left(p_1^{\frac{k-1}{k}} - p^{\frac{k-1}{k}}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (53).$$

This is the equation of the isothermal curve for superheated steam, while that of the isodynamic is given by Equation (51). If both curves start from the same point, the isodynamic approaches the axis of abscissas most rapidly.

The heat Q, which must be imparted to the unit of weight of steam in order to keep the temperature constant during the transit from p_1 to p, is found by the second of Equations (32), when we put $dT = 0$, and integrate. Thus,

$$Q = c_p \frac{k-1}{k} T \log \frac{p_1}{p} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (54).$$

The change of steam heat (inner work measured in heat units) is from Equation (36)

$$J - J_1 = \frac{c_pC}{kB}\left(p_1^{\frac{k-1}{k}} - p^{\frac{k-1}{k}}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad (55).$$

If L is the work of the expanding steam,

$$Q = J - J_1 + AL,$$

while for gases $Q = AL$, and hence we say that in this latter case all the heat imparted is transformed into outer work. We see that this is not so for superheated steam. Here only a portion of the heat Q is transformed into outer work, the rest is required for inner work. This portion is

$$U = \frac{c_pC}{AkB}\left(p_1^{\frac{k-1}{k}} - p^{\frac{k-1}{k}}\right),$$

and the outer work is

$$L = c_p\frac{k-1}{Ak}T\log\frac{p_1}{p} - \frac{c_pC}{AkB}\left(p_1^{\frac{k-1}{k}} - p^{\frac{k-1}{k}}\right).$$

Both quantities may be calculated from the initial and end pressures.

The preceding results agree perfectly with the usual conceptions as to the deportment of steam, but until now it has not been possible to determine that portion of the heat imparted which goes to perform inner work.

EXAMPLE.

If saturated steam of 5 atmospheres expands under constant temperature down to 1 atmosphere, the heat imparted is found from Equation (54) by substituting for $\frac{k-1}{k} = \frac{1}{4}$, $c_p = 0.4805$, and $T = 273 + 152.22$. We thus have $Q = 82.204$ heat units.

From Equation (55), we have

$$J - J_1 = 6.803,$$

and, finally, from the difference of Q and $J - J_1$, the outer work measured in heat units

$$AL = 75.401,$$

or the outer work itself

$$L = 31970 \text{ meter-kilograms.}$$

Generation of Steam under Constant Pressure.—If one unit of weight of saturated or superheated steam is generated under constant pressure p from water at 0°, we have from Equation (39)

$$W = J_0 + c_p\left(T - \frac{C}{B}p^{\frac{k-1}{k}}\right),$$

and the work done is

$$pv = BT - cp^{\frac{k-1}{k}}.$$

If the weight G of steam is generated, we have for the heat necessary

$$Q = G\left[J_0 + c_p\left(T - \frac{C}{B}p^{\frac{k-1}{k}}\right)\right] \quad . \quad . \quad . \quad . \quad . \quad (56).$$

and the work is

$$L = G\left(BT - Cp^{\frac{k-1}{k}}\right).$$

If the temperature of the steam for the same pressure p is T_1, and if the work (or the volume of steam generated) is the same as before, then if G_1 is the weight of steam generated

$$G\left(T - \frac{C}{B}p^{\frac{k-1}{k}}\right) = G_1\left(T_1 - \frac{C}{B}p^{\frac{k-1}{k}}\right) \quad . \quad . \quad . \quad (57).$$

If, therefore, in any given case the pressure p and temperature T_1 are given, and we wish to replace this steam by another of temperature T, we can find by Equation (57) the necessary steam weight G, and then, by Equation (56), the heat requisite Q.

EXAMPLE.

Suppose we have G_1 kilograms of saturated steam of $p_1 = 5$ atmospheres, and $t_1 = 152.22°$, and we wish the same volume of superheated steam at the same pressure and the temperature $t = 200°$. We have from Equation (57) for the weight of steam required (using Tables I. and II.),

$$G = 0.8852\,G_1.$$

The heat required to generate the saturated steam is, from Equation (56),

$$Q_1 = 653.05\,G_1$$

For the superheated steam of same volume and pressure, it is

$$Q = 676.00\,G,$$

or, using the relation between G and G_1,

$$Q = 0.9163\,Q_1\,.$$

The generation of the superheated steam, other things being the same, requires less heat than the generation of the same volume of saturated steam, and *here lies the advantage of engines working with superheated steam.* The example just given serves as a direct comparison between engines of the same size and speed working with saturated and superheated steam of 5 atmospheres and 200°, *provided there is no expansion.*

There would be no difficulty in extending the comparison to expansion engines also. We shall only remark here, that for such engines the advantage of superheating is somewhat diminished, because the adiabatic curve of superheated steam approaches the axis of abscissas somewhat more rapidly than that for saturated steam. None the less, superheating is in all circumstances theoretically of advantage.

Heating under Constant Volume. If the unit of weight of steam is heated under constant volume, only the steam heat changes, because outer work is not performed. We have, therefore,

$$Q = J - J_1$$

when the initial condition is given by p_1, v_1, T_1, or by Equations (84) and (86),

$$Q = \frac{A}{k-1}(p - p_1)v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (58).$$

or

$$Q = \frac{c_p}{k}\left(T - T_1\right) - \frac{c_p}{k}\frac{C}{B}\left(p^{\frac{k-1}{k}} - p_1^{\frac{k-1}{k}}\right) \quad . \quad . \quad . \quad (59).$$

The first equation gives us the relation between the final pressure p and the heat imparted. The second gives us the final temperature T.

The preceding problems might be easily multiplied. We can easily solve for superheated steam all the examples already solved in this book for saturated steam and gases.

Of especial interest are the phenomena of efflux of superheated steam through orifices and the deportment of mixtures of steams. For lack of space we only call attention to one more problem, which is of importance because we have in respect to it experiments which afford a new confirmation of the correctness of our views regarding the deportment of saturated steam.

Let there be in a cylinder A superheated or pure saturated steam of pressure p_1, temperature T_1, and volume v_1. Let it be forced under constant pressure p_1 through a pipe to a second cylinder B, where it drives back a piston under the less but constant pressure p_2. What is the temperature T_2, and specific volume v_2 when equilibrium is attained?

If we follow the unit of weight of steam from A to B, we have at the beginning the steam heat J_1, and at the end J_2. In A the work is p_1v_1, and in B, p_2v_2. The first work causes an increase of the steam heat Ap_1v_1. The second a diminution by Ap_2v_2. We have, therefore, if heat is neither imparted nor abstracted,

$$J_1 + Ap_1v_1 - Ap_2v_2 = J_2.$$

From Equation (84), we have

$$J_1 = J_0 + \frac{A}{k-1}p_1v_1, \quad \text{and} \quad J_2 = J_0 + \frac{A}{k-1}p_2v_2.$$

Hence, after reduction,

$$p_2v_2 = p_1v_1 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (60).$$

from which we can find the specific volume v_2.

If we use this formula in Equation (24), we have

$$BT_2 - Cp_2^{\frac{k-1}{k}} = BT_1 - Cp_1^{\frac{k-1}{k}}$$

and hence, for the fall of temperature,

$$t_1 - t_2 = \frac{C}{B}\left(p_1^{\frac{k-1}{k}} - p_2^{\frac{k-1}{k}}\right). \quad . \quad . \quad . \quad . \quad . \quad (61).$$

and from this we can find the temperature t_2.

EXAMPLE.

If $p_1 = 13$ atmospheres, and $p_2 = 1$ atmosphere, the fall of temperature is, by Equation (61) and Table II.,

$$t_1 - t_2 = 72.357 - 38.251 = 34.25°.$$

If the steam in A is saturated, then by Table I., $t_1 = 192.08°$, and hence $t_2 = 157.83°$.

If the steam in A is superheated, and has the temperature $t_1 = 200°$, $205°$, or $210°$, we have for the corresponding temperatures, since for the same pressures p_1 and p_2 the fall is the same,

$$t_2 = 165.75, \quad 170.75, \quad 175.75.$$

Hirn has found by experiment, for the first case, $t_2 = 155.58$, and for the other three

$$t_2 = 166°, \quad 171.5°, \quad 177°.$$

The agreement with calculation is entirely satisfactory. For less initial pressures p_1, the differences are greater. Thus, when in all cases the final pressure was $p_2 = 1$ atmosphere, we have for

		Calculation.	Hirn.
$p_1 = 5$ atmos.	$t_1 = 152.22$	$t_2 = 133.84$	137.72
5	246	227.12	233.5
3	133.91	121.87	123.4

The deviations may be ascribed, for the most part, to the uncertainties and difficulties of the experiments.

The preceding case occurs when steam escapes from a boiler into the atmosphere. The temperature t_2 is then that of the steam after it has expanded come to rest, and its pressure sunk to one atmosphere. Of course the experiment cannot be so tested, because the cold air cools the steam jet. In order to avoid this, Hirn let the steam escape into a wooden box surrounded by a second box. This second box was inclosed in a third. The steam passed, after filling the first, through a large orifice into the second, from this into the third, and finally into the air. The apertures were so large that the pressure in the inner box, in which the temperature t_1 was observed, hardly differed from the exterior atmospheric pressure. It is very desirable that these fine and ingenious experiments of Hirn should be repeated and extended.

Since for permanent gases $C = 0$ in Equation (61), the fall of temperature in such case is zero. This can be proved from the formulæ in Chapter V., page 154. When, therefore, a permanent gas flows adiabatically and under constant pressure from one vessel into another in which the pressure is also constant, there is no change of temperature. This, of course, holds good only for a perfect gas. Actual gases shows deviations similar to steam, as shown by the experiments of Joule and Regnault.

[The problem of the mixture of superheated steam with saturated steam is one of considerable importance. It is often the case that only a portion of the steam passes through the superheater and then mixes with the wet steam which comes directly from the boiler on its way to the cylinder. This takes place under constant pressure, and heat is neither imparted nor abstracted.

Suppose we mix together

G_1 kilograms of superheated steam in the condition p, v_1, and T_1, and

G_2 kilograms of wet saturated steam of the pressure p, specific volume v_2, temperature T_2, and specific steam weight x,

Required the condition v and T of the mixture, assuming that this mixture is either superheated or dry saturated, *i. e.*, contains no water. (If we assume that the mixture is wet, we can solve the problem in a precisely similar manner, but the case does not occur in practice.)

We have for the total heat in the G_1 kilograms of superheated steam, before mixture, from Equation (39),

$$W_1 = G_1\left[J_0 + c_p\left(T_1 - \frac{C}{B}p^{\frac{k-1}{k}}\right)\right],$$

and for the total heat in the G_2 kilograms of wet steam, before mixture,

$$W_2 = G_2\left[J_0 + c_p\left(T_2 - \frac{C}{B}p^{\frac{k-1}{k}}\right) - (1-x)\,r\right].$$

The total heat in the mixture $G = G_1 + G_2$ is, if it is dry,

$$W = G\left[J_0 + c_p\left(T - \frac{C}{B}p^{\frac{k-1}{k}}\right)\right].$$

Since heat is neither imparted nor abstracted, the total heat after equilibrium must be the same, hence

$$W = W_1 + W_2,$$

or, after reduction,

$$GT = G_1T_1 + G_2T_2 - G_2(1-x)\frac{r}{c_p} \quad . \quad . \quad . \quad . \quad . \quad (1).$$

From Equation (24) we also obtain, after reduction,

$$Gv = G_1 v_1 + G_2 v_2 - BG_2 (1 - x) \frac{r}{pc_p} \quad . \quad . \quad . \quad . \quad . \quad (2).$$

From these two equations we can find the absolute temperature T and specific volume v of the mixture.

For the change of volume after mixture, we have

$$Gv - G_1 v_1 - G_2 v_2 = - BG_2 (1 - x) \frac{r}{pc_p} \quad . \quad . \quad . \quad . \quad (3).$$

For a given pressure then, the change of volume is directly proportional to the originally existing weight of water, $G_2 (1 - x)$, and it is negative, *i. e.*, there is a diminution of volume by mixing.

In most practical cases it is required to find for given values of G, p, T and T_1, how much saturated steam G_2 should be mixed, in order that the resulting mixture may be either superheated or dry saturated.

We find from (1) directly, by substituting $G_1 = G - G_2$

$$G_2 = \frac{(T_1 - T) G}{T_1 - T_2 - (1 - x) \frac{r}{c_p}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

If the G_2 kilograms were also superheated, we should have

$$W_2 = G_2 \left[J_0 + c_p \left(T_2 - \frac{C}{B} p^{\frac{k-1}{k}} \right) \right],$$

and hence

$$GT = G_1 T_1 + G_2 T_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5).$$

$$Gv = G_1 v_1 + G_2 v_2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

$$G_2 = \frac{(T_1 - T) G}{T_1 - T_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (7).$$

If the G_2 kilograms were originally dry saturated, we should have in (1), (2), and (3) $x = 1$, and hence we should have the same equations as above, only in the place of v_2 we should put u, the specific volume of dry saturated steam for the given pressure.

In these two cases there is no change of volume.]

CHAPTER XXIV.

A.—THE MORE IMPORTANT PRINCIPLES WHICH SHOULD GOVERN THE CONSTRUCTION OF THE STEAM ENGINE.

ONE of the most important points in the construction of a steam engine is that it shall give a certain delivery with the least amount of fuel. This depends not only upon the proportions of the engine itself, but also upon those of the boiler. We require from the boiler, first, that it shall absorb as much as possible of the heat of the fuel and transmit it to the water. For this it is necessary not only to give the boiler an appropriate shape, but also to construct it of suitable material. Then the furnace must be so arranged that the fuel is completely consumed, and that but little heat shall be lost by radiation or conduction. Sometimes one of these conditions is in opposition to another.

As to the form of the boiler, that is to be preferred which gives for given capacity the greatest heating surface. But on the other hand, this form should give the necessary strength. The first Watt boilers, the so-called "wagon" boilers, had a tolerably large heating surface, and answered well for the low pressures then in use. At present, when higher pressures are used, they would not be sufficiently strong. Hence cylindrical boilers are now used, either with interior or exterior fire-place.

The boiler should also have such capacity as to furnish the steam required by the engine, and to keep the engine in uniform action. For this reason the steam used per stroke should be but a small part of the boiler capacity. In general, the steam space should be at the very least 12 times the capacity of the cylinder. In order that the heating surface may be large, the water should occupy a certain extent of the boiler.

In general, the water space is $\frac{6}{10}$ of the entire capacity. In order to prevent radiation, the boiler may be covered, where exposed, by poor conductors. Boilers are sometimes constructed now of steel plate as well as iron, because the former is not only stronger but has a greater conducting power.

As to the furnace, care must be taken to secure complete combustion of fuel, that the heat may be absorbed by the boiler sides, and that but little heat is lost. For complete combustion a certain amount of air is essential. But if more air than necessary is used, the excess absorbs a portion of the heat, of which indeed a part is given up to the boiler, but another part escapes at the chimney. A good draught is also necessary. This will be greater the higher the chimney and the greater the difference of temperature of the air in the chimney and the cold air outside. The height of chimney has a limit, both by reason of cost, and because the increased friction diminishes the draught. The temperature in the chimney should not be too great, because then a great part of the heat passes off unutilized. It has been sought to utilize this waste heat in the chimney by making it heat the feed water, when ordinary feed pumps are used. Engines working with superheated steam, of which there are but few, use this heat to superheat the steam.

In stationary engines the grate surface is a certain proportion of the heating surface, about $\frac{1}{12}$ or $\frac{1}{14}$ only. In locomotives this ratio is still less, even as low as $\frac{1}{60}$ or less, but here there is a strong artificial draught. It is thus possible with a boiler of relatively small capacity and weight (weight of boiler with water) to generate in a short time a considerable amount of steam, a property which is of importance in locomotives especially.

The ratio of the heat absorbed by the boiler in a given time, as one hour, and which can be determined evidently by the amount of water vaporized in that time, to that furnished by the fuel, is called the efficiency of the boiler. The heat units furnished by the complete combustion of different fuels have been determined by experiment. We may call this the heating value of the fuel.

The more water is evaporated in a given time by a given weight of fuel the greater the efficiency.

EXAMPLE 1.—In a hot-air engine the heat furnished per hour to the air is 6170 heat units, while in the same time 4.585 kilograms of coal are consumed, whose heating value is 3500 heat units. What is the furnace efficiency ?

We have

$$\frac{6170}{4.585 \times 3500} = 0.38.$$

Therefore 62 per cent. of the heat is lost.

EXAMPLE 2.—The boiler of an expansion engine which uses steam of 5 atmospheres, vaporizes per hour, for every horse power, 30 kilograms of water, and requires for this 5 kilograms of hard coal, whose heating power is 7500 heat units. What is the boiler efficiency ?

If we assume 640 heat units to 1 kilogram of steam at all pressures, we have $30 \times 640 = 19200$ heat units imparted to the water per hour per horse power. The 5 kilograms of coal give $7500 \times 5 = 37500$ heat units. Hence the efficiency is

$$\frac{19200}{37500} = 0.512.$$

As to the engine itself; the first requirement is that for a given power it shall use as little steam as possible. This is accomplished principally by using the steam expansively and having as much expansion as possible. Since the counterpressure upon the piston has considerable influence, this should be as small as may be. Where, then, water is plentiful, condensing engines are of value. In order that the useful effect for a given steam consumption may be a maximum, the prejudicial resistances, friction, work of the pumps, etc., should be a minimum. These conditions require the construction to be simple. If we use high steam (7 or 8 atmospheres) and a high expansion (1 to 6 or 1 to 8) the use of the condenser offers less advantage, as the influence of the back pressure is relatively less, and two pumps must be worked by the engine.

While seeking to reduce the cost of working to a minimum, we should also make the cost of construction small. This, as well as the cost of erection, depends upon the dimensions, which we must therefore make as small as possible. This may be effected by the use of high steam, and also by rapid action. As both these increase, the cylinder volume becomes less. For a rapid engine, the fly-wheel also is lighter and the friction of the shaft is less. Most industrial purposes also require a high velocity, so that high piston speed causes simpler gearing.

Total Delivery—Useful Effect—Efficiency.—By total delivery we mean the work of the effective steam pressure, or pressure of boiler steam minus the back pressure of the air or condenser. From this total effect we have to subtract the losses due to difference between boiler and cylinder pressure, friction of piston, valves, etc. The difference is the calculated or theoretical useful effect. This then is the work actually imparted to the engine. If we measure the work done, by the dynamometer, we have the actual or observed useful effect. The more reliable the coefficients used in determining the losses, the better the agreement between the calculated and the observed useful effect. The division of the useful effect by the total gives the efficiency. A machine is more nearly perfect the nearer this ratio is to unity. The same method of calculation is used when we make use of the principles of the mechanical theory of heat, as when we proceed according to the old method of Pambour, only we have to take into account a new loss, which Zeuner calls *the loss by reason of the incompleteness of the cycle process.* We shall return to this later on.

In any steam engine, the greater the efficiency of the furnace and the engine itself, the better is the machine. If in addition, cost of erection and repairs is small, all conditions are satisfied which can be demanded of the construction. To demand that the furnace shall absorb all, or the greatest part of the heat contained in the fuel, is as unreasonable as to demand that a water-wheel shall receive the entire flow of a river from the source to the sea.

If we divide the useful effect of a steam engine, expressed in units of heat, by the number of heat units given by the combustion of the fuel, we obtain the "*thermal effect*" of the entire apparatus.

Example 1.—The total delivery of a steam engine is 1000 meter-kilograms per second, and the useful effect 537. What is the efficiency ?

We have $$\frac{537}{1000} = 0.537.$$

Example 2.—What is thermal effect of the hot-air engine, page 548, when the hourly delivery is 265680 ?

Since in 1 hour 4.585 × 3500 heat units are set free in the furnace, and 265680 meter-kilograms corresponds to $\frac{265680}{424} = 626.6$ heat units, we have

$$\frac{626.6}{4.585 \times 3500} = 0.039, \text{ or about 4 per cent.}$$

The thermal effect is that which properly informs us as to the economy of steam or hot-air engines.

B.—The Cycle Process of the Perfect Steam Engine, and the "Disposable Work."

In the first part, we have seen that the delivery of every caloric engine is given by

$$L = \frac{QG}{AT_1}\left(T_1 - T\right),$$

where $\frac{QG}{AT_1}$ is the heat weight imparted and $T_1 - T$ the temperature fall, or the difference of the highest and lowest temperatures of the air when compressed adiabatically. We called it there the "useful delivery," because it was that obtained by subtracting from the total delivery, or work of the air on the piston, that required for the compression of the air by the feed piston. We shall now call it the "*disposable work*," since it is that which the air in passing through its cycle puts at our disposition, from which we are to get as much useful effect as we can.

We have also seen what the cycle process is when the *absolute* maximum of work is required. The addition and abstraction of heat must be so regulated that all the heat imparted must be transformed into work. In other words, heat addition and abstraction must take place according to the isothermal curve, and the two others must be adiabatic. Such a cycle process we can call "perfect," and an engine which goes through such a cycle is called "*perfect*," or ideal. It is impossible, with the same expenditure heat and temperature fall, to obtain a greater delivery than such an engine gives. But it has the disadvantage that, other things being the same, it requires a much greater cylinder volume than hot-air engines in which heat addition and abstraction take place according to some other law than the isothermal. For this reason it is not to be recommended in practice.

The case is different in this respect when we use steam instead of air or a permanent gas. Here also such an engine is perfect when the cycle process is perfect. But here such a process is the easiest executed, because the isothermal lines are parallel to the axis of X.

We shall first speak of the perfect cycle process of the steam engine. We shall see, as we proceed, why in our present engines the cycle is incomplete. The work of such a perfect steam engine we call, with Zeuner, the disposable work.

Let EF, Fig. 92, be the steam cylinder with the piston KK. Left of the piston is a certain weight of water of G kilograms. The pressure upon the piston is p, and the back pressure is p_1. In a condensing engine p_1 is about 0.15, and in a non-condensing 1.1 atmosphere. Now let heat be imparted till the water is raised to its boiling point t° for the pressure p. If heat is still further imparted, steam is generated of the pressure p and temperature t, and the piston is driven toward the right. When the distance $HV = AB$ is passed let the greatest part of the water be vaporized. Let the specific steam weight be now x. Then we have now in the cylinder Gx kilograms of steam, and $G(1-x)$ of water. The heat imparted to the water at t°, to generate the steam, is Grx heat units, where r is the total latent heat of vaporization. Let now the mixture expand adiabatically along BC, until the volume is v_1, and the temperature and pressure t_1 and p_1. As we know, steam condenses during the expansion, and the specific steam quantity at the end is less than at the beginning. Sup-

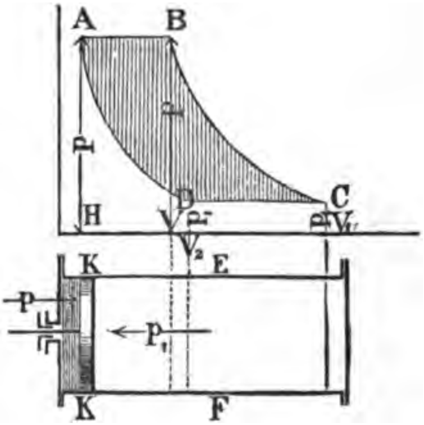

Fig. 92.

pose this quantity is x_1, then we have (page 177) Equation XXV.,

$$\frac{xr}{T} + \tau = \frac{x_1 r_1}{T_1} + \tau_1,$$

where r_1 is the latent heat of vaporization at the temperature t_1. The piston is now at the end of its stroke. Now let the volume v_1, of the temperature t_1, be compressed under the constant pressure p_1 till the volume is v_2, that is, the piston passes through CD. Then let the remaining volume be compressed adiabatically, D being so chosen that during compression from V to H, the remaining steam is converted into water, and we have the original condition again. If the process is thus performed as indicated, we have not only a *complete* cycle process, but also a *perfect* cycle process, that is, one in which the work obtained is a maximum. Whatever other complete cycle process the steam may be made to perform, the work obtained for the same amount of heat imparted will be less. If the specific steam quantity at D is x_2 (of course, less than at C), we have, since at A, $x_2 = 0$,

$$\frac{r_1 x_2}{T_1} + \tau_1 = \tau.$$

At C the steam weight was Gx_1, and at D it is Gx_2, so that from C to D the heat abstracted is

$$Q_1 = Gr_1(x_1 - x_2).$$

The entire process thus is similar to that on page 244 of Part I. where air expanded and was compressed under constant pressure. Just as there the work actually obtained, after subtracting that of the back pressure p, is given by the area $T_1T_2T_3T'_2$, so here the area $ABCD$ is the effective work, or that obtained after subtracting that of the back pressure p_1. If we denote the work by L, we have

$$L = \frac{1}{A}(Q - Q_1) \text{ (see page 199, Part I.)}$$

If for Q and Q_1 we put the values above,

$$L = Grx - Gr_1(x_1 - x_2).$$

Since

$$r_1x_1 = \left(\frac{rx}{T} + \tau - \tau_1\right) T_1, \quad \text{and} \quad r'_1x_2 = (\tau - \tau_1)\, T_1$$

we have

$$L = \frac{Grx}{AT}(T - T_1). \quad . \quad . \quad . \quad \text{(CXIII.)}$$

or

$$L = \frac{Q}{AT}(T - T_1). \quad . \quad . \quad . \quad \text{(CXIV.)}$$

This is the same equation which we found in Part I. for the delivery of the hot-air engine. The quotient $\frac{Q}{AT}$ is the heat weight and $T - T_1$ the temperature fall. Just as in hydraulics we determine the total delivery, or, as we now call it, the disposable work, of a water-wheel from the weight of water entering the wheel in a certain time and the fall, so here the disposable work of a steam engine is given by the product of the heat weight imparted in a certain time and the temperature fall.

Since in every complete cycle process the heat weight abstracted is equal to that imparted, we have also

$$L = \frac{Q_1}{AT_1}(T - T_1) \quad . \quad . \quad . \quad . \quad \text{(CXV.)}$$

Both formulæ give for a certain Q or Q_1 the absolute maximum delivery of a machine. For the same heat Q or Q_1 the delivery is greater the greater the temperature fall. In hot-air engines we could not, on account of practical reasons, have T over 573°, and T_1 cannot be much below 273°. If in the steam engine T were 573°, or $t = 300°$, we should have an enormous steam pressure, since for 230° the pressure is about 28 atmospheres. At present we seldom exceed 10 atmospheres, which corresponds to $t = 180.3°$, or $T = 453.3°$. Whether it is practicable to employ higher pressures can only be determined by practice. Further, we cannot well go below $t_1 = 46.2°$, or $T_1 = 319.2°$, as this temperature corresponds to $\frac{1}{10}$th of an atmosphere. For a lower temperature the amount of condensation water is too great. For engines without condensation, $t_1 = 100°$, and $T_1 = 373°$. If then we regard $t = 180.3°$ as the maximum temperature, we have for the maximum delivery of a perfect steam engine, for condensation,

$$= \frac{424Q}{453.3}(180.3 - 46.2) = 125.38Q \quad . \quad . \quad \text{(CXVI.)}$$

or for English units and Fahrenheit degrees $318.78Q$.
and non-condensing,

$$L = \frac{424Q}{453.3}(180.3 - 100) = 75.08Q \quad . \quad . \quad \text{(CXVII.)}$$

or for English units and Fahrenheit degrees $190.88Q$.

EXAMPLE.

What is the delivery of a perfect steam engine, which uses per hour 100 kilograms (or 220 lbs.) of steam of 10 atmospheres ?

If we assume the steam to be dry, $Q = Gr$ where Q and G are quantities per hour. Now $r = \rho + Apu$ is for 10 atmospheres 478.8 (or 861.8). Hence $Q = 478.8 \times 100 = 47880$ heat units per hour, or 13.3 per second (or 49.61).

For a condensing engine, then,

$$L = 125.38 \times 13.3 = 1667.55 \text{ meter-kilograms},$$

$$\text{or } L = 318.78 \times 49.61 = 15814.7 \text{ foot-lbs.}$$

$$\text{Hence } N = \frac{1667.55}{75} = 22.23 \text{ horse power},$$

$$\text{or } N = \frac{15814.7}{550} = 28.75 \text{ horse power English.}$$

For non-condensing,

$$L = 75.08 \times 13.3 = 998.56 \text{ meter-kilograms},$$

$$\text{or } L = 190.88 \times 49.61 = 9469.56 \text{ foot-lbs.}$$

$$\text{Hence } N = \frac{998.56}{75} = 13.32 \text{ horse power},$$

$$\text{or } N = \frac{9469.56}{550} = 17.21 \text{ horse power English.}$$

Since now, from formulæ CXIV. and CXV., the delivery of an engine depends only upon the heat Q and temperature fall, it is evident that the kind of liquid used makes no difference, whether water, alcohol, ether, or air.

Let us now consider the cycle process of our actual steam engines and find their delivery. We shall then see why their cycle process is not perfect. If we then compare the delivery with that of a perfect engine, we shall have the loss of effect due to the imperfection of the process, to which we have already referred.

C.—CYCLE PROCESS OF THE ACTUAL STEAM ENGINE AND DETERMINATION OF THE LOSS OF EFFECT DUE TO THE IMPERFECTION OF THE PROCESS.

In actual engines we have to do with a *complete* but not a *perfect* cycle process.

Let *A* be the steam cylinder and *K* the boiler. The steam has the pressure p and temperature t. From the boiler it passes through the steam pipe to the valve box on the right of the cylinder *A*. Let the piston have its highest position, and hence the upper part be a little open. Provided that there is no friction in steam pipe, steam of the boiler pressure enters above the piston and forces it down. Let the line Oo represent the pressure p. When the piston has passed through the distance OV, and when, therefore, there have entered V cubic units of steam from the boiler, let expansion commence, and the steam expand according to the adiabatic line xx_1. At the end of expansion let the steam have the pressure p_1 and the temperature t_1 of the condenser *C*. Above the piston we have then steam of the pressure p_1 and volume V_1. By this time the valve has opened the lower port, and steam of the boiler pressure p is below the piston, which now rises. While rising, it forces the steam volume V_1 under the pressure p_1 gradually into the condenser. Thus the back

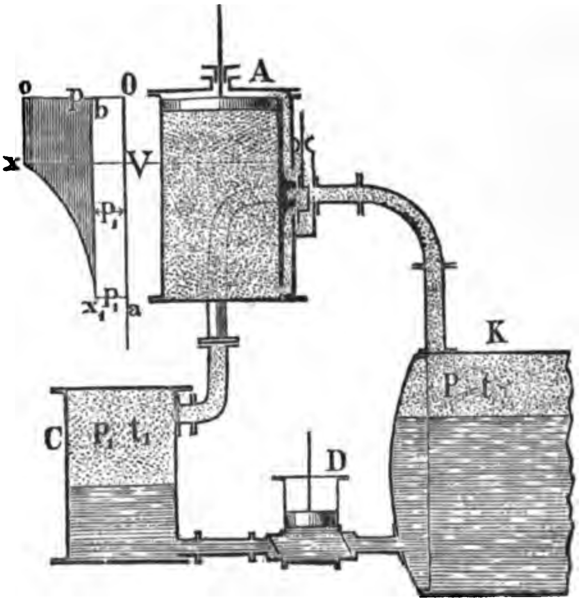

FIG. 98.

pressure is p_1. The line x_1a represents this pressure, while x_1b is the stroke.

If the engine has a surface condenser, the volume V_1 is condensed, and is then forced by the feed pump D into K. Here it is again heated from t_1 to t, then converted into steam, and then admitted to the cylinder. If we have a jet condenser, the pump D must remove from the condenser not only the condensed steam, but also the injection water. But it only has to force into the boiler as much water as before. If there is no condenser, the back pressure p_1 is that of the atmosphere, and it is just the same as if we had a condensing engine in which the condensed steam has a temperature of 100°. Although in this case the same weight of water must be forced into the boiler as before, this water, if there is no feed heater, has a lower temperature, and more heat is required to heat it than in the condensing engine.

Since we thus know the character of the cycle process, we can calculate the delivery. First, it is evident that the area $boxx_1$, gives the delivery per stroke, the work of overcoming the back pressure being deducted from the total. If, then, we deduct the work required for forcing the feed into the boiler, we have the work corresponding to the cycle process of the actual steam engine.

We assume again G as the weight of steam and water per stroke, of which xG kilograms are steam and $(1 - x) G$ water. The steam volume used per stroke is V, or

$$V = (xu + \sigma) G \text{ cubic meters.}$$

The work during full pressure is

$$L_1 = pV = p(xu + \sigma) G \text{ meter-kilograms.}$$

If the specific steam weight at the end of expansion is x_1, we have for the volume V_1

$$V_1 = (x_1u_1 + \sigma) G_1.$$

Since the steam expands adiabatically, we have for the work during expansion (Equation XXVII.)

$$L_2 = (q - q_1 + x\rho - x_1\rho_1)\frac{G}{A}.$$

Hence the delivery per stroke is

$$L_1 + L_2 = \left[p\,(xu + \sigma) + \frac{1}{A}\,(q - q_1 + x\rho - x_1\rho_1)\right] G.$$

From this we must subtract the work in overcoming the back pressure p_1. This is

$$L_3 = p_1\,(x_1u_1 + \sigma)\ G.$$

Therefore the delivery, neglecting the work required for the feed, is

$$A\,(L_1 + L_2 - L_3) = [Ap\,(xu + \sigma) + (q - q_1 + x\rho - x_1\rho_1) - Ap_1 \\ (x_1u_1 + \sigma)]\ G \text{ heat units.}$$

$$L_1 + L_2 - L_3 = L = [q - q_1 + xr - x_1r_1 + \sigma(p - p_1)A]\,\frac{G}{A}\,,\ \text{or}$$

$$AL = [q - q_1 + xr - x_1r_1 + A\sigma\,(p - p_1)]\,G \text{ heat units.} \qquad \text{(CXVIII.)}$$

If the last member in this equation is neglected on account of its smallness, we shall have Equation LV., page 516, which we have already found for the efflux of steam. Hence

$$A\,\frac{w^2}{2g}G = AL,$$

or

$$G\,\frac{w^2}{2g} = L.$$

That is, the total delivery of the actual steam engine per stroke is equal to the living force of the steam G used per stroke when it flows with the velocity w from the boiler. This might at first sight seem to make it advantageous to allow the steam to act by impulse or reaction. When we consider, however, that a reaction wheel only gives its maximum delivery when it revolves with the same velocity as the liquid departs, such a wheel would have to have an enormous velocity, as the velocity of steam is very great when issuing even under low pressures. Such a velocity would consume much of the effect, even if the construction had any practical value. (Zeuner, Wärmetheorie, page 477.)

If we determine x_1r_1 from the known relation

$$\frac{xr}{T} + \tau = \frac{x_1r_1}{T_1} + \tau_1,$$

and insert it in the preceding equation, we have, after reduction,

$$AL = \left[\frac{xr}{T}(T-T_1)+q-q_1-(\tau-\tau_1)\,T_1+A\sigma(p-p_1)\right]G. \quad \text{(CXIX.)}$$

This expression is not yet the outer work of the steam engine. It is rather the entire work obtained up to the point where the G kilograms of steam at the temperature t have become water at the temperature t_1. To complete the cycle process we have still to force this water into the boiler.

Here we have to distinguish between engines condensing and non-condensing. In the first case, we may have either a surface or a jet condenser. In the other, the steam escapes into the air.

(*a.*) *Condensing Engine—Surface Condenser.*—In this case the pump D has to raise per stroke the G kilograms and force them into the boiler. If the pressure of the air is p_0, we have for the work of removing from the condenser

$$G\sigma(p_0 - p_1),$$

and for forcing into the boiler

$$G\sigma(p - p_0).$$

When we add both works, we have for the work of the pump

$L_4 = G\sigma(p - p_1)$ meter-kilograms, or in heat units

$$AL_4 = A\sigma(p - p_1)\,G.$$

Subtracting this from CXIX., we have for the work obtained by the cycle process of a steam engine with surface condenser,

$$AL = \left[\frac{xr}{T}(T-T_1)+q-q_1-(\tau-\tau_1)\,T_1\right]G \text{ heat units.} \qquad \text{(CXX.)}$$

(*b.*) *Condensing Engine—Jet Condenser.*—We suppose that the air pump not only removes the water from the condenser, but also forces it into the boiler. We also neglect the fact that it removes air also. If G_0 is the injection water per stroke, the pump has to remove per stroke $G + G_0$, but only has to force G into the boiler. The work of removing is

$$(G_0 + G)(p_0 - p_1)\,\sigma = G_0\sigma\,(p_0 - p_1) + G\sigma\,(p_0 - p_1).$$

The water G_0 then runs off, and G is forced into the boiler. The work required is

$$G\sigma\,(p - p_0).$$

The total work of the pump is then

$$L_4 = G_0\sigma\,(p_0 - p_1) + G\sigma\,(p - p_1).$$

We have then for the work of the cycle process of a condensing engine with jet condenser

$$AL = \left[\frac{xr}{T}(T - T_1) + q - q_1 - (\tau - \tau_1)\,T_1\right] G - G_0\sigma\,(p_0 - p_1)\,A. \qquad \text{(CXXI.)}$$

The last member is so small that it may be neglected.

(*c.*) *Non-condensing Engine, with Ordinary Force Pump.*—Let the height to which the water is sucked be h. The work is Gh. The work of forcing into the boiler is

$$G\sigma\,(p - p_0).$$

But in the present case $p_0 = p_1$, hence

$$L_4 = Gh + G\sigma\,(p - p_1).$$

Subtracting this work from CXIX., we have

$$AL = G\left[\frac{xr}{T}(T - T_1) + q - q_1 - (\tau - \tau_1)\,T_1 - Ah\right]. \quad \text{(CXXII.)}$$

Here Ah may be neglected, and we have again Equation CXX.

(*d.*) *The Boiler is Fed by a Giffard Injector.*—In this case there is no outer work required for the injector. We can therefore use Equation CXIX. directly, of which we may neglect the last equation. Hence Equation CXX. gives, in all cases, the total delivery measured in heat units of the cycle process of the steam engine.

The question now arises, what is the amount of heat expended? If we know this, and insert it in Equation CXIV. instead of Q, we shall have the delivery of an engine with perfect cycle process. The comparison of this with Equation CXX. will give the loss of effect by reason of the imperfection of the process. This heat can be easily determined.

We assume first that the engine is condensing. The G kilograms of steam used per stroke are removed in liquid state from the condenser and forced into the boiler. The temperature of this water we have indicated by t, and the heat of the liquid is q_1. Hence the G kilograms of water contain Gq_1 heat units. In the boiler the temperature is raised to $t°$, and the heat of the liquid is q. The heat imparted is then $G(q - q_1)$. Of this water xG kilograms are now vaporized, which requires the heat Grx.

The total heat then is

$$G(rx + q - q_1) = G[q - q_1 + (\rho + Apu)x].$$

Let us assume again that the engine is non-condensing, and that the feed is furnished by the injector. If the feed water has a temperature of t_0 the heat of the liquid is q_0, and if we assume the water heated to $t_1°$ by the steam of the injector, this steam must itself lose heat, so that it becomes water at $t_1°$. When now this condensed steam with the feed water enters the boiler, the former must receive as much heat as it lost in condensing in order to be converted into steam of $t°$. We neglect thus, indeed, that heat which, transformed into work, is necessary to raise the feed water and force it into the boiler. On account of its slight comparative amount, this is allowable. If therefore the quantity of water raised is G, and if it is heated to $t_1°$, the heat lost by the steam in thus heating it is

$$G(q_1 - q_0),$$

and this, as remarked, must be again imparted to the condensed steam in the boiler in order to convert it into steam at $t°$. The feed water is now heated in the boiler from $t_1°$ to t, and this requires the heat $G(q - q_1)$. The total heat imparted to the feed water in order to bring it up to the temperature of the boiler water is then

$$Gq_1 - Gq_0 + Gq - Gq_1 = G(q - q_0).$$

Finally this water is to be converted into steam at $t°$. For this we require, assuming that of the total weight of feed water, Gx are steam, Grx heat units. Hence the total heat imparted to the feed water is

$$G(rx + q - q_0) = G[q - q_0 + (\rho + Apu)x] \text{ heat units.}$$

If, finally, the boiler is fed by an ordinary force pump, and the temperature of the feed water is t_0, we have again

$$G(rx + q - q_0) = G[q - q_0 + (\rho + Apu)x] \text{ heat units,}$$

in order to form steam of $t°$. We see, then, that with the injector we have to impart as much heat to the feed water, in order to generate the steam required, as when the ordinary force pump is used. In this respect also, then, the injector possesses no advantage. The single advantage of the injector is that the frictional resistances are less.

From the above it follows that the heat required for the delivery of the cycle process of the steam engine is given by

$$Q_1 = G(rx + q - q_0) = G[q - q_0 + (\rho + Apu)x],$$

in which for condensing engines we put q_1 for q_0.

The heat, then, is less the greater q_0, that is, the hotter the feed water. We see, then, the desirableness of a feed water heater.

If now this heat is used in a perfect engine, we have for the delivery

$$L_m = \frac{G(q - q_0 + rx)}{AT}(T - T_1).$$

This delivery is therefore that which, from the standpoint of the mechanical heat theory, is disposable when the heat $G(q - q_0 + rx)$ is used. It is more convenient to put the formula in the following form,

$$L_m = \frac{G}{A}\left[\frac{rx}{T}(T - T_1) + \frac{(q - q_0)(T - T_1)}{T}\right]. \quad . \quad \text{(CXXIII.)}$$

Now in our actual steam engines we have the work from Equation CXX.,

$$L = \frac{G}{A}\left[\frac{rx}{T}(T - T_1) + q - q_1 - (\tau - \tau_1)\,T_1\right].$$

If we subtract this from the preceding, we have for the loss of work by reason of the imperfection of the cycle process,

$$L_1 = \frac{G}{AT}\left[(q_1 - q_0)\,T - (q - q_0)\,T_1 + (\tau - \tau_1)\,TT_1\right]. \quad \text{(CXXIV.)}$$

For condensing engines we put q_1 in place of q_0. If we divide this by the disposable work, we have the ratio of the loss of effect to the work which is at our disposition in the heat used. This ratio is

$$w = \frac{(q_1 - q_0)\,T - (q - q_0)\,T_1 + (\tau - \tau_1)\,TT_1}{(q - q_0 + rx)(T - T_1)}. \quad \text{(CXXV.)}$$

EXAMPLE.

A non-condensing steam engine works with dry steam of 5 atmospheres. What is w when the feed water has a temperature $t_0 = 15°$?

From our Tables we have for

5 atmospheres	and for	1 atmosphere
$t = 152.2$		$t_1 = 100$
$q = 153.7$		$q_1 = 100.5$
$\tau = 0.45$		$\tau_1 = 0.31$
$r = 499.2$		

hence

$$w = \frac{(100.5 - 15)\,425.2 - (153.7 - 15)\,373 + (0.45 - 0.31)\,425.2 \times 373}{(153.7 - 15 + 499.2)\,52.2}$$

$$= \frac{6005.33}{33298.4} = 0.18.$$

Therefore 18 per cent. of the work at disposal is lost by reason of the imperfection of the cycle process.

Zeuner, to whom this elegant and interesting discussion is due, has investigated by various examples the influences of heating the feed water, and of water contained in the steam, upon the loss of effect. The following tabulation in which the steam is assumed to be dry, shows the influence of heating the feed water. The engines are, of course, non-condensing.

NON-CONDENSING ENGINES.

(*Back Pressure,* 1 *Atmosphere.*)

Boiler Pressure in Atmospheres.	Loss of Effect w for $t_1 = 15°$	and $t_1 = 100°$
1½	0.15	0.01
3	0.16	0.03
4	0.17	0.04
5	0.17	0.05
6	0.18	0.05
8	0.19	0.06
10	0.19	0.07

If, then, the feed water is heated up to 100°, the loss of effect by reason of the imperfection of the cycle process is small, especially for low steam pressures. For condensing engines, in which the back pressure is about $\frac{1}{10}$ of an atmosphere, in which, therefore, the feed-water temperature is $t_1 = 46.2°$, the loss for $1\frac{1}{2}$ atmospheres is 0.05, and for 10 atmospheres 0.10. Here, then, the loss of effect increases with the pressure.

In the following tabulation we see that the loss of effect *increases with the quantity of water in the steam.* The engine is assumed non-condensing and working with steam of 5 atmospheres.

Specific steam quantity.	Loss of effect for feed water temperatures. $t_0 = 15°$	80°	100°
$x = 1$	$w = 0.17$	0.08	0.05
$x = 0.90$	0.19	0.09	0.05
$x = 0.80$	0.21	0.10	0.06

We see from both tabulations how advantageous it is to use hot feed water. This is confirmed by experiment. The waste gases in the chimney may be used for heating the feed. But even then, the loss of effect for high pressures, especially when the steam contains 10 or 20 per cent. of water, is considerable, so that it becomes a question whether the cycle process of our present steam engines can be so altered as to correspond to that of a perfect steam engine. For this purpose, we should evidently not condense all the steam in the condenser, but rather so much should remain, that by adiabatic compression this remaining steam may be converted into water, with the already condensed steam, at the boiler temperature (t). Nevertheless, the preceding discussion shows that on the whole the imperfection of the process is small. The other losses, as that due to imperfect expansion, prejudicial space, etc., are in part greater, at least the loss due to imperfect cycle is but a small part of the total losses. Accordingly, it is by no means correct as Redtenbacher asserts, that the cycle process of our steam engines is exceedingly imperfect, and that therefore some other method of utilizing the steam should be invented. "So long as the fundamental principles of the mechanical theory of heat," says Zeuner, "are regarded as correct, so long we can regard the cycle process of our steam engines as quite perfect, and if there are no losses of work greater than that due to the essential imperfection of the process, we need not search for improvement in the steam engine; at any rate, in those engines which use saturated steam."

CHAPTER XXV.

COMPLETE CALCULATION OF THE STEAM ENGINE.

Indicated Delivery.—We have in the preceding calculated the total delivery, or that obtained by the cycle process of the ordinary steam engine, when we disregard the work required for the feed. We have now to determine more exactly the work of the steam in the cylinder, not only with reference to the back pressure, but also to other prejudicial actions. We have first to calculate the work which the steam actually performs on the piston. Since this work is accurately given by the indicator diagram, we call it the indicated delivery or horse power of the steam or engine. In these calculations we shall proceed, of course, from the principles of the mechanical heat theory. Then we shall show how the prejudicial resistances, such as piston and valve friction, that of fly-wheel and pumps, etc., are to be determined. Finally, we shall show how to determine the dimensions, the consumption of fuel, etc., for an engine of given horse power.

Let us first examine more closely the action of the steam in the cylinder. We assume an engine with ordinary slide valve, moved by an eccentric.

Action of the Steam in the Cylinder.—In Figs. 94, 95, 96, 97, and 98, *AB* is the cylinder, *CD* the piston, *EF* the slide valve box, *I* a portion of the steam pipe, *GH* the slide valve, *ab* and *cd* the steam passages, *a* and *c* the entrance ports, and *e* the exhaust port, through which the steam passes either into the air or into the condenser. In Fig. 94, the piston is at the left end of its stroke; the port *a* is *already a little open*, and steam enters from the boiler and presses upon the left side of the piston. This steam we shall call the "*driving* steam." This opening of the entrance port for the admission of steam, *before* the piston gets to the end of its stroke, is necessary for smooth motion of the engine. While now the driving steam forces the piston to the right, the valve moves also in the same direction, and the port *a* is opened more and more, and fresh steam continually enters. Finally the port *a* is fully opened, and the valve has then reached its extreme position towards the right. It then begins to move towards the left, and thus closes *a* more and more, so that the steam enters with increasing resistance. In Fig. 95, the port *a* is completely closed, and hence no more steam can enter behind the piston. Since the piston has not yet arrived at the end of its stroke, however, the driving steam must now act expansively. Meanwhile the valve still goes towards the left, and in Fig.

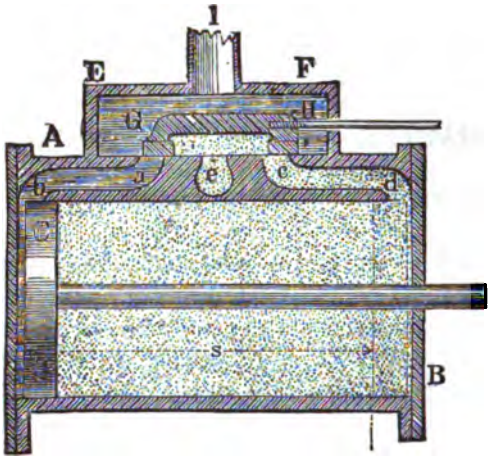

FIG. 94.

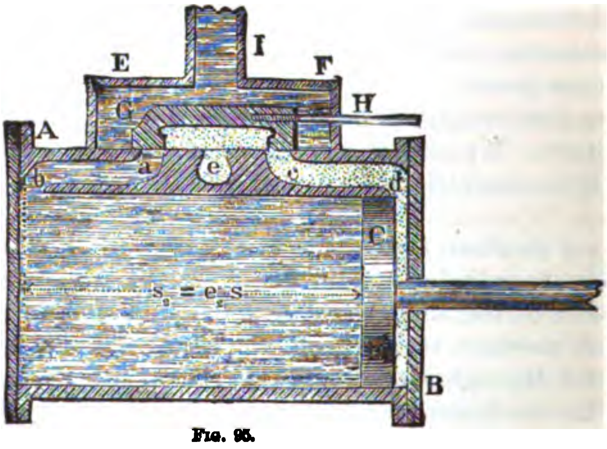

FIG. 95.

96, we have the position of valve and piston when the first is about to open the port *a* for the discharge of the driving steam, while the second is not yet at the end of its stroke towards the right. Up to this moment we have expansion of the driving steam, but of course not after. This is then the point of release, while Fig. 95 is the position where expansion begins. From this point on, the valve opens the port *a* for the release of the steam. The driving steam flows through *a*, *b*, and *e* to the condenser, or out into the air. Fig. 97 shows the position of piston and valve when the port *a* is tolerably open for discharge, while the piston has not yet arrived at the right end of its stroke.

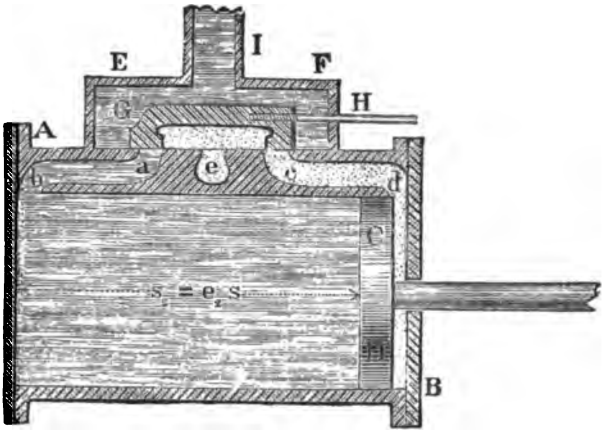

FIG. 96.

In this position the port *c* is closed. While now the piston still moves toward the right and the valve toward the left, the port *c* is opened to admit steam, and the steam enters as before it did in Fig. 94, only on the right side of the piston instead of the left. The driving steam is now on the right of the piston. Thus far we

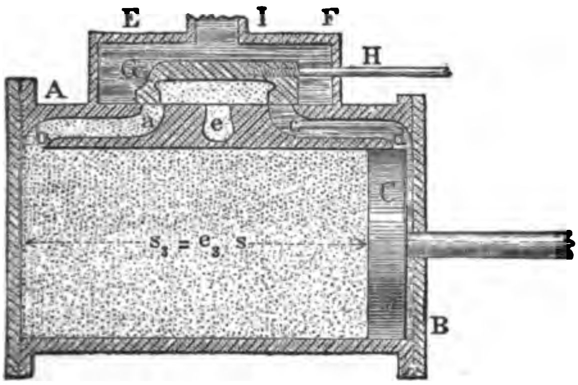

FIG. 97.

have confined our attention to the left side of the piston and have considered the action of the driving steam. Let us now see how the back pressure steam acts. This steam we have indicated by points in our Figures, while the driving steam is indicated by horizontal lines. Let us refer again to Fig. 94. We see here that the port *c* is tolerably wide open, wider than *a*, and hence that the release of the back pressure steam to the condenser takes place *before* the admission of the driving steam. From this point, as the valve moves to the right, *c* is opened more. It is fully opened when the slide is at its extreme right position. Fig. 96 shows the valve returned a good ways toward the left, but still in communication with the condenser. The steam in the cylinder, on right of piston, has then the pressure of the atmosphere, or of the condenser. In Fig. 97 the port *c* is closed, and as the piston still goes toward the right, the steam is compressed, becomes hotter, denser, and has a higher pressure. For this compression a certain work is necessary, which must be deducted from the total delivery. But this work is not lost, since now less fresh steam is required for filling the space back of the piston. Experience shows that this compression, or "cushioning," is also necessary for quiet and smooth working. Fig. 98 shows the end of compression, or the position of the valve when *c* just begins to open for the admission of fresh steam on the right. The action of the steam just described is caused by two things, the "*angle of advance*," and the "*lap*" of the slide valve.

At first, things were so arranged that both ports were closed when the engine was on its dead points; in other words, when the piston was at either end of its stroke.

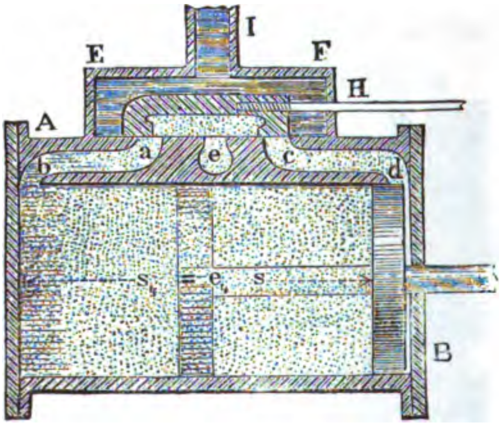

Fig. 98.

The slide was then in its central position. Such a relation between the motion of the slide and piston can be easily attained by so placing the eccentric disc that the line joining its center with the center of the shaft makes an angle of 90° with the line joining the dead points. If thus *B* and *C*, Fig. 99, are the dead points for a horizontal cylinder, the eccentric disc *S* must be so placed on the shaft *D*, that *DE* makes an angle of 90° with *BC*. In this case both ports would be fully opened when the piston is in the middle of its stroke, if the connecting rod were infinitely long. Even for moderate length of this rod, the same is nearly true. Since now, in our present arrangement, we wish both ports to be open when the piston is at either end of its stroke, the valve must be beyond its central position. This is attained by fixing the disc on the shaft so that *DE* makes more than 90° with *BC*. This increase of the angle of 90° is the "angle of advance." The angle of advance, then, is the angle made by the eccentricity with the perpendicular to the valve face when the piston is at a dead point.

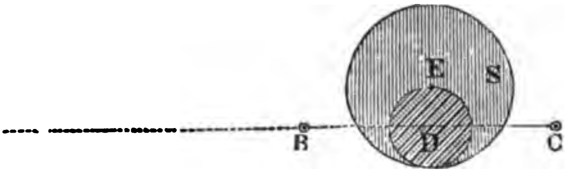

FIG. 99.

If now the steam is required to act in the cylinder with a certain expansion and compression, we must have the following arrangement. We make the slide so long that in its central position it laps over the ports by a certain amount on each side. The amount which it exceeds the port on the outside is called the "*outside lap*," and on the inside, the "*inside lap*." As it is more advantageous, as has been pointed out, for the re-

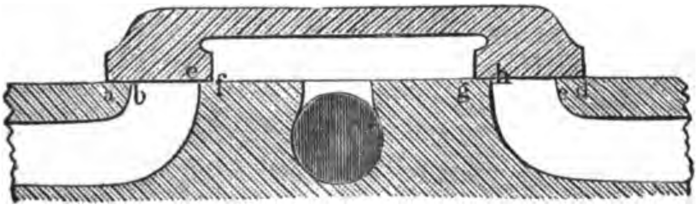

FIG. 100.

lease of the steam to take place somewhat earlier than the admission, the outer lap is always greater than the inner. Thus *ab* and *cd*, Fig. 100, are the outer laps, and *ef* and *gh* the inner, or they are the distances by which the valve, when in its central position, extends beyond the ports.

For this central position of the valve the center *d* of the eccentric disc must be in the perpendicular *cd* to *be*. If, now, when the piston is at the left end of its stroke, the port for the admission of steam on the left is to be opened a little, the point *d* must be somewhere to the right, say at *f*. If the conecting rod and eccentric rods are very long, the travel of the valve toward the right is approximately *fg*.

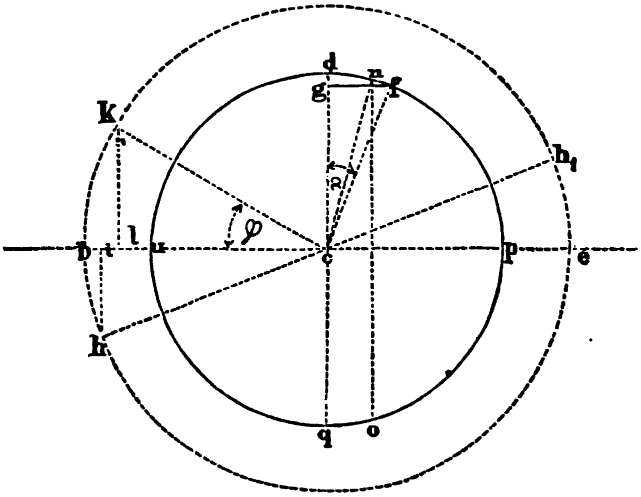

FIG. 101.

The angle *dcf* is the angle of advance. If we wish to know how far the piston is from the left end of its stroke, when the valve has its central position, we have only to lay off the angle *bch* = *dcf*, and let fall *hi*. Then *bi* is the distance, if *bc* = *ce* is the angle of the crank, or the half stroke of the piston.

Let the angle of advance $dcf = \alpha$, and the eccentricity *cd*, that is the distance of the eccentric disc from the center of the shaft be ρ, then

$$gf = \rho \sin \alpha.$$

If, now, the outer lap $ab = cd = a_1$, and the "*lead*," or the opening of the port, when the piston is at the end of its stroke, is b_1, we have

$$gf = a_1 + b_1 = \rho \sin \alpha.$$

In general, for horizontal engines, $a_1 = 0.25\rho$, or $\frac{1}{4}$ of the eccentricity, and $b_1 = \frac{1}{10}\rho$, hence

$$(0.25 + 0.1)\,\rho = \rho \sin \alpha,$$

or

$$\sin \alpha = 0.35, \quad \text{or} \quad \alpha = 20^\circ\ 30'.$$

If further, the inner lap $ef = gh$ is a_2, the opening of the port, when the piston is at the end of its stroke, for the release of the steam, or the *inside lead*, is b_2, we have

$$a_2 + b_2 = \rho \sin \alpha = 0.35\rho.$$

If we make the inside lap $a_2 = 0.05\rho$, that is $\frac{1}{5}$th of the outside lap, we have

$$0.05\rho + b_2 = 0.35\rho, \quad \text{or} \quad b_2 = 0.30\rho.$$

Hence the port is opened for discharge three times as much as for entrance, when the piston is on dead point.

We can now easily find the angle α_1 or α_2 through which the center of the eccentric disc must turn, in order to open the port for entrance or discharge. For the first,

$$\rho \sin \alpha_1 = a_1, \quad \text{or} \quad \sin \alpha_1 = \frac{a_1}{\rho},$$

or inserting value of a_1, viz., 0.25ρ,

$$\sin \alpha_1 = 0.25, \quad \text{or} \quad \alpha_1 = 14^\circ\ 29'.$$

Further, $\rho \sin \alpha_2 = a_2$, or $\sin \alpha_2 = \frac{a_2}{\rho} = \frac{0.05\rho}{\rho} = 0.5$, or

$$\alpha_2 = 2^\circ\ 52'.$$

The eccentric has to turn but a little, therefore, from its central position, in order to open the port for discharge.

If the piston is at the left end of its stroke, the end of the crank is at *b*, or at one dead point. If now *b* passes through the arc *bk*, we can easily find the travel of the piston. Since the connecting rod is very long compared to the eccentricity, this travel is *bl*, and since *cb* is half the stroke, or $\frac{1}{2}s$,

$$bl = bc - cl = \frac{s}{2} - \frac{s}{2}\cos\varphi = \frac{s}{2}(1 - \cos\varphi).$$

For $\varphi = 90°$, $\cos\varphi = 0$ and the travel is $\frac{s}{2}$. For $\varphi = 180°$, $\cos\varphi = -1$, and $\frac{s}{2}(1 - (-1)) = s$. Generally the travel of the piston x for any angle of the crank is

$$x = \frac{s}{2}(1 - \cos\varphi).$$

When the crank is turned through the arc φ from its dead point, the eccentricity makes the angle $\alpha + \varphi$ with *dc*, and the travel w of the valve is

$$w = \rho\sin(\alpha + \varphi).$$

This travel is positive when the motion of the slide is in the same direction as that of the piston, otherwise it is negative.

1. *Travel of the Piston up to the End of Admission or to the Beginning of Expansion.*—Let this travel s_1 be a portion e_1 of s, so that

$$s_1 = e_1 s.$$

We wish to find e_1. The entrance of steam ends when the end *a* of the valve returns to *b* again, on the back stroke. Before the port begins to open, the valve must pass through the distance $ab = a_1$, or the eccentricity through the angle $dcn = \alpha_1$. If we let fall from *u* a perpendicular to *bc* and prolong it to *o*, *oc* is the position of the eccentricity when the valve again closes the port. The eccentric revolves then, from beginning of admission to cut-off, through $npo = 180 - 2\alpha_1$. The crank, of course, goes through the same angle. But when the eccentricity is at *n*, the crank makes an angle with *bc* of $\alpha - \alpha_1$. If we denote the rest of the arc through which the crank must go by y, we have

$$\alpha - \alpha_1 + y = 180 - 2\alpha_1, \quad \text{or}$$

$$y = 180 - (\alpha + \alpha_1).$$

If we insert this in the equation of the travel of the piston,

$$e_1 s = \frac{s}{2}(1 + \cos(\alpha + \alpha_1)), \text{ hence}$$

$$e_1 = \frac{1 + \cos(\alpha + \alpha_1)}{2}.$$

Since $\alpha = 20° 30'$, $\alpha_1 = 14° 29'$, we have $e_1 = 0.910$. The admission of steam, therefore, ceases when the piston has passed through 0.91 of its stroke. Expansion then begins.

2. *Travel of the Piston up to the Beginning of Compression, or the End of Release.*—The compression begins when, on the return of the valve, the corner g meets h. The valve is then distant gh from its centre position, and the eccentricity makes the angle α_2 with cq. From d then it makes the angle $180 - \alpha_2$. The crank has passed through the same angle from h or from b through

$$180 - \alpha_2 - \alpha = 180 - (\alpha + \alpha_2).$$

But $$\cos[180 - (\alpha + \alpha_2)] = -\cos(\alpha + \alpha_2),$$

and denoting the travel up to beginning of compression by $s_2 = e_2 s$,

$$s_2 = e_2 s = x = \frac{s}{2}[1 + \cos(\alpha + \alpha_2)], \text{ or}$$

$$e_2 = \frac{1 + \cos(\alpha + \alpha_2)}{2}.$$

If we put for α and α_2 the numerical values,

$$e_2 = 0.959.$$

This position of the piston is shown in Fig. 97.

3. *Travel of the Piston up to the End of Expansion or to the Point of Release.*—The expansion ends when the valve has moved so far towards the left that f coincides with e. The valve has then moved towards the left ef from its central position. Since ef corresponds to the angle α_2, the eccentricity makes this angle to the left of q. Reckoned from d, the angle is $180 + \alpha_2$. The crank has made the same angle, reckoned from h, or from b it makes

$$180 + \alpha_2 - \alpha = 180 - (\alpha - \alpha_2).$$

If the travel in the present case $s_2 = e_2 s$, we have

$$s_2 = e_2 s = \frac{s}{2}\,[1 + \cos(\alpha - \alpha_2)],$$

and

$$e_2 = \frac{1 + \cos(\alpha - \alpha_2)}{2}.$$

Inserting the numerical values,

$$e_2 = 0.977.$$

Since expansion begins at 0.910, the duration of expansion is $0.977 - 0.910 = 0.067$ of the entire stroke.

The position of valve and piston in this case is shown in Fig. 96.

4. *Travel of the Piston up to the End of Compression of the Back Pressure Steam, or up to the Admission of Driving Steam.*—The compression ends when the point d of the valve coincides with c. The valve has then moved cd from its central position towards the left. The eccentricity makes the angle α_1 with q, or $180 + \alpha_1$ with d, hence the crank makes from b the angle

$$180 + \alpha_1 - \alpha = 180 - (\alpha - \alpha_1).$$

The travel of the piston up to the end of compression is then

$$s_4 = e_4 s = x = \frac{s}{2}\,[1 + \cos(\alpha - \alpha_1)], \text{ or}$$

$$e_4 = \frac{1 + \cos(\alpha - \alpha_1)}{2}.$$

Inserting the numerical values, $e_4 = 0.997$.

The piston has then only a very small distance to go to its dead point. This position is shown in Fig. 98.

Since for the beginning of compression $e_3 = 0.959$, and for the end $e_4 = 0.997$, the entire compression is

$$0.997 - 0.959 = 0.038 \text{ of the stroke, or about}$$

half as great as the expansion. We see also that the compression begins later and ends later than the expansion. It is indispensable for smooth motion of the engine.

The fraction e_1 of the entire stroke s, which is filled with boiler steam, is the "coefficient of fill." For the usual proportions as given, it is 0.91. This value may be regarded as a maximum. If the engine has a special expansion valve, e_1 is less, but the other quantities e_2, e_3, and e_4 are as above.

Steam Volume per Stroke—Degree of Expansion and Compression.—If the area of the piston is F square meters, the volume of the entire stroke is Fs cubic meters, and this would be the volume of steam used per stroke if the steam entered during the entire stroke; since, however, the steam enters during e_1s, we have for the volume of steam used

$$Fe_1s \text{ cubic meters.}$$

Moreover the piston, when at the end of its stroke, does not reach the cylinder cover, but there is a space between, filled with steam. Also the steam passages must each time be filled. In full pressure engines both these steam quantities contribute almost nothing to the work. This space is hence called the "*prejudicial space.*" In expansion engines these steam quantities take part in the expansion, and hence for the same degree of expansion the pressure in the cylinder sinks less than when there is no prejudicial space, while, during the full pressure period, the steam in this space performs no work. Thus the prejudicial space increases indeed the delivery of expansion engines, but not in the ratio of the increased consumption of steam, and hence an engine with a prejudicial space has a less efficiency than without.

Let the prejudicial space be a fraction e of the cylinder volume. Then its volume is Fes cubic meters.

If now steam enters during the entire stroke, the steam used is

$$Fes + Fs = Fs\,(1 + e) \text{ cubic meters.}$$

In reality, however, we have

$$Fes + Fe_1s = Fs\,(e + e_1).$$

The steam volume at the end of expansion is

$$Fes + Fe_2s = Fs\,(e + e_2).$$

Hence the "*degree of expansion*" is

$$\varepsilon_1 = \frac{e + e_1}{e + e_2}.$$

At the moment when communication is closed with the condenser or air, the inclosed steam volume of the condenser or air pressure is

$$Fs + Fes - Fe_3s = (1 + e - e_3)\,Fs.$$

At this moment compression begins. At the end of compression, the inclosed steam volume is

$$Fs + Fes - Fe_4s = (1 + e - e_4)\,Fs.$$

Hence the "*degree of compression*" is

$$\varepsilon_2 = \frac{1 + e - e_3}{1 + e - e_4}.$$

In the ordinary slide valve, we have e about 0.05, that is, the prejudicial space is $\frac{5}{100}$ of the entire cylinder volume. In this case the steam passages are about half the length of the cylinder. In expansion engines with two slides, we have $e = 0.07$ to 0.075. If the passages are very short, as in the Corliss engine, where the prejudicial space is only that between piston and cylinder cover, $e = 0.02$.

EXAMPLE.

What is the degree of expansion when the steam is cut off at half stroke? If we take $s = 0.070$,

$$\varepsilon_1 = \frac{0.070 + 0.5}{0.070 + 0.977} = \frac{0.570}{1.047} = 0.544.$$

Work of the Driving Steam.—This work in every engine consists of two parts; the one is the action of the full pressure steam, the other is that of the expanding steam. If p_1 is the pressure of the full steam, we have for its work

$$L_1 = Fs_1p_1.$$

If we assume that during expansion, the steam follows the law

$$pv^{n_1} = p_1v_1^{n_1},$$

we have for the work during expansion

$$\frac{p_1v}{n_1 - 1}\left[1 - \left(\frac{v}{v_1}\right)^{n_1 - 1}\right],$$

where v is the specific steam volume at the beginning of expansion. If the engine uses per stroke G kilograms of steam, and if the volume of this weight before expansion is V and after V_1, we have for the expansion work

$$L_2 = \frac{p_1V}{n_1 - 1}\left[1 - \left(\frac{V}{V_1}\right)^{n_1 - 1}\right].$$

The value of n_1 varies, as we have seen, and depends upon the amount of water in the steam. On an average we can put $n_1 = 1.125$, when the steam is taken directly from the boiler. If, however, the steam is heated, or even superheated, before it enters the cylinder, n_1 may be greater. According to Grashof, we can take $n_1 = 1.333$, when the steam is still dry after expansion, when, therefore, no steam condenses during expansion.

As to the value of p_1, it is to be taken somewhat less in finding the expansion work, when the cut-off is worked by an ordinary eccentric, because in such case the cut-off is gradual. We have, therefore,

$$p_1 = \beta_1 p_1,$$

where β_1 is a proper fraction, which we can take about 0.95. In Corliss engines, where the ports are suddenly closed, $\beta = 1$, and p_1 has its full value. In engines with large piston velocity, β must be taken much less than 0.95.

The steam volume V before expansion is

$$V = Fes + Fe_1s = Fs(e + e_1),$$

and the volume V_1 after expansion is

$$V_1 = Fes + Fe_2s = Fs(e + e_2).$$

If we insert these values in the above expression for L_2, and put $p_1 = \beta p_1$, we have

$$L_2 = \frac{\beta_1 p_1 Fs(e + e_1)}{n_1 - 1}\left[1 - \left(\frac{e + e_1}{e + e_2}\right)^{n_1 - 1}\right],$$

or putting the degree of expansion $\dfrac{e + e_1}{e + e_2} = \varepsilon_1$,

$$L_2 = \beta_1 p_1 Fs(e + e_1)\frac{1 - \varepsilon_1^{n_1 - 1}}{n_1 - 1}.$$

If we put

$$\beta_1(e + e_1)\frac{1 - \varepsilon_1^{n_1 - 1}}{n_1 - 1} = \lambda,$$

we have

$$L_2 = \lambda_1 p_1 Fs.$$

The expansion ends before the piston arrives at the end of its stroke. Then the exhaust opens, and the pressure sinks rapidly to that of the condenser or of the atmosphere. If in this case the mean pressure is p_3, we have, while the piston passes through $s - s_2$,

$$L_3 = F(s - s_2)p_3 = Fs(1 - e_2)p_3.$$

Hence the total work of the driving steam is

$$\begin{aligned} L_1 + L_2 + L_3 &= Fs_1p_1 + \lambda_1 p_1\, Fs + Fs(1-e_2)p_2 \\ &= e_1p_1Fs + \lambda_1 p_1 Fs + Fs(1-e_2)p_2 \\ &= [(e_1+\lambda_1)p_1 + (1-e_2)p_2]\, Fs. \end{aligned}$$

Work of the Back Pressure.—We must subtract from the work just found that of the back pressure. Let the mean pressure during the travel s_3 be p_3, then

$$L_4 = Fs_3p_3 = Fse_3p_3.$$

Now compression begins. Let the law be

$$pv^{n_2} = p_1v_1^{n_2}.$$

The value of n_2, like that of n_1, can only be determined by exact experiments. In the average $n_2 = 1.15$ (Grashof's Resultate der mechanischen Wärmetheorie). The pressure at the beginning of compression is somewhat greater than p_3. In general we can take it $\beta_3 p_3$, where $\beta_3 = 1.05$. If now we denote the volume at the beginning of compression by v_2 and at the end by v_3, we have for the work of resistance during compression

$$L_5 = \frac{\beta_3 p_3 v_2}{n_2 - 1}\left[\left(\frac{v_2}{v_3}\right)^{n_2-1} - 1\right].$$

But $v_2 = Fs(1+e-e_3)$ and $v_3 = Fs(1+e-e_4)$, hence

$$L_5 = \frac{\beta_3 p_3 Fs(1+e-e_3)}{n_2 - 1}\left[\left(\frac{1+e-e_3}{1+e-e_4}\right)^{n_2-1} - 1\right].$$

Since the quantity in parenthesis, the degree of compression, is ε_2,

$$L_5 = \beta_3 p_3 Fs(1+e-e_3)\frac{\varepsilon_2^{n_2-1} - 1}{n_2 - 1}.$$

If we put here

$$\beta_3(1+e-e_3)\frac{\varepsilon_2^{n_2-1} - 1}{n_2 - 1} = \lambda_3$$

$$L_5 = \lambda_3 p_3 Fs.$$

When compression is ended, the piston has not reached the end of its stroke, but has still to go $s - s_4$. While it goes this distance, steam enters from the boiler, and the pressure is on the average greater than the mean pressure of the driving steam, especially in high piston speed. Let it be p_4, then

$$L_6 = F(s - s_4)\,p_4 = Fs\,(1 - e_4)\,p_4.$$

The total work of the back pressure is then

$$\begin{aligned} L_4 + L_5 + L_6 &= Fse_3\,p_3 + Fs\lambda_3 p_3 + Fs\,(1 - e_4)p_4 \\ &= Fs\,[p_3(e_3 + \lambda_3) + p_4(1 - e_4)]. \end{aligned}$$

If we subtract this from the work of the driving steam, we have the indicated work per stroke, that is, without reference to friction, working of pumps, etc. Let this be L_i, then

$$L_i = Fs\,[p_1(e_1 + \lambda_1) + (1 - e_2)\,p_2 - p_3(e_3 + \lambda_3) - p_4\,(1 - e_4)].$$

Since the value of p_4 for ordinary stationary engines, especially when not moving rapidly, is but little different from p_1,

$$L_i = Fs\,[p_1(e_1 + \lambda_1 + e_4 - 1) + (1 - e_2)\,p_2 - (e_3 + \lambda_3)\,p_3].$$

As to the mean pressure p_2 from s_2 to s, or through $s - s_2$, this depends upon the pressure at the end of expansion. But this depends upon the coefficient of fill e_1 of the cylinder. The greater this, so much the greater is p_2, other things being the same. Further, p_2 depends upon the manner in which the port is opened for discharge. If opened quickly, p_2 is less; if slowly, p_2 is greater. Grashof gives

$$p_2 = \lambda e_1 p_1 + p_3 - \lambda p_3,$$

where p_3 is the mean back pressure. Under ordinary circumstances $\lambda = 0.80$.

If we insert this value of p_2, we have

$$L_i = Fs\,[p_1\,(e_1 + \lambda_1 + e_4 - 1) + (1 - e_2)\,(\lambda e_1 p_1 + p_3 - \lambda p_3) - p_3\,(e_3 + \lambda_3)].$$

If we divide this by Fs, we have for the mean effective pressure

$$\frac{L_i}{Fs} = p_i = p_1\,(e_1 + \lambda_1 + e_4 - 1) + (1 - e_2)\,(\lambda e_1 p_1 + p_3 - \lambda p_3) - p_3\,(e_3 + \lambda_3).$$

If for brevity we put

$$e_1 + \lambda_1 + e_4 - 1 + \lambda e_1 (1 - e_2) = f_1 \quad \text{and}$$

$$e_3 + \lambda_3 - (1 - \lambda)(1 - e_2) = f_3, \quad \text{we have}$$

$$\frac{L_i}{Fs} = p_i = p_1 f_1 - p_3 f_3.$$

Hence the indicated work per stroke is

$$L_i = Fsp_i = Fs\,(p_1 f_1 - p_3 f_3).$$

If p_i is given in atmospheres, then

$$L_i = 10334\,Fsp_i = 10334 Fs\,(p_1 f_1 - p_3 f_3).$$

Values of p, p_1 and p_3.—The pressure p_1 in the cylinder can be very different from the pressure p in the boiler. This depends upon how wide the valve in the steam pipe is open, as also the throttle valve; also upon the cross-section and length of the steam passages in comparison with the piston speed, which we shall denote by c; upon the curves and bends of the steam pipe; upon its length and radiation. If the steam pipe is completely opened, and the passages have the required cross-section, the mean pressure in the cylinder during admission varies but little from the boiler pressure. In other cases the difference may be considerable. An exact calculation of this difference of pressure, from the diameter of the partially opened steam pipe, the cross-section and length of the passages, etc., is indeed hardly practicable. It is evident that in calculating the delivery we must have regard to the cylinder pressure, and when we wish an exact determination of the delivery, we must find this pressure by the indicator. A long and narrow steam pipe, with bends and angles, and not protected from radiation, can make the pressure in the valve-box less than the boiler pressure. We must give to this pipe either the same or a somewhat less cross-section than the steam passages.

The cross-section of these last depends not only upon that of the cylinder, but also upon the mean piston speed c. Through these passages the same amount of steam must pass in a certain time as the cylinder uses. Since the velocity of the steam is considerable, we can make the cross-section of the passages less than that of the cylinder. But steam, when it flows through relatively narrow passages, experiences a great loss of velocity, which may be quite considerable when the valve nearly closes the parts. For a mean piston speed of 1 meter, experience gives for the cross-section of the canals $\frac{1}{30}$th of that of the piston. In this case the steam pressure in the cylinder remains constant during the entire admission. If then the cross-section of the steam passages is f, and that of the cylinder is F, we have

$$\frac{f}{F} = \frac{1}{30}.$$

We can further assume that the resistance, for the same cross-section, increases with the velocity; that for 2, 3, $\cdots c$ meters velocity, it is 2, 3, $\cdots c$ times as great. In order, then, to have only the same resistance as for 1 meter, we must make the passages 2, 3, $\cdots c$ times as wide. Hence

$$\frac{f}{F} = \frac{c}{30}.$$

From this formula we have the following tabulation

CROSS-SECTION OF STEAM PASSAGES.

Piston speed in meters =	1	1.2	1.5	2	3	4	5
$\frac{f}{F} =$	$\frac{1}{30}$	$\frac{1}{25}$	$\frac{1}{20}$	$\frac{1}{15}$	$\frac{1}{10}$	$\frac{1}{7.5}$	$\frac{1}{6}$

Engines with high piston speed, as locomotives, require therefore wide steam passages. Thus for $c = 2.3$ meters,

$$\frac{f}{F} = \frac{1}{30} \times 2.3 = \frac{1}{13}, \text{ and for } c = 3.1 \ \frac{f}{F} = \frac{1}{30} \times 3.1 = \frac{1}{9.7}.$$

If the steam passages are calculated for a certain piston speed, and we let the engine work more rapidly, the pressure in the cylinder is less than in the valve box, and especially in engines working with little expansion, the indicator will show a noticeable fall of pressure near the middle of the stroke, because here the velocity of the piston is greatest.

In the calculation of projected engines, we must consider the pressure p_1 in the cylinder as given, and consider the boiler pressure as always somewhat greater. In engines which are required to give great delivery, the boiler pressure must be taken still greater. It is different with the mean pressure p_3 of the back pressure steam. This is not given in advance. If, however, the steam passages are proportioned as above, we have for stationary engines

a, when non-condensing, $p_3 = 1.15$ atmospheres;
b, when condensing, $p_3 = 0.2$ atmospheres.

Locomotives which have a blast pipe, may have $p_3 = 1.15$ to 1.27 atmospheres.

Shorter Form for the Expressions $\mathfrak{f}_1$ *and* $\mathfrak{f}_3$.—The quantities e, e_2, e_3, and e_4 are constant so long as the angles of advance and the laps remain the same. Also β_3, n_2 and λ are to be regarded as constant. We can, therefore, give to the expressions in which these quantities occur, a simpler form.

Thus we have found

$$\varepsilon_2 = \frac{1 + e - e_3}{1 + e - e_4}.$$

Since now $e = 0.05$, $e_3 = 0.959$, and $e_4 = 0.997$,

$$\varepsilon_2 = \frac{1 + 0.05 - 0.959}{1 + 0.05 - 0.997} = \frac{0.91}{0.53} = 1.717.$$

Also $\beta_3 = 1.05$, and $n_2 = 1.15$, hence

$$\lambda_3 = \beta_3 (1 + e - e_3) \frac{\varepsilon_2^{n_2 - 1} - 1}{n_2 - 1} = 0.0538.$$

Since λ can be taken 0.8,

$$\begin{aligned} \mathfrak{f}_3 &= e_3 + \lambda_3 - (1 - \lambda)(1 - e_3) \\ &= 0.959 + 0.0538 - (1 - 0.8)(1 - 0.977) = 1.0082. \end{aligned}$$

Finally,

$$f_1 = e_1 + \lambda_1 + e_4 - 1 + \lambda e_1 (1 - e_2) = e_1 + \lambda_1 + 0.997 - 1 + 0.8 e_1 (1 - 0.977)$$
$$= 1.0184 e_1 + \lambda_1 - 0.003.$$

Shortest Form of the Formula for the Indicated Delivery.—The formula already found for the effective pressure p_i becomes now

$$\frac{L_i}{Fs} = p_i = p_1 (1.0184 e_1 + \lambda_1 - 0.003) - p_3 \times 1.0082,$$

or

$$L_i = Fsp_i = Fs\,[p_1 (1.0184 e_1 + \lambda_1 - 0.003) - 1.0082 p_3],$$

where p_1 and p_3 are the pressures in kilograms. If p_1 and p_3 are given in atmospheres,

$$L_i = 10334 Fs\,[p_1 (1.0184 e_1 + \lambda_1 - 0.003) - 1.0082 p_3].$$

So soon then as we know for any engine (which must have the assumed angle of advance, etc.), the coefficient of fill e_1 and λ_1, we can find L_i. Also, $f_1 = 1.0184 e_1 + \lambda_1 - 0.003$.

Grashof, in his "Resultaten der mechanischen Wärmetheorie," has found ε_1, λ_1, f_1 for different values of e_1, as given by the table below. The table also contains $\beta_1 \varepsilon_1^{n_1}$, which gives the pressure $\beta_1 p_1 \varepsilon_1^{n_1}$ at the end of expansion.

e_1	ε_1	λ_1	f_1	$\beta_1 \varepsilon_1^{n_1}$
0.1	0.1623	0.2627	0.3615	0.123
0.15	0.2100	0.2963	0.4461	0.164
0.2	0.2578	0.3199	0.5206	0.207
0.25	0.3055	0.3351	0.5867	0.250
0.3	0.3533	0.3428	0.6453	0.295
0.35	0.4010	0.3444	0.6978	0.340
0.4	0.4488	0.3404	0.7448	0.386
0.45	0.4965	0.3312	0.7865	0.432
0.5	0.5443	0.3171	0.8233	0.479
0.55	0.5920	0.2987	0.8558	0.527
0.6	0.6398	0.2765	0.8845	0.575
0.65	0.6875	0.2501	0.9091	0.623
0.7	0.7353	0.2206	0.9305	0.672
0.75	0.7830	0.1876	0.9484	0.721
0.8	0.8308	0.1514	0.9631	0.771
0.85	0.8785	0.1119	0.9745	0.821
0.9	0.9263	0.0700	0.9836	0.872
0.91	0.9358	0.0618	0.9855	0.882

From these values the values of p_i in the following tables are calculated for different values of e_1 and p_1. The first table applies to non-condensing engines, the other to condensing. In the first, the mean back pressure p_3 is 1.1; in the other, 0.2 atmospheres. Cases in which $\beta_1 p_1 \varepsilon_1^{n_1} < p_3$ are excluded.

p_i FOR NON-CONDENSING ENGINES

	$p_1 = 3$	$p_1 = 4$	$p_1 = 5$	$p_1 = 6$	$p_1 = 7$	$p_1 = 8$	$p_1 = 9$
$e_1 = 0.1$	....	....	....	....	....	....	2.145
0.15	....	....	....	....	2.014	2.460	2.906
0.2	....	....	....	2.015	2.585	3.056	3.577
0.25	....	....	1.825	2.411	2.998	3.585	4.171
0.3	....	1.472	2.118	2.763	3.408	4.054	4.699
0.35	....	1.682	2.380	3.078	3.776	4.474	5.171
0.4	1.126	1.870	2.615	3.360	4.105	4.850	5.594
0.45	1.251	2.037	2.824	3.610	4.397	5.183	5.970
0.5	1.361	2.184	3.008	3.831	4.654	5.478	6.301
0.55	1.459	2.314	3.170	4.026	4.882	5.738	6.593
0.6	1.545	2.429	3.314	4.198	5.083	5.967	6.852
0.65	1.618	2.528	3.437	4.346	5.255	6.164	7.073
0.7	1.683	2.613	3.544	4.474	5.405	6.335	7.266
0.75	1.736	2.685	3.633	4.582	5.530	6.478	7.427
0.8	1.780	2.744	3.707	4.670	5.633	6.596	7.559
0.85	1.815	2.789	3.764	4.738	5.718	6.687	7.662
0.9	1.842	2.826	3.809	4.798	5.776	6.760	7.744
0.91	1.848	2.833	3.819	4.804	5.790	6.775	7.761

p_i FOR CONDENSING ENGINES.

	$p_1 = 1.5$	$p_1 = 2$	$p_1 = 2.5$	$p_1 = 3$	$p_1 = 4$	$p_1 = 5$	$p_1 = 6$
$e_1 = 0.1$	....	0.521	0.702	0.883	1.244	1.606	1.967
0.15	0.468	0.691	0.914	1.137	1.583	2.029	2.475
0.2	0.579	0.840	1.100	1.360	1.881	2.401	2.922
0.25	0.678	0.972	1.265	1.558	2.145	2.732	3.319
0.3	0.766	1.089	1.412	1.734	2.380	3.025	3.670
0.35	0.845	1.194	1.543	1.892	2.590	3.287	3.985
0.4	0.916	1.288	1.660	2.033	2.778	3.522	4.267
0.45	0.978	1.371	1.765	2.158	2.944	3.731	4.517
0.5	1.033	1.445	1.857	2.268	3.092	3.915	4.738
0.55	1.082	1.510	1.938	2.366	3.222	4.077	4.933
0.6	1.125	1.567	2.010	2.452	3.336	4.221	5.105
0.65	1.162	1.617	2.071	2.526	3.435	4.344	5.253
0.7	1.194	1.659	2.125	2.590	3.520	4.451	5.381
0.75	1.221	1.695	2.169	2.644	3.592	4.540	5.489
0.8	1.243	1.725	2.206	2.688	3.651	4.614	5.577
0.85	1.260	1.747	2.235	2.722	3.696	4.671	5.645
0.9	1.274	1.766	2.257	2.749	3.733	4.716	4.700
0.91	1.277	1.769	2.262	2.755	3.740	4.726	5.711

From these two tables we can take the mean effective pressure for different coefficients of fill, and then can find the indicated work per stroke from the formula,

$$L_i = 10334\,Fsp_i.$$

If u is the number of revolutions per minute, and hence $2u$ the number of strokes, the indicated work per second is

$$\frac{2uL_i}{60} = \frac{10334 \times 2up_iFs}{60}$$

or

$$\frac{uL_i}{30} = 10334p_i\,\frac{u}{30}\,Fs \text{ meter-kilograms.}$$

In horse powers

$$N_i = \frac{uL_i}{30 \times 75} = \frac{10334}{30 \times 75}\,up_i\,Fs.$$

Since the engine makes in 1 minute $2u$ strokes, and each stroke is s meters long, the distance passed over by the piston per minute is $2us$ meters, and in one second

$$\frac{2us}{60} = \frac{us}{30}$$

This is therefore the mean velocity c of the piston.

We have then c in place of $\frac{us}{30}$, and

$$N_i = \frac{10334}{75}\,p_iFc.$$

EXAMPLE 1.—The diameter of the cylinder of a non-condensing engine is 0.47 meters, the stroke $s = 1.044$ meters, the number of revolutions per minute is 24, and the pressure in the cylinder $p_1 = 4$ atmospheres. What is the delivery per second when the proportions of the slide valve are as has been assumed in our discussion?

First, $F = \frac{\pi d^2}{4} = 0.7854d^2 = 0.7854 \times (0.47)^2 = 0.1734.$

Now from our tables, for a pressure of 4 atmospheres and $e_1 = 0.91$, $p_t = 2.833$. Hence

$$L_t = 10334 \times 2.833 \times 0.1734 \times 1.044 = 5301 \text{ m. kil.}$$

and

$$N_t = 4.593 \times 24 \times 2.833 \times 0.173 \times 1.044 = 56.4 \text{ horse power.}$$

EXAMPLE 2.—What would be the delivery, if $e_1 = 0.4$?

In this case

$$p_t = 1.870, \quad \text{and}$$

$$N_t = 4.593 \times 24 \times 1.87 \times 0.173 \times 1.044 = 37.23 \text{ horse power.}$$

The delivery is then $\frac{37.23}{56.40}$, or 0.66 of the first, while only about 0.4 as much steam is used, and hence not half as much fuel.

Work of the Engine when Disconnected.—From the indicated work of the engine, just calculated, we must subtract that required to work the pumps, eccentrics, and overcome the various frictional resistances. We obtain this work if we determine that of the disconnected engine, and then increase this latter by a certain amount, as given by experiment; because the work required by the friction of a *working* engine is greater by a certain amount than that of an engine running without overcoming useful resistance.

The greatest part of the work in question is required to run the fly-wheel. Let us estimate it first.

Theory of the Crank.—The fly-wheel serves not only to convert reciprocating motion into circular or rotary, and to carry

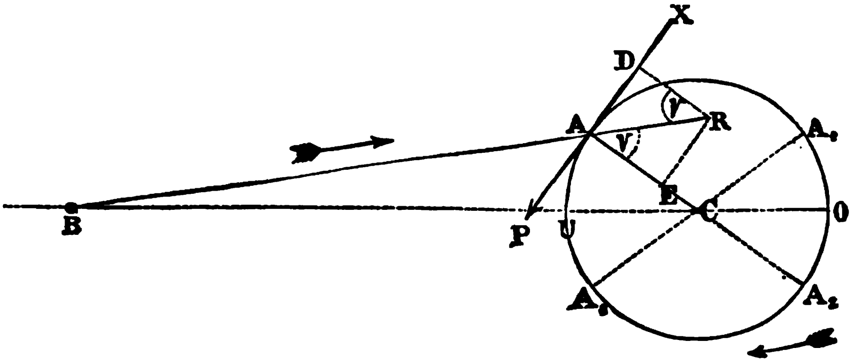

Fig. 102.

the motion past the dead points U and O, but also to make the motion uniform.

If BA, Fig. 102, is the connecting rod, CA the crank, the rod exerts either compression or tension upon the crank. If the rod has the position, and rotation takes place as shown by the arrows, it acts to cause compression. If the crank is at A_1, or A_3, the rod causes tension. Let $AR = Q$ be the pressure. This pressure only acts in part to cause rotation. That part, namely, which is perpendicular to the direction of the crank, or which acts in the direction AX. By the other component of Q, the crank is forced against the bearings and the friction increased. If now we decompose $AR = Q$ into the directions AX and AC, AD is the force causing rotation, and AE is that causing compression. If the angle $DRA = RAC = \gamma$, we have

$$AD = Q \sin \gamma . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1).$$

and

$$AE = Q \cos \gamma .$$

The resistance opposes the force $Q \sin \gamma$. Let it be $AP = P$, and constant at all points of the crank circle. We assume that the work of rotation is equal to that of the resistance. There are then four points in which the driving force $Q \sin \gamma$ is equal to the resistance P. These points can be easily found. For this purpose we assume, for the sake of simplicity, that the connecting rod is very long compared to the crank. The results thus obtained deviate, as we shall see later, but little from those obtained under the assumption of a finite rod. If, therefore, the rod is assumed very long, it is always parallel to UO, and we have

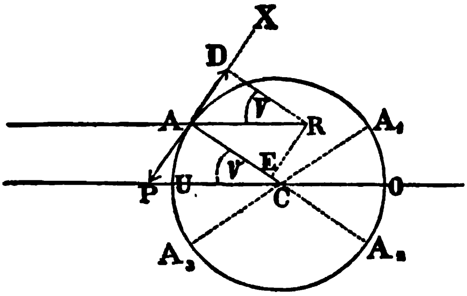

FIG. 103.

$$Q \sin \gamma = Q \sin ACU = P \quad . \quad . \quad . \quad . \quad (2).$$

and

$$\sin \gamma = \sin ACU = \frac{P}{Q} . \quad . \quad . \quad . \quad . \quad (3).$$

Let us assume the engine to work with full pressure. The force $Q = AR$ remains then constant during the whole distance $UO = 2r$. The work of this force is, for an entire revolution, $4rQ$.

The work of the resistance P in the same time is $2\pi rP$, and since both works are equal,

$$2Q = \pi P \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4).$$

or

$$P = \frac{2}{\pi} Q = 0.6366Q \quad . \quad . \quad . \quad . \quad (5).$$

The resistance cannot be greater for uniform motion. If we insert this value in (1) we have

$$\sin \gamma = \sin ACU = 0.6366,$$

or the angle

$$\gamma = ACU = 39^\circ \, 32'.$$

When, then, the crank makes the angle 39° 32′ with UO, the moving or tangential force is equal to the resistance P. For every other position of the crank the tangential moving force varies with the angle. If we make A_1CO, A_2CO, A_3CU equal to ACU, we have at the points A_1, A_2, A_3, the moving force $Q \sin \gamma$ equal to the resistance P. Between A and A_1, and between A_2 and A_3 the moving force is greater, while between A_1 and A_2, and A_3 and A, it is less than the resistance. But since the work of the force must be equal to that of the resistance, the excess, in the first portions, must equal the deficiencies in the other two. In order to accomplish this, it is necessary to attach a heavy weight to the axle. Such a weight is the fly wheel. The object of the fly wheel, then, is to receive the excess of work on AA_1 and A_2A_3, and give it up along A_1A_2, and A_3A.

Calculation of the Weight of Fly Wheel.—We can now easily find the weight S of the fly wheel. Suppose, first, a weight S_1 upon the crank pin, which has the same living force as the fly wheel, or, more simply, of the fly wheel rim. If the mean velocity of the weight is v and that of rim V, we must have, when the one mass is replaced by the other, $S_1v^2 = SV^2$, or

$$S = S_1 \left(\frac{v}{V}\right)^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6).$$

If the mean radius of the rim is R, and of the crank $r = \frac{s}{2}$, we have

$$v : V :: r : R, \quad \text{or} \quad \left(\frac{v}{V}\right)^2 = \left(\frac{r}{R}\right)^2 . \quad . \quad . \quad (7).$$

Hence from (6)

$$S = S_1 \left(\frac{r}{R}\right)^2 . \quad . \quad . \quad . \quad . \quad . \quad . \quad (8).$$

We have now to determine the weight S_1.

Since from A to A_1 there is an excess of force, there is an increase of velocity. If the velocity at A is v_1 and at A_1, v_2, the living force at A is $\frac{v_1^2}{2g} S_1$, and at A_1, $\frac{v_2^2}{2g} S_1$.

Hence from A to A_1 the living force stored is

$$\frac{v_2^2 - v_1^2}{2g} S_1 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (9).$$

This work is given up from A_1 to A_2, absorbed again from A_2 to A_3, and so on. If now the mean velocity is v, then

$$\frac{v_1 + v_2}{2} = v, \quad \text{or} \quad v_1 + v_2 = 2v. \quad . \quad . \quad . \quad (10).$$

We denote the ratio of the difference between the greatest and least velocities $(v_2 - v_1)$, to the mean velocity, by the term "*coefficient of irregularity*," and represent it by δ. Thus

$$\delta = \frac{v_2 - v_1}{v}, \quad \text{or} \quad v\delta = v_2 - v_1 . \quad . \quad . \quad (11).$$

This coefficient must be taken less, according as more uniformity is required. Thus the coefficient should be much less for an engine required to run a cotton mill, for instance, than for working pumps, etc.

From (10) and (11)

$$v_2^2 - v_1^2 = 2v^2\delta.$$

Inserting this in (9) we have for the increase of living force from A to A_1,

$$\frac{v^2\delta}{g} S_1 . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (12).$$

We can now obtain another expression for this work. The force Q performs from A to A_1 the work

$$2Qr\cos\gamma = 2Qr\cos 39^\circ\ 32' = 1.5424Qr.$$

The resistance $P = 0.6366Q$ is overcome through AA_1, or through an arc of $180 - 2(39^\circ\ 39') = 100^\circ\ 56'$. The length of this arc is $\frac{100^\circ\ 56'}{180}\pi v = 1.7615r$.

Hence the work of the resistance is

$$1.7615r \times 0.6366Q = 1.1214Qr.$$

The excess of the work of the force is then

$$(1.5424 - 1.1214)\ Qr = 0.4210Qr.$$

This excess must be equal to that of the living force. Hence

$$\frac{v^2\delta}{g}S_1 = 0.4210\ Qr, \quad \text{or} \quad v^2S_1 = 0.4210\ \frac{Qr}{\delta}g.$$

Since $\quad S_1v^2 = SV^2 = S\left(\frac{R}{r}v\right)^2 = S\left(\frac{R}{r}\right)^2v^2$, we have

$$S\left(\frac{R}{r}\right)^2v^2 = 0.4210\frac{Qr}{\delta}g, \text{ or}$$

$$S = 0.4210\left(\frac{r}{R}\right)^2\frac{Qr}{v^2\delta}g \quad . \quad . \quad . \quad . \quad . \quad (13).$$

For a finite connecting rod v^2S_1 is somewhat greater. If, for example, the connecting rod is 6 times as long as the crank, or $\frac{r}{l} = \frac{1}{6}$, we find by similar calculation

$$S = 0.4978\left(\frac{r}{R}\right)^2\frac{Qr}{v^2\delta}g.$$

For $\frac{r}{l} = \frac{1}{6}$

$$S = 0.5154 \left(\frac{r}{R}\right)^2 \frac{Qr}{v^2\delta} g.$$

For $\frac{r}{l} = \frac{1}{4}$

$$S = 0.5436 \left(\frac{r}{R}\right)^2 \frac{Qr}{v^2\delta} g.$$

If the engine works expansively, then for the same delivery and otherwise similar circumstances, the above expressions must be multiplied by the expression

$$0.77 + 0.23 \frac{1}{e_1} - 0.017 \left(\frac{1}{e_1}\right)^2 \quad . \quad . \quad . \quad . \quad (14).$$

where e_1 is the coefficient of fill of the cylinder. Thus for $e_1 = 0.5$, or for cut-off at $\frac{1}{2}$ the length of cylinder,

$$0.77 + 0.23 \times 2 - 0.017 \times 4 = 1.162.$$

We shall now express Qr in terms of the indicated work L_i. The force Q represents the mean effective pressure upon the piston, hence

$$Q = Fp_i$$

where p_i is given in kilograms per sq. meter. Hence

$$Qr = rFp_i = \tfrac{1}{2}sFp_i = \tfrac{1}{2}L_i.$$

Substituting this in the above expressions, and inserting the value of $g = 9.81$ meters, we have for

$\frac{r}{l} = \frac{1}{6}$

$$S = 2.439 \left(\frac{r}{R}\right)^2 \frac{L_i}{v^2\delta}$$

$\frac{r}{l} = \frac{1}{5}$

$$S = 2.525 \left(\frac{r}{R}\right)^2 \frac{L_i}{v^2\delta}$$

$\frac{r}{l} = \frac{1}{4}$

$$S = 2.664 \left(\frac{r}{R}\right)^2 \frac{L_t}{v^2\delta}.$$

The weight S is that which when applied at the distance R from the center C of the shaft, will cause the required degree of uniformity of motion.

Now every fly wheel consists of a rim and 4 or 8 arms. If we let S refer to the rim alone, then by reason of the inertia of the arms the degree of irregularity will be less than that assumed in the calculation. We may, therefore, take about 0.9 of the above values for S and consider this as the weight of the rim. We have then

(1.) for $\frac{r}{l} = \frac{1}{6}$

$$S_r = 2.195 \left(\frac{r}{R}\right)^2 \frac{L_t}{v^2\delta},$$

(2.) for $\frac{r}{l} = \frac{1}{5}$

$$S_r = 2.273 \left(\frac{r}{R}\right)^2 \frac{L_t}{v^2\delta},$$

(3.) for $\frac{r}{l} = \frac{1}{4}$

$$S_r = 2.398 \left(\frac{r}{R}\right)^2 \frac{L_t}{v^2\delta}.$$

These formulæ apply to full pressure engines. For engines working expansively, we must multiply by the coefficient given by (14).

If the arms are $\frac{3}{10}$ths of the weight of the rim, and if we denote the weight of the fly wheel shaft by S_w, and the total weight of shaft and wheel by S, we have

$$S = S_w + 1.3 S_r.$$

EXAMPLE.

What weight must the fly wheel have for the engine of page 586, of 56.4 horse power, or $L_t = 5301$ meter-kilograms ? The coefficient of irregularity δ is taken $\frac{1}{32}$, and $\frac{r}{l} = \frac{\frac{1}{2}s}{l}$ and $\frac{r}{R} = \frac{1}{6}$.

The circumference of the crank circle is $2\pi r = \pi s$. Since there are u revolutions per minute, the mean velocity v is $\frac{\pi us}{60} = \frac{3.1416 \times 1.044 \times 24}{60} = 1.32$ meters. Hence

$$S_r = 2.273 \left(\tfrac{1}{5}\right)^2 \frac{5301}{(1.32)^2 \times \frac{1}{32}} = 8862 \text{ kilograms.}$$

If with the same delivery the coefficient of fill is $e_1 = 0.5$, we must multiply this weight by $0.77 + 0.23 \times 2 - 0.017 \times 4 = 1.62$, and hence

$$S_r = 10298 \text{ kilograms.}$$

The weight of the entire fly wheel, in the first case, is $1.3 \times 8862 = 11521$, and in the second $1.3 \times 10298 = 13387$ kilograms.

Since the friction of the fly wheel journals consumes a considerable amount of the work, we should have the weight as small as possible, and make the radius as great as possible.

Dimensions of the Rim and Arms.—If we know the weight of the rim, we can easily find its dimensions. We denote the cross-section by F_1 and the weight of 1 cubic meter of cast iron by γ. (Since the specific weight of cast iron is 7.4, 1 cubic meter weighs $1000 \times 7.4 = 7400$ kilograms.) The volume of the rim is then

$$2\pi R F_1 \text{ cubic meters,}$$

and its weight

$$2\pi R F_1 \gamma.$$

We have then

$$S_r = 2\pi R F_1 \gamma,$$

or putting γ, 7400 kilograms,

$$S_r = 46472 R F_1,$$

and hence

$$F_1 = \frac{S_r}{46472 R}.$$

In general, we make the radial depth of rim 1 to 2 times the thickness. If the first is d and the second b, we have $F_1 = bd$.

If we denote the number of arms by n, and the cross-section by F_2, the weight is

$$7400nF_2R,$$

and we have

$$\tfrac{1}{3}S_r = 7400nF_2R.$$

To smaller wheels we may give 4 arms, to larger 6 or even 8. If n is given, we have

$$F_2 = \frac{S_r}{22200nR}$$

EXAMPLE.

What must be the cross-sections F_1 and F_2 for the fly wheel of the full pressure engine, already mentioned, for which the weight of fly wheel rim has been found 8862 kilograms, and the mean radius $R = 2.61$ meters ?

We have $F_1 = \frac{8862}{46472 \times 2.61} = 0.0729$ square meters. If we take $b = \frac{2}{3}d$, we have

$F_1 = 0.0729 = \frac{2}{3}d^2$, or $d = \sqrt{0.1094} = 0.331$ meters, or 33.3 centimeters, and $b = \frac{2}{3} \times 33.3 = 22.2$ centimeters.

For the arms, if we have 8 of them,

$$F_2 = \frac{8862}{22200 \times 8 \times 2.61} = 0.015 \text{ sq. meters.}$$

Diameter of the Journals—Weight of the Shaft.—We can now find the weight of the shaft. The journals, according to Morin, should have the diameter

$$d_1 = 20\sqrt[3]{\frac{N_i}{u}} \text{ centimeter}$$

where N_i is the indicated horse power and u the number of revolutions per minute. The diameter of the shaft can be 1 or 2 centimeters greater.

For our full pressure engine, $N_i = 56.4$ horse power, $u = 24$; hence

$$d_1 = 20\sqrt[3]{\frac{56.4}{24}} = 26.62 \text{ centimeters.}$$

If we make the diameter of the shaft 28.62 centimeters, and assume it is 3 meters long, the weight is approximately

$$7400 \times 3.14\,(0.143)^2 \times 3 = 1421 \text{ kilograms.}$$

Hence the weight of fly wheel and shaft is

$$S = 1421 + 11521 = 12942 \text{ kilograms.}$$

Mean Effective Pressure Necessary for Overcoming the Resistance of Friction.—Since we now know the diameter d_1 of the shaft and journals, and the weight of fly wheel and shaft, we can find the work required to overcome the journal friction.

The circumference of the journal is πd_1. If we put coefficient of friction $= c_1$, we have for the work per stroke (per half revolution)

$$\tfrac{1}{2}\pi d_1 c_1 S.$$

If p_s is the mean effective pressure per square meter of the piston, required for this work, the work of the steam per stroke is

$$10334 p_s F s = 10334 \frac{\pi d^2}{4} s p_s,$$

where d is the diameter of the piston. We have thus p_s,

$$p_s = \frac{4 \times \frac{1}{2}\pi d_1 c_1 S}{10334 \pi d^2 s} = \frac{2 d_1 c_1 S}{10334 d^2 s}.$$

If we take the coefficient of friction $c_1 = 0.1$, we have

$$p_s = 0.00002 \frac{d_1 S}{d^2 s}.$$

For the mean effective pressure p_t, required to overcome the friction of the piston, piston-rod, cross-head, crank-pin, eccentric, slide valve, and feed pump, Grashof gives

$$p_t = \frac{0.0227}{d}.$$

We have accordingly for the mean effective pressure of the engine when disconnected,

$$p_m = 0.00002\frac{d_1S}{d^2s} + \frac{0.0227}{d}.$$

For condensing engines, we have also the air and cold water pumps.

Mean Effective Pressure required for working the Cold Water and Air Pumps.—If we assume that the engine requires per hour D kilograms of steam, and n times as much cold water for condensing this steam, the cold water pump must furnish per hour nD kilograms of cold water. If the height is h meters, the work required to furnish this water is per hour nDh meter-kilograms.

We may allow that at each stroke $\frac{1}{10}$th of the water falls back, that therefore not nD, but $(1 + 0.1)\,nD = 1.1nD$ kilograms must be raised the distance h. Hence the work per hour is

$$1.1nDh.$$

If we allow $\frac{1}{3}$d of this for resistance of friction, we have for the actual work required per hour,

$$\frac{4}{3} \times \frac{11}{10}nDh,$$

and per minute, about

$$\frac{1.5nDh}{60}.$$

If now p_q is the mean effective pressure upon the piston required to perform this work, we have for the work of the steam per minute (u revolutions),

$$10334 \times 2uFsp_q.$$

This work must equal the preceding, hence

$$10334 \times 2uFsp_q = \frac{1.5nDh}{60},$$

or

$${}_q = 0.00015\,\frac{nDh}{120Fsu} \text{ very nearly.}$$

Let us now determine the work required by the air pump. This has to remove the water weight

$$(n+1)D$$

kilograms from the condenser. This requires the space $0.001\,(n+1)\,D$ cubic meters. In removing the water from the condenser, the pressure overcome is that of the atmosphere less the condenser pressure. Upon the return stroke, the condenser pressure is overcome. In each double stroke the average pressure is then that of the atmosphere. The work is then

$$10334 \times 0.001\,(n+1)\,D \text{ meter-kilograms per hour,}$$

or

$$\frac{10334}{60} \times 0.001\,(n+1)\,D \text{ per minute.}$$

If p_e is the mean effective pressure which performs this work, we have

$$10334 p_e F s 2u = \frac{10334}{60} \times 0.001\,(n+1)\,D,$$

or

$$p_e = 0.001\,\frac{(n+1)\,D}{120 F s u}.$$

Since the pump must also move the air from the condenser, and the frictional resistance must be overcome, we should, according to Grashof, at least double this pressure, and thus have

$$p_e = 0.0022\,\frac{(n+1)\,D}{120 F s u}.$$

If we neglect 1 in comparison to n,

$$p_e = 0.0022\,\frac{nD}{120 F s u}.$$

We have, then, for the mean effective pressure in the cylinder, required for working the cold water and air pumps,

$$p_q + p_e = p'_m = 0.00015\,\frac{nDh}{120Fsu} + 0.0022\,\frac{nD}{120Fsu}$$

$$= 0.00015n\,(h + 15)\,\frac{D}{120Fsu}.$$

As a rule, $n = 20$, and

$$p'_m = (0.003h + 0.045)\,\frac{D}{120Fsu}.$$

Hence the mean effective pressure p_m of the engine, when disconnected, if condensing, is

$$p_m = 0.00002\,\frac{d_1S}{d^2s} + \frac{0.0227}{d} + p'_m,$$

or

$$p_m = 0.00002\,\frac{d_1S}{d^2s} + \frac{0.0227}{d}$$

$$+ \,(0.003h + 0.045)\,\frac{D}{120Fsu}$$

Useful Delivery.—The work, therefore, of the engine when disconnected, per stroke is $L_m = Fsp_m$, where p_m is to be found as above for condensing or non-condensing engines. If we subtract this work from the indicated delivery, we have the useful work per stroke. For this, then,

$$L_u = L_i - L_m.$$

Now it is evident that the work L_m absorbed by the different frictional resistances and by the working of the pumps, is greater when the engine is at work than when disconnected. According to Pambour, we must increase the work of the resistances by a part of the useful work, and then subtract from L_i. Thus he takes this part at 0.12 to $0.14L_u$, and hence

$$L_u = L_i - (L_m + 0.13L_u),$$

or

$$L_u + 0.13L_u = L_i - L_m,$$

or

$$L_u = \frac{L_i - L_m}{1.13}.$$

Hence, if p_u is the mean effective pressure required for the useful work,

$$Fsp_u = \frac{Fsp_i - Fsp_m}{1.13},$$

or

$$p_u = \frac{p_i - p_m}{1.13}.$$

If we have u revolutions per minute, the horse power of useful work is

$$N_u = \frac{uL_u}{30 \times 75} = \frac{Fcp_u}{75},$$

where p_u is in kilograms per square meter.

For the indicated efficiency (m_i) of the engine, we have

$$m_i = \frac{L_u}{L_i} = \frac{N_u}{N_i} = \frac{p_u}{p_i}.$$

Steam Weight per Hour.—According to Völkers, the steam weight per hour D is given by

$$D = 120Fsu\,[(e + e_1)\gamma_1 - e\gamma_2] + 450d\sqrt{p_i}.$$

Here, γ_1 denotes the weight in kilograms of 1 cubic meter of steam at the pressure p_1 atmospheres, which, for saturated steam is given by Table II.

For non-condensing engines, $\gamma_2 = 1.32$; and for condensing, 0.264.

The second term includes the loss of steam and heat, as also the moisture of the steam, for engine in average good condition.

Finding thus D, we can find p_m from the formula already given for that quantity.

Dimensions of the Condenser and Pumps.—We have seen, page 510, that the ratio n of the weight of cooling water and condensed steam is given by

$$n = \frac{600 - t}{t_1 - t_0},$$

where t_1 is the temperature of the condenser water, and t_0 that of the injected cold water.

If we take as a mean temperature $t_1 = 46°$, corresponding to a pressure of $\frac{1}{10}$th atmosphere, and $t_0 = 18°$, we have

$$n = \frac{600 - 46}{46 - 18} = \frac{554}{28} = 20.$$

If t_0 is less than 18°, n may be less than 20.

The volume of the condenser is taken

$$C = \frac{Fs}{4} \text{ to } \frac{Fs}{3},$$

that is, $\frac{1}{3}$d or $\frac{1}{4}$th of the cylinder volume.

Let now V_1 be the volume described by the piston of the cold water pump per stroke, then

$$2uV_1$$

is the space described per minute. If the pump is double acting, this is the water quantity furnished per minute. If it is single acting, the quantity is uV_1 cubic meters.

In the first case, the water per hour is $2 \times 60uV_1$, and in the second $60uV_1$ cubic meters.

Now the steam weight per hour is D, and the water weight required for condensation is nD. The entire weight $(n + 1)D$ takes the space $\frac{(n + 1)D}{1000}$ cubic meters.

We have then for V_1

(*a.*) *For a double acting suction pump,*

$$V_1 = \frac{(n + 1)D}{1000 \times 2 \times 60 \times u} = \frac{(n + 1)D}{120000u} \text{ cubic meters.}$$

If about 10 per cent. of water falls back, we must increase V_1 by $\frac{1}{10}$th. Then

$$V_1 = 1.1 \frac{(n+1)D}{120000u} = \frac{(n+1)D}{109091u},$$

for which we may put

$$V_1 = \frac{nD}{108000u} \text{ cubic meters.}$$

(*b.*) *For single acting pump,*

we have double this, or

$$V_1 = \frac{nD}{54000u} \text{ cubic meters.}$$

Quantity of Fuel (*B*) *per Hour.*—When we know the steam weight D per hour, we can easily find the amount of fuel required for the generation of this steam, when the heating value of the fuel and the efficiency of the boiler and grate are known.

We know that 1 kilogram of steam requires for its generation

$$W = 606.5 + 0.305t \text{ heat units.}$$

D kilograms then require WD.

If 1 kilogram of fuel furnishes by complete combustion K heat units, we have for WD

$$\frac{WD}{K} \text{ kilograms of fuel necessary,}$$

when *all* the heat goes to generate steam.

But only a part of the fuel is completely consumed, even if the greatest part. Then, the hot gases of combustion carry off a large amount of heat, and only a part of K is effective to heat the boiler plates. This part, the ratio of which to the total heating power R of the fuel we call the efficiency of the grate, we denote by W_1. Further, not all the heat which enters the boiler plates goes to heat the water. A part is lost by reason of radiation and imperfect conduction. The heat which the water actually receives, compared to that received by the boiler, is the efficiency of the boiler. We denote it by W_2.

It is evident, then, that for vaporizing D kilograms of water we must have

$$B = \frac{WD}{W_1 W_2 K} \text{ kilograms of fuel.}$$

Cost of a Horse Power per Hour.—Let us now determine the cost of a horse power per hour. Let the price of the boiler be P and of the engine P_1. Also the price of all the masonry, chimney, boiler, and engine house, etc., be P_2. Let the interest on the price of boiler and engine be x per cent. The interest yearly is then

$$\frac{x}{100}(P + P_1).$$

Let the interest upon the capital P_2 be y per cent. Then we have yearly for this

$$\frac{y}{100} P_2.$$

Let the engine work z hours per year, and each hour consume B kilograms of fuel at a price of P_3 per kilogram. Then the yearly expenditure for fuel is

$$zBP_3.$$

If the price for attendance is A for each hour for each horse power, and for lubrication per year is A_1, and yearly repairs A_2, we have the yearly expenditure

$$zA + N_u A_1 + A_2.$$

The total expenditure per year is then

$$zA + N_u A_1 + A_2 + zBP_3 + \tfrac{y}{100} P_2 + \tfrac{x}{100}(P + P_1),$$

and the expenditure per hour for each horse power is

$$A_h = \frac{zA + N_u A_1 + A_2 + zBP_3 + \frac{y}{100} P_2 + \frac{x}{100}(P + P_1)}{zN_u}.$$

The expenditure per hour for each horse power is then, in general, less,

1. The greater the number of working hours per year. This is evident, as for many interruptions a large amount of fuel is wasted.

2. The greater N_u, or the larger the engine. It is also evident that two or more engines which give the same work as one are more costly in construction and maintenance than the single one. When, indeed, the size passes a certain limit, the difficulties of construction may be so great that two or more may be cheaper than one.

3. The higher the temperature of the feed water. The heating of the feed water, the importance of which we have shown on page 602, is a general practice. For this purpose the chimney gases are used, or the heat of the escaping cylinder steam.

4. The greater the mean effective pressure p_i and the less the mean back pressure p_m. The boiler pressure should, therefore, be high (6 to 8 atmospheres). The higher this pressure, the less is the advantage of condensation.

5. The greater the velocity of the engine. But this has evidently a limit. Radinger has shown that for a certain degree of fill and a certain pressure, the usually received mean velocity can be exceeded without danger of irregular action. The motive force at the beginning of each stroke must be so great as to be able to overcome the friction of the piston and the inertia of the moving masses, such as piston, piston and connecting rods. This governs the extent of compression (the advantage of which has been referred to) as also the amount of lead.

Hrabák (see Grashof's Resultate der mechanischen Wärmetheorie) gives for the following useful deliveries the corresponding mean velocities :

$N_u = 1$	10	20	45	80	150
$30c = 30$	35	40	45	50	55

Under the assumption that $x = 10$, $z = 3600$, $D = 7B$ and $P_8 = 30$ cents about, the coefficient of fill e_1 has been calculated for different sizes and pressures, when the quantity

$$zBP_8 + \frac{x}{100}(P + P_1)$$

is a minimum. This quantity is the principal yearly cost. The following table gives the degree of fill e_1:

	Engine without Condensation.				Condensing Engines.			
$N_u =$	7	20	60	180	7	20	60	180
$p_1 = 2$	—	—	—	—	0.33	0.30	0.25	0.23
$p_1 = 3$	0.41	0.40	0.39	0.38	0.30	0.25	0.30	0.20
$p_1 = 4$	0.33	0.32	0.31	0.30	0.25	0.22	0.20	0.15
$p_1 = 6$	0.30	0.25	0.23	0.20	0.24	0.20	0.18	0.13

We see, therefore,

a. That the coefficient of fill, for the same delivery N_u and the same steam pressure p_1 is less for condensing engines than for non-condensing. That, however, the coefficient for both systems is more nearly the same as the pressure increases.

b. Both condensing and non-condensing engines have a less coefficient of fill, the greater the useful delivery N_u and the greater the pressure p_1.

For a non-condensing engine, for example, of 7 horse power, it would be, under the assumed conditions, not advantageous, for a pressure of 4 atmospheres, to have the degree of fill of the cylinder e_1 greater or less than 0.33.

Calculation of a Projected Steam Engine.—Let us now conclude by showing how to proceed in order to find the dimensions of the more important parts of a steam engine of any required useful horse power. We cannot find these dimensions directly from the preceding formulæ, but we will show how by their aid we can find approximate values, and then, from these values can find the more exact proportions. Let us take an example which will illustrate the general method of procedure.

EXAMPLE.

Required to build a steam engine, working expansively, whose effective delivery shall be 60 horse power.

We assume that water is abundant, and hence the engine may be a condensing engine. We also assume that the work required of the engine is of such character that a coefficient of irregularity of $\delta = \frac{1}{32}$ will be sufficient. What dimensions must we give the engine? what amount of water and fuel is required per hour? etc.

If, instead of the *effective* delivery, the indicated delivery had been given, our formulæ would enable us to find with ease the dimensions of the various parts, and then we could estimate the resistances owing to the motion of these parts. The work corresponding to these resistances, deducted from the indicated delivery, would then give at once the effective delivery. We see at once that the indicated delivery must be much greater than the actual. We have called the quotient of the actual by the indicated delivery

$$m_i = \frac{N_u}{N_i} = \frac{p_u}{p_i}$$

the indicated efficiency.

This efficiency m_i we must seek to determine by experiment and calculation. It is, of course, more for large engines and less for smaller ones. Also for the same delivery it is somewhat less for expansion engines than for full pressure, and for condensing engines least of all.

According to Grashof, we have

1. *For non-condensing engines without expansion*, when

a. N_u is from 5 to 25 horse power,

$$m_i = \frac{N_u + 35}{N_u + 50}.$$

b. N_u from 25 to 80 horse power,

$$m_i = \frac{N_u + 75}{N_u + 100}.$$

2. *For non-condensing engines with expansion.*

a. N_u from 10 to 40 horse power,

$$m_i = \frac{N_u + 32}{N_u + 50}.$$

b. N_u from 40 to 100 horse power,

$$m_i = \frac{N_u + 72}{N_u + 100}.$$

3. *For condensing and expansion engines.*

a. N_u from 15 to 46 horse power,

$$m_t = \frac{N_u + 26}{N_u + 50}.$$

(*b.*) N_u from 46 to 180 horse power,

$$m_t = \frac{N_u + 86}{N_u + 130}.$$

From these formulæ the following table is calculated, which contains values rather too small than too large:

$N_u =$	5	10	15	20	25	30	40
1	0.727	0.750	0.769	0.786	0.800	0.808	0.821
2	....	0.700	0.723	0.743	0.760	0.775	0.800
3	....	....	0.631	0.657	0.680	0.700	0.733

$N_u =$	50	60	80	100	120	150	180
1	0.833	0.844	0.861	....	....	....	....
2	0.813	0.825	0.844	0.860	....	....	....
3	0.756	0.768	0.790	0.809	0.824	0.843	0.858

Since our engine is a condensing engine with expansion, we have from the table $m_t = 0.768$.

Hence

$$0.768 = \frac{N_u}{N_t} = \frac{60}{N_t}, \text{ or}$$

$$N_t = \frac{60}{0.768} = 78 \text{ horse power.}$$

Let the mean steam pressure of the driving steam be $p_t = 4$ atmospheres, and the coefficient of fill be $e_1 = 0.2$, as given from the table, page 647. Further, from page 646 let the velocity be given by

$$30c = 47, \quad \text{or} \quad c = 1.567 \text{ meters.}$$

Then from page 628

$$N_i = \frac{10334}{75} p_i Fc.$$

From the table, page 627, we have for p_i, for $e_1 = 0.20$, and $p_1 = 4$ atmospheres,

$$p_i = 1.881.$$

Hence

$$Fc = \frac{75N_i}{10334p_1} = \frac{75 \times 78}{10334 \times 1.881} = 0.301,$$

and since $c = 1.567$ meters, we have for the cross-section F of the cylinder, not including that of the rod,

$$F = \frac{0.301}{1.567} = 0.192 \text{ sq. meters.}$$

Since the velocity of the piston (c) is 1.567 meters, we must give to the steam passages, according to page 624, a cross-section of about $\frac{1}{15}$th of the cylinder cross-section. We may make them 4 to 5 times as long as broad.

If we make the diameter of the piston rod $\frac{1}{10}$th of that of the piston, we have for d

$$\frac{\pi d^2}{4} - \tfrac{1}{4}\pi\left(\tfrac{1}{10}d\right)^2 = F = 0.192;$$

hence

$$d = 0.498 = \text{diameter of the cylinder,}$$

and

$$0.0498 \text{ meters} = 4.98 \text{ cm.} = \text{diameter of piston rod.}$$

The stroke s of the piston is best given by

$$\frac{s}{d} = 2.8 - d,$$

hence

$$\frac{s}{0.498} = 2.8 - 0.498 = 2.302,$$

or

$$s = 1.146 \text{ meters.}$$

We can now find the weight S_r of the fly wheel rim. Suppose we make the connecting rod 5 times as long as the crank, then $\frac{r}{l} = \frac{1}{5}$, and from page 635,

$$S_r = 2.273 \left(\frac{r}{R}\right)^2 \frac{L_t}{v^2\delta}.$$

If we make R four times r, in which case the mean diameter R of the fly wheel rim will be $4 \times 1.146 = 4.584$ meters, we have, since $L_t = 75 \times 78 = 5850$ meter-kilograms, and $\delta = \frac{1}{32}$.

$$S_r = 2.273 \times \tfrac{1}{16} \times \frac{5850 \times 32}{v^2} \text{ kilograms.}$$

For v we have

$$v = \frac{\pi c}{2} = \frac{3.1416 \times 1.567}{2} = 2.462 \text{ meters.}$$

Hence

$$S_r = 4390 \text{ kilograms.}$$

Since our engine works expansively, and the coefficient of fill is $0.20 = \frac{1}{5}$, the above weight must be multiplied by a coefficient given by the equation, page 634,

$$0{\cdot}77 + 0.23 \times 5 - 0.017 \times 25 = 1.495.$$

Hence the weight of the fly-wheel rim is

$$S_r = 4390 \times 1.495 = 6563 \text{ kilograms.}$$

If we make the arms $\frac{3}{10}$ths of the weight of the rim, they will weigh $\frac{3}{10} \times 6563 = 1968.9$, or in round numbers,

weight of fly wheel = 1970 kilograms.

The dimensions of the rim and arms can be easily found from the formulæ of page 636.

We can now determine the diameter of the fly wheel journal.

If we denote it by d_1, we have, according to Morin,

$$d_1 = 02\sqrt[3]{\frac{N_i}{u}} \text{ centimeters,}$$

where u is the number of revolutions per minute.

For u, we have from page 196,

$$u = \frac{30c}{s} = \frac{30 \times 1.567}{1.146} = 41.02,$$

hence

$$d_1 = 20\sqrt[3]{\frac{78}{41.02}} = 20\sqrt[3]{1.902} = 24.8 \text{ centimeters.}$$

If we make the diameter of the shaft 26.8 centimeters, and make it 3 meters long, since the weight of 1 cubic meter of cast iron is 7400 kilograms, the weight of the shaft is

$$\frac{3.1416 \times (0.268)^2}{4} \times 3 \times 7400 = 1250 \text{ kilograms.}$$

The total weight of fly wheel and shaft is then

$$S = S_w + 1.3\,S_r = 1250 + 6563 + 1970 = 9783 \text{ kil.}$$

The mean effective pressure p_s required for overcoming the friction of the fly wheel shaft is, from page 638,

$$p_s = 0.00002\,\frac{d_1 S}{d^2 s}.$$

In the present case

$$p_s = 0.00002\,\frac{0.248 \times 9783}{(0.498)^2 \times 1.146} = 0.170.$$

For the mean pressure (effective) p_t required to overcome the friction of the piston, piston rod, etc., and to work the feed pump, we have (page 638)

$$p_t = \frac{0.0227}{d} = \frac{0.0227}{0.498} = 0.046.$$

Hence the mean effective pressure p_m of the engine running loose, is, *when non-condensing*,

$$p_m = 0.170 + 0.046 = 0.216.$$

For a condensing engine we must add a term which includes the working of the cold water and air pumps (page 639'. Since, however, the steam weight D used by the engine per hour occurs in this term, we must first find D.

From page 642, we have for D

$$D = 120Fsu\,[(e + e_1)\,\gamma_1 - e\gamma_2] + 450d\sqrt{p_i}\,,$$

where γ_1 is the weight of 1 cubic meter of steam at the pressure of 4 atmospheres, which from Table II. is 2.23 kilograms, $\gamma_2 = 0.264$, $e = 0.07$ (page 618), $e_1 = 0.20$, $s = 1.146$ and $u = 41.02$. Hence

$$D = 120 \times 0.192 \times 1.146 \times 41.02\,[(0.07 + 0.20)\,2.23 - 0.07 \times 0.264] + 450 \times 0.498\sqrt{1.881} = 632.47 + 307.35 = 939.82 \text{ kilgrs.}$$

Hence (page 641),

$$p'_m = (0.003h + 0.045)\frac{939.82}{120 \times 0.192 \times 1.146 \times 41.02}\,.$$

If we take $h = 2$ meters, we have

$$p'_m = (0.006 + 0.045)\,\frac{939.82}{1083} = 0.051 \times 0.868 = 0.044.$$

Hence

$$p_m = 0.216 + 0.044 = 0.260.$$

From page 642 we have the effective pressure which gives the useful work

$$p_u = \frac{p_i - p_m}{1.13} = \frac{1.881 - 0.260}{1.13} = 1.435.$$

Hence N_u (page 642) is

$$N_u = \frac{10334 \times 0.192 \times 1.567 \times 1.435}{75} = 59.47 \text{ horse power.}$$

The result coincides then so exactly with the required power of 60 horse power that another and closer computation is not necessary. This is principally not only because we have taken $\frac{r}{l} = \frac{1}{5}$, but also have made $\frac{r}{R} = \frac{1}{4}$. If, for example, we had taken $\frac{r}{R} = \frac{1}{5}$, the fly wheel would have been much lighter, and hence the work required for its motion less. We should then have found for N_u a greater value than 60 horse power. In such case the area of the piston would have to be reduced somewhat in order to obtain the desired result.

Let us determine now the dimensions of the condenser and pumps.

From page 643, the volume of the condenser is $C = \frac{Fs}{4}$ to $\frac{Fs}{3}$

Let us take then $C = \frac{Fs}{3.5}$, then

$$C = \frac{0.192 \times 1.146}{3.5} = 0.063 \text{ cubic meters.}$$

Let the cold water pump be single acting. The volume V_1 of the same is, from page 644,

$$V_1 = \frac{nD}{54000u}$$

or taking $n = 20$,

$$V_1 = \frac{20 \times 939.82}{54000 \times 41.02} = 0.0085 \text{ cubic meters.}$$

If we make the stroke of the pump one half that of the cylinder, we have for the cross-section F_1,

$$F_1 \frac{s}{2} = 0.0085,$$

$$F_1 \times 0.573 = 0.0085,$$

or

$$F_1 = 0.0148.$$

Hence we have the diameter d_2

$$\frac{\pi d_2^2}{4} = 0.0148,$$

or

$$d_2 = 0.138 \text{ meters} = 13.8 \text{ centimeters.}$$

If we denote the volume described by the piston of the air pump per stroke by V_2, we can, in general, take

$$\frac{V_2}{V_1} = 4 \text{ to } 4.5.$$

Taking the first value,

$$V_2 = 0.0085 \times 4 = 0.0340 \text{ cubic meters.}$$

As soon as we fix upon the stroke, we can find the diameter.

The volume described by the piston of the feed pump per stroke, can also be easily calculated. The feed water per hour is $D = 939.82$ kilograms $= 0.9398$ cubic meters. If $\frac{1}{10}$th of the water falls back, we have per hour the water volume $0.9398 + 0.09398 = 1.0337$, and per minute, 0.0172 cubic meters. In order to feed the boiler rapidly, the pump must furnish in this time, 3 to 6 times this volume. If we say 4 times, we have the quantity per minute 0.0688 cubic meters. If the pump is single acting, it makes, in 41.02 revolutions of the engine, 41.02 strokes, or feeds the boiler 41.02 times per minute. Hence the volume which the piston of the feed pump must describe per stroke is, since for each time it feeds the boiler it rises and falls

$$\frac{0.0688}{41.02 \times \frac{1}{2}} = 0.00334 \text{ cubic meters.}$$

If now (page 646)

$$D = 7B, \quad \text{or} \quad B = \tfrac{1}{7}D,$$

we have for the weight of fuel per hour

$$B = \frac{939.82}{7} = 134.26 \text{ kilograms.}$$

It is thus assumed that the heating value of the fuel is tolerably great, and therefore that good hard coal is used.

We can now recapitulate the dimensions of our engine as calculated, or given.

1.	Diameter of cylinder (d)	0.498	meters.
2.	Length of stroke (s)	1.146	“
3.	Cross-section of steam passages	0.0101	sq. m.
4.	Mean velocity of piston (c)	1.567	meters.
5.	Coefficient of fill (e_1)	0.20	
6.	Revolutions per minute (u)	41.02	
7.	Diameter of the piston rod	4.98	centim.
8.	Ratio of length of crank to connecting rod $\frac{r}{R}$	$\frac{1}{5}$	
9.	Length of crank $(r) = \frac{s}{2}$	0.573	meters.
10.	Weight of fly wheel rim	6563	kilogrs.
11.	Weight of fly wheel arms	1970	“
12.	Weight of fly wheel shaft	1250	“
13.	Diameter of journals	24.8	centim.
14.	Diameter of shaft	26.8	“
15.	Steam consumption per hour	939.82	kilogrs.
16.	Condensing water per hour 20 × 939.82	1879.64	“
17.	Consumption of coal per hour . . 134.26 kil. =	18.7964	cub. m.
18.	Volume of condenser (C)	0.063	“
19.	Volume described by piston of cold water pump per stroke	0.0085	“
20.	Volume described by the piston of the air pump per stroke	0.0340	“
21.	Volume described by the piston of the feed pump per stroke	0.00334	“

EXAMPLES FOR PRACTICE.

1. What is the pressure of saturated steam whose temperature is 20° Fah.? What is the temperature for a pressure of 10 atmospheres? What is the pressure for 140° C.?

2. What is the mean specific heat of water between 10° and 25° C.? Between 25° and 70° Fah.? How much heat is required to raise 10 lbs. of water from 60° to 80° Fah.? How much to raise 2 kilograms from 30° to 70° C.? What is the specific heat of water at 212° Fah.? At 140° C.?

3. How much heat is required to convert 2 lbs. of water at 80° Fah. into saturated steam at 300° Fah.? How much to convert 1 kilogram of water at 10° C. into saturated steam of 120° C.?

4. How much heat is required to vaporize 1 lb. of water at 300° Fah. into steam of the same temperature? What is the pressure? What is the outer work?

5. What is the volume of a quantity of steam and water at 212° Fah. whose weight is 1 lb., and which consists of 0.2 lb. of steam and 0.8 lb. of water?

6. One pound of steam and water has a temperature of 250° Fah., of which 2 cubic feet are steam. How much does the steam weigh? How much does the water weigh?

7. What is the specific volume of steam at 80° C.? At 240° Fah.?

8. What is the outer work performed in converting 2 lbs. of water, at 250° Fah. into steam, under a constant pressure equal to the steam tension? What is the steam tension? What is the steam volume?

9. What is the density of steam at 4 atmospheres' pressure? What is its temperature? If the volume is 8 cubic feet, what volume and weight of water were necessary to form it?

10. How many heat units must be imparted to 1 lb. of saturated steam, in order to keep it all saturated while it expands, performing work, till the temperature sinks 1° Fah.? If the initial temperature is 222° Fah., what is the initial volume? What is the final volume? The initial pressure? The final pressure? The outer work done?

11. How many heat units must be imparted, as before, to 1 lb. of saturated steam when it expands, performing work, from 5 atmospheres down to 1 atmosphere? What are the initial and final volumes? Initial and final temperatures? Work done during expansion?

12. A full-pressure non-condensing engine has a stroke of 3 feet, cross-section of piston, 1.5 sq. feet. The steam pressure is 5 atmospheres, and it makes 25 revolutions per minute. What is the theoretical work per second, and how much heat is required?

13. If a condensing engine forces 3 cubic feet of steam at a pressure of $\frac{1}{10}$th of an atmosphere into the condenser, what work is necessary, and how much heat is taken from the steam ?

14. What work is performed by the adiabatic expansion of 1 lb. of saturated steam from 4 atmospheres to 1 atmosphere ? How much steam is condensed ?

15. What would the work be if, to start with, we had only water and no steam? How much steam would be formed ?

16. If 1 lb. of a mixture of 0.8 lb. steam and 0.2 lb. water, expands adiabatically from 8 atmospheres down to 1 atmosphere, what is the work performed? What is the initial volume ? Final volume ? Heat disappearing ? How much steam is condensed ?

17. If a mixture of 10 lbs. is composed of 0.8 steam and 0.2 water, and has a pressure of 1.5 atmospheres, what will be the amount of steam and water when the mixture is cooled, under constant volume, until the pressure is $\frac{1}{10}$th of an atmosphere ? What amount of heat must be abstracted ?

18. A boiler has 170 sq. feet of heating surface, and contains 300 cubic feet, of which 0.6 are water and the rest steam. In ordinary use, the boiler generates per hour 5 lbs. of steam for every sq. foot of heating surface, of 5 atmospheres' tension. In how many minutes will the pressure rise to 10 atmospheres, the temperature of the feed water being 60° Fah. ?

19. If a vessel containing G lbs. of pure saturated steam, at 1.5 atmospheres, communicates with another containing $25G$ lbs. of a mixture of water and steam, at $\frac{1}{10}$th atmosphere, of which 0.02 of a pound are steam, what is the condition of the mixture after the cock is opened ?

20. Given 10,000 lbs. of feed water at 190° Fah. evaporated at 70 lbs. steam gauge pressure, and the steam containing 2.75 per cent. of moisture. Find the heat units required for evaporation. Also suppose 1200 lbs. of coal were consumed, find the efficiency of the boiler. Efficiency $= \dfrac{\text{Actual Evaporation}}{\text{Theoretical Evaporation}}$; also, find the equivalent evaporation at and from 212° Fah.

21. Suppose a calorimeter used for determining the moisture of steam, which not only condenses but retains the steam and spray admitted.

Further, let

$W = W_1 + C'\,W_2$ = the sum of the original weight of condensing water, and the product of the specific heat of the testing vessel by its weight.

w = weight of mixture of steam and spray.

x = weight of steam in the mixture.

$w - x$ = weight of spray in the mixture.

r = total latent heat of steam.

p_s = steam gauge pressure (= excess above atmosphere).

$D = q_s - q = (t_s^\circ - t^\circ$ nearly$)$ = difference between the heat of liquid at temperature of the steam and at final temperature of water in condensing vessel.

$d = q - q_1 = (t^\circ - t_1^\circ$ nearly$)$ = difference between heat of liquid at final temperature of the water, and at initial temperature of condensing water. Prove that if no external work is done while the steam is condensing, the percentage of moisture is

$$\frac{w - x}{w} = 1 - \frac{W(q - q_1) - w(q_s - q)}{wr}.$$

By experiment, we find $W_1 = 5.796$ lbs., $W_2 = 3.858$ lbs., $C' = 0.11$, $w = 0.25$ lbs., $p_s = 36\frac{3}{4}$ lbs., $t_1 = 65.5°$ Fah., $t = 108.3°$ Fah., what is the percentage of moisture in the steam ?

22. In a surface condenser the water enters with a temperature of 60° Fah., and departs at 80° Fah. The mean temperature of the condenser is 115° Fah. How much more condensing water than steam, by weight, must be used ?

23. An engine using steam of 5 atmospheres has a jet condenser in which the average pressure is 0.1 atmosphere. The cooling water has a temperature of 60° Fah. How much more water than steam must be used ?

24. A boiler contains steam at a pressure of 5 atmospheres. When the safety valve is opened, what is the velocity of efflux, disregarding friction, and supposing the steam to be dry ? How much steam is condensed during efflux ?

25. What diameter should the safety valve of a steam boiler have, which generates per hour 500 lbs. of steam at 5 atmospheres, for 20-fold security ?

26. Hot water is allowed to flow from the test cock of a boiler under the pressure of 5 atmospheres. What is the specific steam weight at the orifice ? With what velocity does the mixture issue ? What is the discharge per second ? How much steam and water are contained in the mixture ?

27. The steam pressure in a boiler is 5 atmospheres, the height of suction 8 feet. The condensing chamber is at the water level. The engine uses 20 lbs. of steam per minute. What should be the area of the mouthpiece of a Giffard injector ? What of the suction pipe and the feed pipe when the feed water has a temperature of 60° Fah., and the mixture of water and steam 120° Fah. ?

28. If one pound of dry saturated steam at 3 atmospheres expands in vacuo down to 1 atmosphere, what is the temperature ? How many degrees must saturated steam of one atmosphere be heated under constant pressure, in order that for the same temperature it may have the same volume ?

29. An engine works with superheated steam of 5 atmospheres and temperature 360° Fah. What is the expansion ratio when the steam at the end of expansion is just in the saturated condition ?

30. A vessel contains one pound of pure saturated steam at 5 atmospheres. Let the steam in this vessel expand into another in which is a vacuum, whose volume is 4 times as large. What is the final pressure and temperature ? And is the steam superheated ?

31. If saturated steam of 5 atmospheres expands under constant temperature down to 1 atmosphere, what is the heat imparted ? The outer work done ?

32. Suppose we have 10 lbs. of saturated steam of 5 atmospheres. What is the heat required to generate it ? How much heat is required to generate the same volume of superheated steam of the same pressure ?

33. In a hot-air engine the heat furnished per hour to the air is 6200 heat units, while in the same time 10 lbs. of coal are consumed, whose heating value is 700 heat units. What is the efficiency of the furnace ?

34. The boiler of an expansion engine, which furnishes steam of 5 atmospheres, vaporizes per hour, for every horse power, 60 lbs. of water, and requires for this 10 lbs. of coal, whose heating power is 700 heat units. What is the boiler efficiency ?

35. What would be the delivery of a perfect steam engine using per hour 200 lbs. of steam of 10 atmospheres ?

TABLE I.

EXPANSIVE FORCE OF STEAM FOR TEMPERATURES FROM — 32° UP TO + 230° C OR — 25.6° UP TO + 446° FAH. ACCORDING TO REGNAULT.

TEMPERATURE CENTIGRADE.	TENSION OF SATURATED STEAM OF WATER.					TEMPERATURE FAHRENHEIT.
	Kilograms per Square Meter.	Millimeters of Mercury.	Atmospheres.	Inches of Mercury.	Pounds per Sq. inch.	
−32°	4.351	0.320	0.0004	0.0126	0.00618	−25.6°
31	4.786	0.352	0.0005	0.0138	0 00680	23·8
30	5.248	0.386	0.0005	0.0152	0.00746	22
29	5.765	0.424	0.0006	0.0167	0.00819	20.2
28	6.309	0.464	0.0006	0.0182	0.00897	18.4
27	6.907	0.508	0.0007	0.0200	0.00982	16.6
26	7.546	0.555	0.0007	0.0218	0.01073	14.8
25	8.226	0.605	0.0008	0.0238	0.01169	13
24	8.974	0.660	0.0009	0.0259	0.01276	11.2
23	9.776	0.719	0.0009	0.0283	0.01390	9.4
22	10.646	0.783	0.0010	0.0308	0.01514	7.6
21	11.598	0.853	0.0011	0.0336	0.01649	5.8
20	12.605	0.927	0.0012	0.0365	0.01792	4
19	13.706	1.008	0.0013	0.0397	0.01949	2.2
18	14.889	1.095	0.0014	0.0431	0.02117	0.4
17	16.167	1.189	0.0015	0.0468	0.02299	+ 1.4
16	17.540	1.290	0.0017	0.0507	0.02494	3.2
15	19.036	1.400	0.0018	0.0551	0.02707	5
14	20.640	1.518	0.0020	0.0597	0.02935	6.8
13	22.380	1.646	0.0022	0.0648	0.03182	8.6
12	24.244	1.783	0.0024	0.0702	0.03447	10.4
11	26.283	1.933	0 0025	0.0761	0.03737	12.2
10	28.459	2.093	0.0027	0.0824	0.04047	14
9	30.825	2.267	0.0030	0.0892	0.04383	15.8
8	33.381	2.455	0.0032	0.0966	0.04746	17.6
7	36.141	2.658	0.0035	0.1046	0.05139	19.4
6	39.105	2.876	0.0038	0.1132	0.05560	21.2
5	42.328	3.113	0.0041	0.1225	0.06019	23
4	45.795	3.368	0.0044	0.1326	0.06512	24.8
3	49.548	3.644	0.0048	0.1434	0.07046	26.6
2	53.587	3.941	0.0052	0.1552	0.07620	28.4
1	57.965	4.263	0.0056	0.1678	0.08243	30.2
0	62.547	4.600	0.0061	0 1811	0.08894	32
+1	67.170	4.940	0.0065	0.1945	0.09552	33.8
2	72.093	5.302	0.0070	0.2087	0.10252	35.6
3	77.323	5.687	0.0075	0.2239	0.10996	37.4
4	82.902	6.097	0.0080	0.2400	0.11789	39.2
5	88.845	6.534	0.0086	0.2572	0.12634	41
6	95.154	6.998	0.0092	0.2755	0.13531	42.8
7	101.871	7.492	0.0099	0.2949	0.14486	44.6
8	109.009	8.017	0.0107	0.3156	0.15502	46.4
9	116.583	8.574	0.011	0.3375	0.16578	48.2
10	124.619	9.165	0.012	0.3608	0.17721	50

TEMPERATURE CENTIGRADE.	TENSION OF SATURATED STEAM OF WATER.					TEMPERATURE FAHRENHEIT.
	Kilograms per Square Meter.	Millimeters of Mercury.	Atmospheres.	Inches of Mercury.	Pounds per Sq. Inch.	
+11°	133.145	9.792	0.013	0.3855	0.18934	+51.8°
12	142.186	10.457	0.014	0.4117	0.20219	53.6
13	151.773	11.162	0.015	0.4394	0.21583	55.4
14	161.916	11.908	0.016	0.4688	0.23025	57.2
15	172.672	12.699	0.017	0.4999	0.24555	59
16	184.053	13.536	0.018	0.5329	0.26178	60.8
17	196.086	14.421	0.019	0.5677	0.27884	62.6
18	208.814	15.357	0.020	0.6046	0.29694	64.4
19	222.261	16.346	0.022	0.6435	0.31606	66.2
20	236.471	17.391	0.023	0.6847	0.33627	68
21	251.482	18.495	0.024	0.7281	0.35762	69.8
22	267.309	19.659	0.026	0.7739	0.38012	71.6
23	284.018	20.888	0.028	0.8224	0.40389	73.4
24	301.642	22.184	0.029	0.8734	0.42895	75.2
25	320.216	23.550	0.031	0.9272	0.45536	77
26	339.769	24.988	0.033	0.9838	0.48316	78.8
27	346.799	25.505	0.034	1.0041	0.49316	80.6
28	382.097	28.101	0.037	1.1063	0.54336	82.4
29	404.955	29.782	0.039	1.1725	0.57586	84.2
30	428.967	31.548	0.042	1.2420	0.61001	86
31	454.231	33.406	0.044	1.3152	0.64594	87.8
32	480.787	35.359	0.047	1.3921	0.68370	89.6
33	508.688	37.411	0.049	1.4728	0.72338	91.4
34	537.977	39.565	0.052	1.5577	0.76503	93.2
35	568.734	41.827	0.055	1.6467	0.80876	95
36	601.014	44.201	0.058	1.7402	0.85467	96.8
37	634.871	46.691	0.061	1.8382	0.90282	98.6
38	670.374	49.302	0.065	1.9410	0.95330	100.4
39	707.590	52.039	0.068	2.0488	1.00622	102.2
40	746.573	54.906	0.072	2.1616	1.06166	104
41	787.419	57.910	0.076	2.2799	1.11975	105.8
42	830.183	61.055	0.080	2.4037	1.18055	107.6
43	874.932	64.346	0.085	2.5333	1.24419	109.4
44	921.761	67.790	0.089	2.6689	1.31079	111.2
45	970.725	71.391	0.094	2.8116	1.38041	113
46	1021.946	75.158	0.099	2.9590	1.45286	114.8
47	1075.451	79.093	0.104	3.1139	1.52984	116.6
48	1132.349	83.204	0.109	3.2757	1.60883	118.4
49	1189.750	87.499	0.115	3.4448	1.69188	120.2
50	1250.707	91.982	0.121	3.6214	1.77856	122
51	1314.328	96.661	0.127	3.8055	1.86903	123.8
52	1380.710	101.543	0.134	3.9978	1.96343	125.6
53	1449.961	106.636	0.140	4.1983	2.06191	127.4
54	1522.149	111.945	0.147	4.4073	2.16456	129.2
55	1597.383	117.478	0.155	4.6251	2.27155	131
56	1675.775	123.244	0.163	4.8522	2.38304	132.8
57	1757.464	129.251	0.170	5.0886	2.49919	134.6
58	1842.502	135.505	0.178	5.3349	2.62012	136.4
59	1931.020	142.015	0.187	5.5912	2 74600	138.2
60	2023.156	148.791	0.196	5.8579	2.87702	140
61	2118.987	155.839	0.205	6.1354	3.01330	141.8
62	2218.671	163.170	0.215	6.4241	3.15505	143.6

TEMPERATURE CENTIGRADE.	TENSION OF SATURATED STEAM OF WATER.					TEMPERATURE FAHRENHEIT.
	Kilograms per Square Meter.	Millimeters of Mercury.	Atmospheres.	Inches of Mercury.	Pounds per Sq. Inch.	
+63°	2322.296	170.791	0.225	6.7241	3.30241	+145.4°
64	2430.027	178.714	0.235	7.0360	3.45561	147.2
65	2541.947	186.945	0.246	7.3601	3.61477	149
66	2658.217	195.496	0.257	7.6974	3.78011	150.8
67	2778.962	204.376	0.267	8.0464	3.9518	152.6
68	2904.328	213.596	0.281	8.4093	4.1301	154.4
69	3034.435	223.165	0.294	8.7861	4.3152	156.2
70	3169.435	233.093	0.306	9.1769	4.5071	158
71	3309.487	243.393	0.320	9.5824	4.7062	159.8
72	3454.707	254.073	0.334	10·0029	4.9127	161.6
73	3605.283	265.147	0.349	10.4389	5.1268	163.4
74	3761.339	276.624	0.364	10.8908	5.3488	165.2
75	3923.052	288.517	0.380	11.3590	5.5787	167
76	4090.584	300.838	0.396	11.8441	5.8170	168.8
77	4264.113	313.600	0.414	12.3465	6.0637	170.6
78	4443.747	326.811	0.430	12.8667	6.3192	172.4
79	4629.717	340.488	0.448	13.4051	6.5836	174.2
80	4822.187	354.643	0.466	13.9624	6.8573	176
81	5021.306	369.287	0.486	14.5389	7.1405	177.8
82	5227.278	384.435	0.506	15.1354	7.4334	179.6
83	5439.993	400.101	0.526	15.7521	7.7363	181.4
84	5660.528	416.298	0.548	16.3898	8.0525	183.2
85	5888.188	433.041	0.570	17.0490	8.3732	185
86	6123.462	450.344	0.593	17.7302	8.7078	186.8
87	6366.541	468.221	0.616	18.4341	9.0535	188.6
88	6617.629	486.687	0.640	19.1611	9.4105	190.4
89	6876.957	505.759	0.665	19.9119	9.7793	192.2
90	7144.701	525.450	0.691	20.6872	10.1600	194
91	7421.107	545.778	0.719	21.4875	10.5531	195.8
92	7706.865	566.757	0.746	22.3134	10.9588	197.6
93	8000.733	588.406	0.774	23.1657	11.3774	199.4
94	8304.415	610.740	0.804	24.0451	11.8092	201.2
95	8617.669	633.778	0.834	24.9521	12.2547	203
96	8940.700	657.535	0.865	25.8874	12.7141	204.8
97	9273.753	682.029	0.897	26.8518	13.1877	206.6
98	9617 098	707.280	0.931	27.8459	13.6759	208.4
99	9970.97	733.305	0.965	28.8705	14.1792	210.2
100	10333.95	760.000	1.000	29.9215	14.6954	212
101	10701.07	787.590	1.036	31.0077	15.2288	213.8
102	11095.53	816.010	1.074	32.1266	15.7784	215.6
103	11493.52	845.28	1.112	33.2790	16.3448	217.4
104	11903.21	875.41	1.152	34.4653	16.9269	219.2
105	12324.73	906.41	1.193	35.6857	17.5263	221
106	12795.19	938.31	1.235	36.9416	18.1432	222.8
107	13204.88	971.14	1.278	38.2342	18.7779	224.6
108	13664.0	1004.91	1.322	39.5637	19.4309	226.4
109	14136.4	1039.65	1.368	40.9314	20.1027	228.2
110	14622.1	1075.37	1.415	42.3377	20.7933	230
111	15132.4	1112.09	1.463	43.7834	21.5034	231.8
112	15634.5	1149.83	1.513	45.2693	22.2331	233.6
113	16161.8	1188.61	1.564	46.7961	22.9829	235.4
114	16703.8	1228.47	1.616	48.3654	23.7537	237.2

TEMPERATURE CENTIGRADE.	TENSION OF SATURATED STEAM OF WATER.					TEMPERATURE FAHRENHEIT.
	Kiliograms per Square Meter.	Millimeters of Mercury.	Atmospheres.	Inches of Mercury.	Pounds per Sq. Inch.	
+115°	17260.5	1269.41	1.670	49.9772	24.5453	+239
116	17833.5	1311.47	1.726	51.6331	25.3586	240.8
117	18419.7	1354.66	1.782	53.3335	26.1987	242.6
118	19022.9	1399.02	1.841	55.0800	27.0514	244.4
119	19641.9	1444.55	1.901	56.8725	27.9318	246.2
120	20277.4	1491.28	1.962	58.7128	28.8354	248
121	20929.6	1539.25	2.025	60.6009	29.7629	249.8
122	21598.9	1588.47	2.091	62.5387	30.7146	251.6
123	22285.4	1638.96	2.157	64.5265	31.6909	253.4
124	22989.8	1690.76	2.225	66.5659	32.6925	255.2
125	23712.0	1743.88	2.295	68.6578	33.7196	257
126	24452.7	1798.35	2.366	70.8017	34.7729	258.8
127	25212.1	1854.20	2.430	73.0006	35.8528	260.6
128	25990.8	1911.47	2.515	75.2554	36.9602	262.4
129	26788.7	1970.15	2.592	77.5656	38.0948	264.2
130	27606.8	2030.28	2.671	79.9829	39.257	266
131	28444.7	2091.94	2.753	82.3605	40.450	267.8
132	29302.6	2155.03	2.836	84.8444	41.669	269.6
133	30181.3	2219.69	2.921	87.3901	42.920	271.4
134	31082.3	2285.92	3.008	89.9976	44.200	273.2
135	32004.4	2353.73	3.097	92.6678	45.512	275
136	32948.4	2423.16	3.188	95.4008	46.854	276.8
137	33914.8	2494.23	3.282	98.1989	48.228	278.6
138	34904.8	2567.00	3.378	101.0639	49.635	280.4
139	35916.4	2641.44	3.476	103.9946	51.075	282.2
140	36952.4	2717.63	3.576	106.9942	52.548	284
141	38012.2	2795.57	3.678	110.0627	54.050	285.8
142	39096.3	2875.30	3.783	113.2018	55.597	287.6
143	40206.3	2956.86	3.890	116.4128	57.174	289.4
144	41339.8	3040.26	4.000	119.6963	58.786	291.2
145	42499.0	3125.55	4.113	123.054	60.442	293°
146	43684.6	3212.74	4.227	126.427	62.117	294.8
147	44896.5	3301.87	4.344	129.996	63.675	296.6
148	46135.4	3392.98	4.464	133.583	65.600	298.4
149	47401.4	3486.09	4.587	137.249	67.408	300.2
150	48695.0	3581.23	4.712	140.994	69.244	302
151	50016.7	3678.43	4.840	144.821	71.126	303.8
152	51367.1	3777.74	4.971	148.731	73.051	305.6
153	52746.1	3879.18	5.104	152.725	75.005	307.4
154	54154.9	3982.77	5.240	156.803	77.004	309.2
155	55598.4	4088.56	5.380	160.968	79.061	311
156	57062.3	4196.59	5.522	165.126	81.148	312.8
157	58561.9	4306.88	5.667	169.565	83.279	314.6
158	60092.6	4419.45	5.815	173.993	85.454	316.4
159	61655.1	4534.36	5.966	178.511	87.673	318.2
160	63249.5	4651.62	6.120	183.119	89.936	320
161	64875.5	4771.28	6.278	187.847	92.257	321.8
162	66536.5	4893.36	6.439	192.656	94.624	323.6
163	68230.0	5017.91	6.603	197.572	97.034	325.4
164	69957.7	5144.97	6.770	202.568	99.483	327.2
165	71719.5	5274.54	6.940	207.655	101.986	329
166	73516.4	5406.69	7.114	212.861	104.543	330.8

TEMPERATURE CENTIGRADE.	TENSION OF SATURATED STEAM OF WATER.					TEMPERATURE FAHRENHEIT.
	Kilograms per Square Meter.	Millimeters of Mercury.	Atmospheres.	Inches of Mercury.	Pounds per Square inch.	
+167°	75338.5	5541.48	7.291	218.157	107.144	+332.6°
168	77216.6	5678.82	7.472	223.573	109.804	334.4
169	79121.3	5818.90	7.656	229.079	112.508	336.2
170	81062.5	5961.66	7.844	234.704	115.271	338
171	83041.3	6107.19	8.036	240.449	118.092	339.8
172	85057.6	6255.48	8.231	246.284	120.958	341.6
173	87112.5	6406.60	8.430	252.238	123.882	343.4
174	89205.8	6560.55	8.632	258.282	126.851	345.2
175	91338.9	6717.43	8.839	264.476	129.825	347
176	93511.6	6877.22	9.049	270.759	132.978	348.8
177	95724.6	7039.97	9.263	277.163	136.123	350.6
178	97978.3	7205.72	9.481	283.686	139.327	352.4
179	100273.6	7374.52	9 703	290.328	142.589	354.2
180	102610.5	7546.39	9.929	297.090	145.910	356
181	104989.8	7721.37	10.150	303.703	149.158	357.8
182	107412.2	7899.52	10.394	311.004	152.744	359.6
183	109877.6	8080.84	10.633	318.155	156.256	361.4
184	112387.1	8265.40	10.876	325.426	159.827	363.2
185	114941.1	8453.23	11.123	332.817	163.457	365
186	117539.8	8644.35	11.374	340.327	167.145	366.8
187	120184.1	8838.82	11.630	347.987	170.907	368.6
188	122874.5	9036.38	11.885	355.617	174.655	370.4
189	125611.2	9237.95	12.155	363.696	178.622	372.2
190	128395.2	9442.70	12.425	371.775	182.590	374
191	131226.6	9650.93	12.699	379.973	186.617	375.8
192	134106.2	9862.71	12.977	388.291	190.702	377.6
193	137034.8	10078.04	13.261	396.789	194.876	379.4
194	140011.7	10297.01	13.549	405.406	199.108	381.2
195	143038.8	10519.63	13.842	414.173	203 414	383
196	146111.7	10745.95	14.139	423.059	207.778	384.8
197	149232.6	10975.00	14.441	432.095	212.216	386.6
198	152415.4	11209.83	14.749	441.312	228.147	388.4
199	155649.9	11447.46	15.062	450.678	221.342	390.2
200	159576.8	11688.96	15.380	460.193	226.015	392
201	162914.7	11984,37	15.703	469.857	230.762	393.8
202	165663.5	12183.69	16.081	479.671	235.582	395.6
203	169104.7	12437.00	16.364	489.635	240.475	397.4
204	172607.9	12694.30	16.703	499.779	245.457	399.2
205	176162.8	12955.66	17.047	510.072	250.512	401
206	179769.4	13221.12	17.396	520.514	255.641	402.8
207	183437.9	13490.75	17.751	531.136	260.858	404.6
208	187158.2	13764.53	18.111	541.908	266.148	406.4
209	190940.4	14042.52	18.477	552.859	271.527	408.2
210	194774.3	14324.80	18.848	563.960	276.978	410
211	198680.5	14611.32	19.226	575.271	282.534	411.8
212	202628.1	14902.22	19.608	586.701	288.147	413.6
213	206648.0	15197.48	19.997	598.340	293.864	415.4
214	210719.6	15497.17	20.391	610.129	299.654	417.2
215	214853.1	15801.33	20.791	622.098	305.532	419
216	219048.7	16109.94	21.197	634.246	311.498	420.8
217	224143.4	16423.15	21.690	648.997	318.743	422.6
218	227625.9	16740.90	22.027	659.080	323.695	424.4

TEMPERATURE CENTIGRADE.	TENSION OF SATURATED STEAM OF WATER.					TEMPERATURE FAHRENHEIT.
	Kilograms per Square Meter.	Millimeters of Mercury.	Atmospheres.	Inches of Mercury.	Pounds per Square inch.	
+219°	232017.8	17063.29	22.452	671.797	329.941	+426.2°
220	236461.5	17390.86	22.882	684.664	336.260	428
221	240977.4	17722.13	23.319	697.789	342.682	429.8
222	245544.9	18058.64	23.761	710.965	349.177	431.6
223	250184.9	18399.94	24.210	724.399	355.776	433.4
224	254907.2	18746.07	24.666	738.044	362.477	435.2
225	259671.5	19097.04	25.128	751.867	369.266	437
226	264507.8	19452.92	25.596	765.871	376.143	438.8
227	269516.4	19813.76	26.071	780.084	383.124	440.6
228	274387.0	20179.61	26.552	794.476	390.192	442.4
229	279430.0	20550.48	27.040	809.077	397.364	444.2
230	284545.8	20926.40	27.535	823.888	404.638	446

TABLE II.

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

French Units.

1	2	3	4	5	6
Steam pressure.			Temperature C. t (page 390) (Regnault.)	Heat of liquid q (page 390) (Regnault.)	Inner latent heat. ρ (page 407.) [$\rho = 575.40 - 0.791t$.]
Atmospheres.	In millimeters of barometer.	Kilograms per sq. meter. p			
0.1	76	1033.4	46.21	46.282	538.848
0.2	152	2066.8	60.45	60.589	527.584
0.3	228	3100.2	69.49	69.687	520.433
0.4	304	4133.6	76.25	76.499	515.086
0.5	380	5167.0	81.71	82.017	510.767
0.6	456	6200.4	86.32	86.662	507.121
0.7	532	7233.8	90.32	90.704	503.957
0.8	608	8267.2	93.88	94.304	501.141
0.9	684	9300.6	97.08	97.543	498.610
1.0	760	10334.0	100.00	100.500	496.300
1.1	836	11367.4	102.68	103.216	494.180
1.2	912	12400.8	105.17	105.740	492.210
1.3	988	13434.2	107.50	108.104	490.367
1.4	1064	14467.6	109.68	110.316	488.643
1.5	1140	15501.0	111.74	112.408	487.014
1.6	1216	16534.4	113.69	114.389	485.471
1.7	1292	17567.8	115.54	116.269	484.008
1.8	1368	18601.2	117.30	118.059	482.616
1.9	1444	19634.6	118.99	119.779	481.279
2.0	1520	20668.0	120.60	121.417	480.005
2.1	1596	21701.4	122.15	122.995	478.779
2.2	1672	22734.8	123.64	124.513	477.601
2.3	1748	23768.2	125.07	125.970	476.470
2.4	1824	24801.6	126.46	127.386	475.370
2.5	1900	25835.0	127.80	128.753	474.310
2.6	1976	26868.4	129.10	130.079	473.282
2.7	2052	27901.8	130.35	131.354	472.293
2.8	2128	28935.2	131.57	132.599	471.328
2.9	2204	29968.6	132.76	133.814	470.387
3.0	2280	31002.0	133.91	134.989	469.477
3.1	2356	32035.4	135.03	136.133	468.591
3.2	2432	33068.8	136.12	137.247	467.729
3.3	2508	34102.2	137.19	138.341	466.883
3.4	2584	35135.6	138.23	139.404	466.060

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

French Units.

7	8	9	10	11	12
	Values of				
Outer latent heat Apu, page 407.	u Cub. metres per kil., page 405.	$\frac{p}{u}$ page 407.	Difference.	γ Kil. per cub. meter., page 410.	Difference.
35.464	14.5508	37.03		0.0687	
36.764	7.5421	69.95	32.92	0.1326	0.0639
37.574	5.1388	101.27	31.32	0.1945	619
38.171	3.9154	131.55	30.28	0.2553	608
38.637	3.1705	161.10	29.55	0.3153	600
39.045	2.6700	189.93	28.83	0.3744	591
39.387	2.3086	218.29	28.36	0.4330	586
39.688	2.0355	246.20	27.91	0.4910	580
39.957	1.8216	273.72	27.52	0.5487	577
40.200	1.6494	300.90	27.18	0.6059	572
40.421	1.5077	327.77	26.87	0.6628	569
40.626	1.3891	354.35	26.58	0.7194	566
40.816	1.2882	380.66	26.31	0.7757	563
40.993	1.2014	406.73	26.07	0.8317	560
41.159	1.1258	432.58	25.85	0.8874	557
41.315	1.0595	458.22	25.64	0.9430	556
41.463	1.0007	483.66	25.44	0.9983	553
41.602	0.9483	508.93	25.27	1.0534	551
41.734	0.9012	534.03	25.10	1.1084	550
41.861	0.8588	558.94	24.91	1.1631	547
41.981	0.8202	583.72	24.78	1.2177	546
42.096	0.7851	608.34	24.62	1.2721	544
42.207	0.7529	632.82	24.48	1.3264	543
42.314	0.7234	657.14	24.32	1.3805	541
42.416	0.6961	681.36	24.22	1.4345	540
42.515	0.6709	705.43	24.07	1.4883	538
42.610	0.6475	729.42	23.99	1.5420	537
42.702	0.6257	753.24	23.82	1.5956	536
42.791	0.6054	776.97	23.73	1.6490	534
42.876	0.5864	800.61	23.64	1.7024	534
42.960	0.5686	824.18	23.52	1.7556	532
43.040	0.5518	847.57	23.44	1.8088	532
43.119	0.5361	870.88	23.31	1.8618	530
43.196	0.5213	894.09	23.21	1.9147	529

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

French Units.

1	2	3	4	5	6
	Steam pressure.		Temperature C. t (page 390) (Regnault.)	Heat of liquid q (page 390) (Regnault.)	Inner latent heat ρ (page 407) [$\rho = 575.40 - 0.791t$.]
Atmospheres.	In millimeters of barometer.	Kilograms per sq. meter. p			
3.5	2660	36169.0	139.24	140.438	465.261
3.6	2736	37202.4	140.23	141.450	464.478
3.7	2812	38235.8	141.21	142.453	463.703
3.8	2888	39269.2	142.15	143.416	462.959
3.9	2964	40302.6	143.08	144.368	462.224
4.0	3040	41336.0	144.00	145.310	461.496
4.1	3116	42369.4	144.89	146.222	460.792
4.2	3192	43402.8	145.76	147.114	460.104
4.3	3268	44436.2	146.61	147.985	459.431
4.4	3344	45469.6	147.46	148.857	458.759
4.5	3420	46503.0	148.29	149.708	458.103
4.6	3496	47536.4	149.10	150.539	457.462
4.7	3572	48569.8	149.90	151.360	456.829
4.8	3648	49603.2	150.69	152.171	456.204
4.9	3724	50636.6	151.46	152.961	455.595
5.0	3800	51670.0	152.22	153.741	454.994
5.1	3876	52703.4	152.97	154.512	454.401
5.2	3952	53736.8	153.70	155.262	453.823
5.3	4028	54770.2	154.43	156.012	453.246
5.4	4104	55803.6	155.14	156.741	452.684
5.5	4180	56837.0	155.85	157.471	452.123
5.6	4256	57870.4	156.54	158.181	451.577
5.7	4332	58903.8	157.22	158.880	451.039
5.8	4408	59937.2	157.90	159.579	450.501
5.9	4484	60970.6	158.56	160.259	449.979
6.0	4560	62004.0	159.22	160.938	449.457
6.1	4636	63037.4	159.37	161.607	448.943
6.2	4712	64070.8	160.50	162.255	448.444
6.3	4788	65104.2	161.14	162.915	447.938
6·4	4864	66137.6	161.76	163.553	447.448
6.5	4940	67171.0	162.37	164.181	446.965
6.6	5016	68204.4	162.98	164.810	446.483
6.7	5092	69237.8	163.58	165.428	446.008
6.8	5168	70271.2	164.18	166.047	445.534
6.9	5244	71304.6	164.76	166.645	445.075

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

French Units.

7	8	9	10	11	12
Outer latent heat Apu, page 407.	Values of u Cub. meters per kil., page 405.	$\frac{\rho}{u}$ page 407.	Difference.	γ Kil. per cub. meter, page 410.	Difference.
43.269	0.5072	917.2		1.9676	
43.342	0.4940	940.3	23.1	2.0203	0.0527
43.413	0.4814	963.2	22.9	2.0729	526
43.480	0.4695	986.1	22.9	2.1255	526
43.548	0.4581	1008.9	22.8	2.1780	525
43.614	0.4474	1031.6	22.7	2.2303	523
43.677	0.4371	1054.2	22.6	2.2826	523
43.739	0.4273	1076.8	22.6	2.3349	523
43.799	0.4179	1099.3	22.5	2.3871	522
43.859	0.4090	1121.7	22.4	2.4391	520
43.918	0.4004	1144.0	22.3	2.4911	520
43.975	0.3922	1166.3	22.3	2.5430	519
44.030	0.3844	1188.5	22.2	2.5949	519
44.085	0.3768	1210.6	22.1	2.6467	518
44.139	0.3696	1232.7	22.1	2.6984	517
44.192	0.3626	1254.7	22.0	2.7500	516
44.243	0.3559	1276.6	21.9	2.8016	516
44.293	0.3495	1298.5	21.9	2.8531	515
44.343	0.3433	1320.3	21.8	2.9046	515
44.392	0.3373	1342.1	21.8	2.9560	514
44.441	0.3315	1363.8	21.7	3.0073	513
44.487	0.3259	1385.4	21.6	3.0586	513
44.533	0.3205	1407.0	21.6	3.1098	512
44.579	0.3153	1428.5	21.5	3.1610	512
44.623	0.3103	1450.0	21.5	3.2122	512
44.667	0.3054	1471.5	21.5	3.2632	510
44.710	0.3007	1492.9	21.4	3.3142	510
44.753	0.2962	1514.2	21.3	3.3652	510
44.794	0.2917	1535.5	21.3	3.4161	509
44.836	0.2874	1556.7	21.2	3.4670	509
44.876	0.2833	1577.9	21.2	3.5178	508
44.916	0.2792	1599.0	21.1	3.5685	507
44.956	0.2753	1620.1	21.1	3.6192	507
44.994	0.2715	1641.2	21.1	3.6699	507
45.032	0.2678	1662.2	21.0	3.7206	507

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

French Units.

1	2	3	4	5	6
	Steam pressure.		Temperature C.	Heat of liquid	Inner latent heat
Atmospheres.	In millimeters of barometer.	Kilograms per sq. meter. p	t (page 390) (Regnault.)	q (page 390) (Regnault.)	ρ (page 407) [$\rho = 575.40 - 0.791t$.]
7.00	5320	72338.0	165.34	167.243	444.616
7.25	5510	74921.5	166.77	168.718	443.485
7.50	5700	77505.0	168.15	170.142	442.393
7.75	5890	80088.5	169.50	171.535	441.325
8.00	6080	82672.0	170.81	172.888	440.289
8.25	6270	85255.5	172.10	174.221	439.269
8.50	6460	87839.0	173.35	175.514	438.280
8.75	6650	90422.5	174.57	176.775	437.315
9.00	6840	93006.0	175.77	178.017	436.366
9.25	7030	95589.5	176.94	179.228	435.440
9.50	7220	98173.0	178.08	180.408	434.539
9.75	7410	100756.5	179.21	181.579	433.645
10.00	7600	103340.0	180.31	182.719	432.775
10.25	7790	105923.5	181.38	183.828	431.928
10.50	7980	108507.0	182.44	184.927	431.090
10.75	8170	111090.5	183.48	186.005	430.267
11.00	8360	113674.0	184.50	187.065	429.460
11.25	8550	116257.5	185.51	188.113	428.661
11.50	8740	118841.0	186.49	189.131	427.886
11.75	8930	121424.5	187.46	190.139	427.119
12.00	9120	124008.0	188.41	191.126	426.368
12.25	9310	126591.5	189.35	192.104	425.624
12.50	9500	129175.0	190.27	193.060	424.896
12.75	9690	131758.5	191.18	194.007	424.177
13.00	9880	134342.0	192.08	194.944	423.465
13.25	10070	136925.5	192.96	195.860	422.769
13.50	10260	139509.0	193.83	196.766	422.080
13.75	10450	142092.5	194.69	197.662	421.400
14.00	10640	144676.0	195.53	198.537	420.736

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

French Units.

7	8	9	10	11	12
Outer latent heat Apu, page 407.	Values of u Cub. meters per kil., page 405.	 $\frac{\rho}{u}$ page 407.	Difference.	γ Kil. per cub. meter, page 410.	Difference.
45.070	0.2642	1683.0		3.7711	
45.162	0.2556	1735.2	52.2	3.8974	0.1263
45.250	0.2475	1787.1	51.9	4.0234	1260
45.337	0.2400	1838.7	51.6	4.1490	1256
45.420	0.2329	1890.1	51.4	4.2745	1255
45.501	0.2263	1941.2	51.1	4.3997	1252
45.578	0.2200	1992.1	50.9	4.5248	1251
45.654	0.2141	2042.8	50.7	4.6495	1247
45.727	0.2085	2093.3	50.5	4.7741	1246
45.798	0.2031	2143.5	50.2	4.8985	1244
45.868	0.1981	2193.5	50.0	5.0226	1241
45.935	0.1933	2243.3	49.8	5.1466	1240
46.001	0.1887	2293.0	49.7	5.2704	1238
46.064	0.1844	2342.5	49.5	5.3941	1237
46.127	0.1802	2391.7	49.2	5.5174	1233
46.189	0.1763	2440.7	49.0	5.6405	1231
46.247	0.1725	2489.5	48.8	5.7636	1231
46.306	0.1689	2538.2	48.7	5.8864	1228
46.362	0.1654	2586.8	48.6	6.0092	1228
46.417	0.1621	2635.2	48.4	6.1318	1226
46.471	0.1589	2683.4	48.2	6.2543	1225
46.524	0.1558	2731.4	48.0	6.3765	1222
46.576	0.1529	2779.3	47.9	6.4986	1221
46.626	0.1500	2827.0	47.7	6.6206	1220
46.676	0.1473	2874.5	47.5	6.7424	1218
46.724	0.1447	2922.0	47.5	6.8642	1218
46.772	0.1421	2969.3	47.3	6.9857	1215
46.818	0.1397	3016.5	47.2	7.1072	1215
46.864	0.1373	3063.4	46.9	7.2283	1211

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

English Units.

1	2	3	4	5	6
Atmospheres.	Steam pressure. Inches of mercury.	Pounds per square inch.	Temperature Fahrenheit (page 390) (Regnault).	Heat of liquid q (page 390) (Regnault).	Inner latent heat ρ (page 407) $[\rho=1035.72-0791\,(t-32)]$
0.1	2.9921	1.4696	115.4876	83.3076	969.9264
0.2	5.9843	2.9392	140.8100	109.0602	949.6512
0.3	8.9764	4.4088	156.282	125.4366	936.7794
0.4	11.9686	5.8784	169.25	137.6982	927.1548
0.5	14.9607	7.3479	179.078	147.6306	919.3806
0.6	17.9529	8.8175	187.376	155.9916	912.8178
0.7	20.9450	10.2872	194.756	163.2672	907.1226
0.8	23.9372	11.7567	200.984	169.7472	902.0538
0.9	26.9293	13.2264	206.744	175.5774	897.498
1.0	29.9215	14.6959	212.	180.9	893.34
1.1	32.9136	16.1656	216.824	185.7888	889.524
1.2	35.9058	17.6352	221.306	190.332	885.978
1.3	38.8979	19.1047	225.5	194.5872	882.6606
1.4	41.8900	20.5743	229.424	198.5688	879.5574
1.5	44.8822	22.0439	233.132	202.3344	876.6252
1.6	47.8744	23.5135	236.642	205.9002	873.8478
1.7	50 8665	24.9831	239.972	209.2842	871.2144
1.8	53.8587	26.4527	243.14	212.5062	868.7088
1.9	56.8508	27.9223	246.182	215.6022	866.3022
2.0	59.8430	29.3919	249.08	218.5516	864.009
2.1	62.8352	30.8615	251.87	221.391	861.8022
2.2	65.8273	32.3311	254.552	224.1234	859.6818
2.3	68.8195	33.8007	257.126	226.746	857.646
2.4	71.8116	35.2703	259.628	229.2948	855.666
2.5	74.8038	36.7399	262.04	231.7554	853.758
2.6	77.7959	38.2095	264.38	234.1422	851.9076
2.7	80.7881	39.6791	266.63	236.4372	850.1374
2.8	83.7802	41.1487	268.826	238.6782	848.3904
2.9	86.7724	42.6183	270.968	240.8652	846.6966
3.0	89.7645	44.0879	273.038	242.9802	845.0586
3.1	92.7567	45.5575	275.054	245.0394	843.4638
3.2	95.7488	47.0271	277.016	247.0446	841.9122
3.3	98.7410	48.4967	278.942	249.0138	840.3894
3.4	101.7332	49.9663	280.814	250.9272	838.908

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

English Units.

7	8	9	10	11	12
	Values of				
Outer latent heat Apu (page 407).	u Cubic feet per pound (page 407).	$\frac{p}{u}$ (page 405).	Differences.	r Pounds per cubic foot (page 405).	Differences.
63.8352	233.0879	4.161		0.004290	
66 1752	120.8162	7.860	3.699	0.008277	0.003987
67.6332	82.318	11.379	3.519	0.012142	0.003865
68.7078	62.7204	14.782	3.403	0.015936	0.003794
69.5466	50.788	18.102	3.320	0.019683	0.003747
70.281	42 7765	21.342	3.240	0.023372	0.003689
70.8956	36.9812	24.528	3.186	0.027030	0.003658
71.4384	32.6064	27.664	3.136	0.030651	0.003621
71.9226	29.180	30.757	3.093	0.034253	0.003602
72.36	26.4216	33.812	3.055	0.037824	0.003571
72.7578	24.1517	36.83	3.018	0.041376	0.003552
73.1268	22.252	39.819	2.989	0.044909	0.003533
73.4688	20.6855	42.774	2.955	0.048423	0.003514
73.7874	19.2451	45.703	2.929	0.051919	0.003486
74.0862	18.034	48.608	2.905	0.055397	0.003478
74.367	16.972	51.489	2.871	0.058868	0.003471
74.6334	16.0301	54.347	2.858	0.062320	0.003452
74.8836	15.1907	57.187	2.840	0.065759	0.003439
75.1212	14.4362	60.007	2.820	0.069193	0.003434
75.3498	13.757	62.806	2.799	0.072608	0.003415
75.5658	13.1387	65.591	2.785	0.076016	0.003408
75.7728	12.5764	68.357	2.766	0.079412	0.003396
75.9726	12.0606	71.108	2.751	0.082802	0.003390
76.1652	11.5888	73.841	2.733	0.086179	0.003377
76.3488	11.1507	76.562	2.721	0.089550	0.003371
76.527	10.747	79.267	2.705	0.092909	0.003359
76.698	10.3722	81.963	2.696	0.096261	0.003352
76.8636	10.028	84.639	2.676	0.099607	0.003346
77.0238	9.6978	87.306	2.667	0.10294	0.003333
77.1768	9.3985	89.962	2.656	0.10627	0.00333
77.328	9.1083	92.605	2.643	0.10959	0.00333
77.472	8.8394	95.239	2.634	0.11292	0.00333
77.6052	8.5877	97.858	2.619	0.11622	0.00330
77.7528	8.3507	100.466	2.608	0.11953	0.00330

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

English Units.

1 Atmospheres.	2 Steam Pressure. Inches of mercury.	3 Pounds per square inch.	4 Temperature Fahrenheit. (page 390) (Regnault.)	5 Heat of liquid q (page 390) (Regnault.)	6 Inner latent heat ρ (page 407) $[\rho = 1035.72 - 0.791\,(t - 32)]$.
3.5	104.7253	51.4359	282.632	252.7884	837.4698
3.6	107.7174	52.9055	284.414	254.610	836.0604
3.7	110.7096	54.3751	286.178	256.4154	834.6654
3.8	113.7017	55.8447	287.87	258.1488	833.3262
3.9	116.6939	57.3143	289.544	259.8624	832.0052
4.0	119.6860	58.7839	291.2	261.558	830.6928
4.1	122.6782	60.2535	292.802	263.1996	829.4256
4.2	125.6704	61.7231	294.368	264.8052	828.1872
4.3	128.6625	63.1927	295.898	266.378	826.9758
4.4	131.6547	64.6623	297.324	267.9426	825.7662
4.5	134.6468	66.1319	298.922	269.4744	824.5854
4.6	136.639	67.6015	300.38	270.9	823.4316
4.7	140.6311	69.0711	301.82	272.448	822.2922
4.8	143.6233	70.5407	303.242	273.9078	821.1672
4.9	146.6154	72.0103	304.628	275.3298	820.071
5.0	149.6076	73.4799	305.996	276.7338	818.9892
5.1	152.5997	74.9495	307.346	278.1216	817.9218
5.2	155.5919	76.4191	308.66	279.4716	816.8814
5.3	158.5840	77.8887	309.974	280.8216	815.8428
5.4	161.5762	79.3582	311.252	282.1338	814.8312
5.5	164.5683	80.8279	312.53	283.4478	813.8214
5.6	167.5605	82.2975	313.772	284.7258	812.8386
5.7	170.5526	83.7671	314.996	285.984	811.8702
5.8	173.5448	85.2367	316.22	287.2422	810.9018
5.9	176.5369	86.7063	317.408	288.4662	809.9622
6.0	179.5291	88.1758	318.596	289.6884	809.0226
6.1	182.5212	89.6455	319.766	290.8926	808.0974
6.2	185.5134	91.1151	320.9	292.059	807.1992
6.3	188.5056	92.5847	322.052	293.247	806.2884
6.4	191.4977	94.0543	323.168	294.3954	805.4064
6.5	194.4899	95.5238	324.266	295.5258	804.537
6.6	197.4820	96.9935	325.364	296.658	803.6694
6.7	200.4724	98.4631	326.444	297.7704	802.8144
6.8	203.4663	99.9326	327.524	298.8846	801.9612
6.9	206.4585	101.4022	328.568	299.961	801.135

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

English Units.

7	8	9	10	11	12
	Values of				
Outer latent heat Apu (page 407)	u Cubic feet per pound. (page 407)	$\frac{p}{u}$ (page 405)	Differences.	γ Pounds per cubic foot. (page 410)	Differences.
77.8842	8.1248	103.062	2.596	0.12283	0.00330
78.0156	7.9133	105.658	2.596	0.12612	0.00329
78.1434	7.7115	108.232	2.574	0.12940	0.00328
78.264	7 5208	110.806	2.574	0.13268	0.00328
78.3864	7.3383	113.367	2.561	0.13596	0.00328
78.5052	7.1663	115.92	2.553	0.13923	0.00327
78.6186	7.0019	118.457	2.537	0.14249	0.00326
78.7302	6.8449	120.994	2.537	0.14576	0.00326
78.8382	6.6943	123.525	2.528	0.14902	0.00326
78.9462	6.5517	126.042	2.517	0.15226	0.00325
79.0524	6.4140	128.548	2.506	0.15551	0.00325
79.155	5.2827	131.054	2.506	0.15875	0.00324
79.254	6.1577	133.548	2.494	0.16199	0.00324
79.353	6.0359	136.031	2.483	0.16522	0.00323
79.4502	5.9206	138.515	2.484	0.16845	0.00323
79.5456	5.8084	140.987	2.472	0.17167	0.00322
79.6374	5.7011	143.448	2.462	0.17489	0.00322
79.7274	5.5986	145.91	2.462	0.17811	0.00322
79.8174	5.4993	148.36	2.450	0.18133	0.00322
79.9056	5.4031	150.808	2.448	0.18453	0.00320
79.9938	5.3102	153.246	2.438	0.18773	0 00320
80.0766	5.2205	155.673	2.427	0.19093	0.00320
80.1594	5.1341	158.10	2.427	0.19413	0.00320
80.2422	5.0507	160.516	2.416	0.19733	0.00320
80.3214	4.9707	162.932	2.416	0.20052	0.00319
80.4006	4.8922	165.348	2.416	0.20371	0.00319
80.478	4.8169	167.753	2.405	0.20689	0.00318
80.5554	4.7448	170.146	2.393	0.21007	0.00318
80.6292	4.6711	172.54	2.394	0.21325	0.00318
80.7048	4.6038	174.921	2.381	0.21643	0.00318
80.7768	4.5381	177.304	2.383	0.21960	0.00317
80.8488	4.4724	179.675	2.371	0.22277	0.00317
80 9208	4.410	182.046	2.371	0.22593	0.00316
80.9892	4.3492	184.42	2.374	0.22909	0.00316
81.0576	4.2899	186.776	2.356	0.23226	0.00316

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

English Units.

1 Atmospheres.	2 Steam pressure. Inches of mercury.	3 Pounds per square inch.	4 Temperature Fahrenheit. (page 390) (Regnault.)	5 Heat of liquid q (page 390) (Regnault.)	6 Inner latent heat ρ (page 407) $[\rho = 1085.72 - 0.791\ (t - 32)]$.
7.0	209.4506	102.8718	329.612	301.0374	800.3088
7.25	216.981	106.5458	332.186	303.6924	798.273
7.50	224.4114	110.2198	334.67	306.2556	796.3074
7.75	231.8918	113.8938	337.1	308.763	794.385
8.0	239.3722	117.5678	339.458	311.1984	792.5203
8.25	246.8526	121.2418	341.78	313.5978	790.6843
8.50	254.333	124.9158	344.03	315.9252	788.904
8.75	261.8134	128.5898	346.226	318.195	787.167
9.0	269.2938	132.2638	348.386	320.4309	785.4588
9.25	276.7743	135.9378	350.492	322.6104	783.792
9.50	284.2569	139.6118	352.544	324.7844	782.1703
9.75	291.7351	143.2858	354.578	326.8422	780.561
10.0	299.2154	146.9598	356.558	328.8942	778.995
10.25	306.6958	150.6338	358.484	330.8904	777.4704
10.50	314.1763	154.3078	360.392	332.8686	775.963
10.75	321.6567	157.9818	362.264	334.809	774.4806
11.0	329.1371	161.6557	364.1	336.717	773.028
11.25	336.6175	165.3298	365.918	338.6034	771.5898
11.50	344.0979	169.0037	367.682	340.4358	770.1948
11.75	351.5783	172.6778	369.428	342.2502	768.8143
12.0	359.0587	176 3517	371.138	344.0268	767.4624
12.25	366.5391	180.0257	372.83	345.7872	766.1283
12.50	374.0195	183.6997	374.486	347.508	764.8128
12.75	381.5	187.3737	376.124	349.2126	763.5186
13 0	388.98	191.0477	377.744	350.8992	762.237
13.25	396.4607	194.7217	379.328	352.548	760.9843
13.50	403.9411	198.3957	380.894	354.1788	759.744
13.75	411.4215	202.0697	382.442	355.7916	758.53
14.0	418.9019	205.7437	383.954	357.3666	757.3248

TABLE II.—*continued.*

PRINCIPAL TABLE FOR SATURATED STEAM OF WATER.

English Units.

7	8	9	10	11	12
	Values of				
Outer latent heat. Apu (page 407)	u Cubic feet per pound. (page 405)	$\frac{p}{u}$ (page 407)	Differences.	γ Pounds per cubic foot. (page 410)	Differences.
81.126	4.2322	189.114	2.338	0.23541	0.00815
81.2916	4.0944	194.979	5.865	0.24329	0.00788
81.45	3.9646	200.811	5.832	0.25116	0.00787
81.6066	3.8445	206.61	5.799	0.25900	0.00784
81.756	3.7308	212.374	5.764	0.26684	0.00784
81.9018	3.6250	218.127	5.753	0.27465	0.00781
82.0404	3.5242	223.85	5.723	0.28246	0.00781
82.1772	3.4296	229.543	5.693	0.29025	0.00780
82.3086	3.3399	235.22	5.677	0.29803	0.00778
82.4364	3.2534	240.86	5.640	0.30579	0.00776
82.5624	3.1733	246.477	5.617	0.31354	0.00775
82.6830	3.0964	252.073	5.596	0.32128	0.00774
82.8018	3.0228	257.66	5.587	0.32901	0.00773
82.9152	2.9539	263.22	5.560	0.33673	0.00772
83.0286	2.8862	268.75	5.530	0.34443	0.00770
83.1402	2.8241	274.254	5.504	0.35211	0.00768
83.2446	2.7633	279.74	5.486	0.35979	0.00768
83.3508	2.7056	285.21	5.47	0.36746	0.00767
83.4516	2.6495	290.67	5.46	0.37513	0.00767
83.5506	2.5966	296.11	5.44	0.38278	0.00765
83.6478	2.5454	301.526	5.41	0.39043	0.00765
83.7432	2.4958	306.92	5.40	0.39806	0.00763
83.8368	2.4493	312.30	5.38	0.40568	0.00762
83.9268	2.4028	317.66	5.36	0.41329	0.00761
84.0168	2.3596	323.00	5.34	0.42090	0.00761
84.1032	2.3179	328.337	5.34	0.42850	0.00760
84.1896	2.2763	333.652	5.31	0.43609	0.00759
84.2724	2.2378	338.955	5.30	0.44367	0.00758
84.3552	2.1994	344.225	5.27	0.45123	0.00756

TABLE IIa.

SATURATED STEAM OF WATER.

French Units.

1 Temperature C. t	2 Absolute temperature T	3 Pressure in millimeters of barometer p	4 Total heat W (Regnault)	Differences.	5 Heat of liquid q (Regnault)	Differences.	6 Total latent heat $r = W - q$	Differences.
0	273	4.600	606.500		0.000		606.500	
5	278	6.534	608.025	1.525	5.000	5.000	603.025	3.475
10	283	9.165	609.550	"	10.002	5.002	599.548	3.477
15	288	12.699	611.075	"	15.005	5.003	596.070	3.478
20	293	17.391	612.600	"	20.010	5.005	592.590	3.480
25	298	23.550	614.125	"	25.017	5.007	589.108	3.482
30	303	31.548	615.650	"	30.026	5.009	585.624	3.484
35	308	41.827	617.175	"	35.037	5.011	582.138	3.486
40	313	54.906	618.700	"	40.051	5.014	578.649	3.489
45	318	71.390	620.225	"	45.068	5.017	575.157	3.492
50	323	91.980	621.750	"	50.087	5.019	571.663	3.494
55	328	117.475	623.275	"	55.110	5.023	568.165	3.498
60	333	148.786	624.800	"	60.137	5.027	564.663	3.502
65	338	186.938	626.325	"	65.167	5.030	561.158	3.505
70	343	233.082	627.850	"	70.201	5.034	557.649	3.509
75	348	288.500	629.375	"	75.239	5.038	554.136	3.513
80	353	354.616	630.900	"	80.282	5.043	550.618	3.518
85	358	433.002	632.425	"	85.329	5.047	547.096	3.522
90	363	525.392	633.950	"	90.381	5.052	543.569	3.527
95	368	633.692	635.475	"	95.438	5.057	540.037	3.532
100	373	760.000	637.000	"	100.500	5.062	536.500	3.537
105	378	906.410	638.525	"	105.568	5.068	532.957	3.543
110	383	1075.370	640.050	"	110.641	5.073	529.409	3.548
115	388	1269.410	641.575	"	115.721	5.080	525.854	3.555
120	393	1491.280	643.100	"	120.806	5.085	522.294	3.560
125	398	1743.880	644.625	"	125.898	5.092	518.727	3.567
130	403	2030.280	646.150	"	130.997	5.099	515.153	3.574
135	408	2353.730	647.675	"	136.103	5.106	511.572	3.581
140	413	2717.630	649.200	"	141.215	5.112	507.985	3.587
145	418	3125.550	650.725	"	146.334	5.119	504.391	3.594
150	423	3581.230	652.250	"	151.462	5.128	500.788	3.603
155	428	4068.560	653.775	"	156.598	5.136	497.177	3.611
160	433	4651.620	655 300	"	161.741	5.143	493.559	3.618
165	438	5274.540	656.825	"	166.892	5.151	489.933	3.626
170	443	5961.660	658.350	"	172.052	5.160	486.298	3.635
175	448	6717.430	659.875	"	177.220	5.168	482.655	3.643
180	453	7546.390	661.400	"	182.398	5.178	479.002	3.653
185	458	8453.230	662.925	"	187.584	5.186	475.341	3.661
190	463	9442.700	664.450	"	192.780	5.196	471.670	3.671
195	468	10519.630	665.975	"	197.985	5.205	467.990	3.680
200	473	11688.960	667.500	"	203.200	5.215	464.300	3.690

TABLE IIa.

SATURATED STEAM OF WATER.

French Units.

7		8		9		10	11	1
Outer latent heat $A p u$	Difference.	Steam heat $J = W - A p u$	Difference.	Inner latent heat $\rho = r - A p u$	Difference.	Value of		Temperature. C. t
						u	$\frac{\rho}{u}$	
31.071		575.43		575.43		210.66	2.732	0
31.475	0.404	576.55	1.12	571.55	3.88	150.23	3.805	5
31.892	0.417	577.66	1.11	567.66	3.87	108.51	5.231	10
32.318	0.426	578.76	1.10	563.75	3.91	79.346	7.104	15
32.755	0.437	579.84.	1.08	559.83	3.92	58.720	9.532	20
33.201	0.446	580.92	1.08	555.91	3.92	43.963	12.645	25
33.656	0.455	581.99	1.07	551.97	3.94	33.266	16.592	30
34.119	0.463	583.06	1.07	548.02	3.95	25.436	21.545	35
34.588	0.469	584.11	1.05	544.06	3.96	19.644	27.696	40
35.064	0.476	585.16	1.05	540.09	3.97	15.315	35.264	45
35.544	0.480	586.21	1.05	536.12	3.97	12.049	44.492	50
36.027	0.483	587.25	1.04	532.14	3.98	9.5613	55.646	55
36.512	0.485	588.29	1.04	528.15	3.99	7.6531	69.020	60
36.996	0.484	589.33	1.04	524.16	3.99	6.1711	84.938	65
37.478	0.482	590.37	1.04	520.17	3.99	5.0139	103.746	70
37.955	0.477	591.42	1.05	516.18	3.99	4.1024	125.825	75
38.425	0.470	592.47	1.05	512.19	3.99	3.3789	151.587	80
38.885	0.460	593.54	1.07	508.21	3.98	2.8003	181.482	85
39.332	0.447	594.62	1.08	504.24	3.97	2.3344	216.003	90
39.762	0.430	595.71	1.09	500.27	3.97	1.9566	255.687	95
40.200	0.438	596.79	1.08	496.29	3.98	1.6496	300.856	100
40.631	0.431	597.89	1.10	492.33	3.96	1.3978	352.218	105
41.048	0.417	599.00	1.11	488.36	3.97	1.1903	410.292	110
41.457	0.400	600.12	1.12	484.40	3.96	1.0184	475.653	115
41.858	0.401	601.24	1.12	480.44	3.96	0.8752	548.902	120
42.250	0.392	602.37	1.13	476.48	3.96	0.7555	630.685	125
42.634	0.384	603.52	1.15	472.52	3.96	0.6548	721.607	130
43.010	0.376	604.66	1.14	468.56	3.96	0.5698	822.321	135
43.377	0.367	605.82	1.16	464.61	3.95	0.4977	933.475	140
43.735	0.358	606.99	1.17	460.66	3.95	0.4363	1055.726	145
44.086	0.351	608.16	1.17	456.70	3.96	0.3839	1189.736	150
44.428	0.342	609.35	1.19	452.75	3.95	0.3388	1336.166	155
44.761	0.333	610.54	1.21	448.80	3.95	0.3001	1495.686	160
45.086	0.325	611.74	1.20	444.85	3.95	0.2665	1668.926	165
45.403	0.317	612.95	1.21	440.89	3.96	0.2375	1856.538	170
45.711	0.308	614.16	1.21	436.94	3.95	0.2122	2059.147	175
46.012	0.299	615.39	1.23	432.99	3.95	0.1901	2277.366	180
46.304	0.292	616.62	1.23	429.04	3.95	0.1708	2511.787	185
46.589	0.285	617.86	1.24	425.08	3.96	0.1538	2762.974	190
46.864	0.275	619.11	1.25	421.13	3.95	0.1389	3031.464	195
47.133	0.269	620.37	1.26	417.17	3.96	0.1257	3317.795	200

TABLE II*a*.—*continued.*

SATURATED STEAM OF WATER.

English Units.

1	2	3	4		5		6	
Temperature Fahrenheit t	Absolute temperature T	Pressure in inches of mercury p	Total heat W (Regnault).	Differences.	Heat of liquid q (Regnault).	Differences.	Total latent heat $r = W - q$	Differences.
32	491.4	0.1811	1091.7	2.745	0.0000		1091.7	
41	500.4	0.2572	1094.445	"	9.0000	9.	1085.445	6.255
50	509.4	0.3608	1097.19	"	18.0036	9.0036	1079.1864	6.2586
59	518.4	0.5000	1099.935	"	27.0090	9.0054	1072.926	6.2604
68	527.4	0.6847	1102.68	"	36.0180	9.009	1066.662	6.264
77	536.4	0.9272	1105.425	"	45.0306	9.0126	1060.3944	6.2676
86	545.4	1.2420	1108.170	"	54.0468	9.0162	1054.1232	6.2712
95	554.4	1.6467	1110.915	"	63.0666	9.0198	1047.8484	6.2748
104	563.4	2.1617	1113.66	"	72.0918	9.0252	1041.5682	6.2802
113	572.4	2.8106	1116.405	"	81.1224	9.0306	1035.2826	6.2856
122	581.4	3.6213	1119.15	"	90.1566	9.0342	1028.9934	6.2892
131	590.4	4.6250	1121.895	"	99.1988	9.0414	1022.697	6.2964
140	599.4	5.8577	1124.64	"	108.2466	9.0478	1016.3934	6.3036
149	608.4	7.3598	1127.385	"	117.3006	9.0540	1010.0844	6.3090
158	617.4	9.1765	1130.13	"	126.3618	9.0612	1003.7682	6.3162
167	626.4	11.3588	1132.875	"	135.4302	9.0684	997.4448	6.3234
176	635.4	13.9614	1135.62	"	144.5076	9.0774	991.1124	6.3324
185	644.4	17 0475	1138.365	"	153.5922	9.0926	984.7728	6.3396
194	653.4	20.6849	1141.11	"	162.6858	9.0936	978.4242	6.3486
203	662.4	24.9487	1143.855	"	171.7884	9.1026	972.0666	6.3576
212	671.4	29.9215	1146.6	"	180.9000	9.1116	965.7000	6.3666
221	680.4	35.6857	1149.345	"	190 0224	9.1224	959.3226	6.3774
230	689.4	42.3878	1152.09	"	199.1538	9.1314	952.9362	6.3864
239	698.4	49.9372	1154.835	"	208.2978	9.1440	946.537	6.3992
248	707.4	58.7128	1157.58	"	217.4508	9.1530	940.1292	6.4078
257	716.4	68.6578	1160.325	"	226.6164	9.1656	933.7086	6.4206
266	725.4	79.9329	1163.07	"	235.7946	9.1782	927.2754	6.4332
275	734.4	92.6663	1165.815	"	244.9854	9 1908	920.8296	6.4458
284	743.4	106.9942	1168.56	"	254.1870	9.2016	914.373	6.4566
293	752.4	123.0542	1171.305	"	263.4012	9.2142	907.9038	6.4692
302	761.4	140.9945	1174.05	"	272.6316	9.2304	901.4184	6.4854
311	770.4	160.9683	1176.795	"	281.8764	9.2448	894.9186	6.4998
320	779.4	183.1362	1179.54	"	291.1338	9.2574	888.4062	6.5124
329	788.4	207.6609	1182.285	"	300.4056	9.2718	881.8794	6.5268
338	797.4	234.7131	1185.03	"	309.6936	9.2880	875.3364	6.5430
347	806.4	264.4681	1187.775	"	318.996	9.3024	868.779	6.5574
356	815.4	297.1046	1190.52	"	328.3164	9.3204	862.2036	6.5754
365	824.4	332.8073	1193.265	"	337.6512	9.3348	855.6138	6.5898
374	833.4	371.7631	1196.01	"	347.004	9.3528	849.006	6.6068
383	842.4	414.1623	1198.755	"	356.373	9.3690	842.382	6.6240
392	851.4	460.1993	1201.5	"	365.76	9.3870	835.74	6.6420

TABLE IIa.—*continued.*

SATURATED STEAM OF WATER.

English Units.

7 Outer latent heat Apu.	Differences.	8 Steam heat $J=W-Apu$.	Differences.	9 Inner lat. heat $\rho=r-Apu$	Differences.	10 u	11 $\frac{p}{u}$	12 Temperature Fahrenheit t
55.9278		1035.774		1035.774		3374.5441	0.3069	32
56.655	0.7272	1037.79	2.016	1028.790	6.984	2406.5212	0.4275	41
57.4056	0.7506	1039.788	1.998	1021.780	7.010	1738.2122	0.5878	50
58.1724	0.7668	1041.768	1.980	1014.753	7.027	1271.0366	0.7982	59
58.959	0.7866	1043.712	1.944	1007.703	7.050	940.6305	1.0711	68
59.7618	0.8028	1045.656	1.944	1000.632	7.071	704.2394	1.4198	77
60.5808	0.8190	1047.582	1.926	993.542	7.090	532.8851	1.8453	86
61.4142	0.8334	1049.501	1.919	986.434	7.108	407.4570	2.4209	95
62.2584	0.8442	1051.402	1.901	979.309	7.125	314.6755	3.102	104
63.1152	0.8568	1053.289	1.887	972.166	7.143	245.3296	3.9624	113
63.9792	0.8640	1055.170	1.881	965.014	7.152	193.01187	4.9903	122
64.8486	0.8694	1057.046	1.876	957.848	7.166	153.1606	6.2517	131
65.7216	0.8730	1058.918	1.872	950.671	7.177	122.5943	7.6556	140
66.5928	0.8712	1060.792	1.874	943.491	7.180	98.8543	9.4532	149
67.4604	0.8676	1062.663	1.871	936.305	7.186	80.31722	11.6576	158
68.319	0.8586	1064.556	1.893	929.125	7.180	65.71598	14.1886	167
69.165	0.8460	1066.455	1.899	921.947	7.178	54.1263	17.0884	176
69.993	0.8280	1068.372	1.917	914.779	7.168	44.8577	20.3926	185
70.7976	0.8046	1071.3124	1.940	907.626	7.153	37.39454	24.2616	194
71.5716	0.7740	1072.2834	1.971	900.495	7 131	31.3426	28.7298	203
72.369	0.7974	1074.231	1.947	893.331	7.164	26.42479	33.8063	212
72.1358	0.7668	1076.209	1.978	886.186	7.145	22.39123	29.5777	221
73.8864	0.7506	1078.203	1.994	879.049	7.137	19.06731	46.0942	230
74.6226	0.7362	1080.202	1.999	871.914	7.135	16.31366	53.4467	239
75.3444	0.7218	1082.235	2.033	864.774	7.140	14.01977	61.5879	248
76.05	0.7056	1084.275	2.040	857.658	7.116	12.10228	69.8673	257
76.7412	0.6912	1086.328	2.053	850.534	7.124	10.48918	81.0839	266
77.418	0.6768	1088.397	2.069	843.411	7.123	9.12757	92.4019	275
78.0786	6.6606	1090.481	2.084	836.294	7.117	7.97261	104.8919	284
78.723	0.6444	1092.582	2.101	829.180	7.114	6.98905	118.6289	293
79.3548	0.6318	1094.695	2.113	822.063	7.117	6.14966	132.6759	302
79.9704	0.6156	1096.824	2.129	814.948	7.115	5.42720	150.1311	311
80.5698	0.5994	1098.970	2.146	807.836	7.112	4.80727	168.0649	320
81.1548	0.5850	1101.130	2.160	800.724	7.112	4.26904	186.3415	329
81.7254	0.5706	1103.304	2.174	793.611	7.113	3.80449	208.6039	338
82.2798	0.5544	1105.495	2.191	786.5	7.111	3.39921	231.3805	347
82.8216	0.5418	1107.698	2.203	779.382	7.118	3.04519	255.9011	356
83.3472	0.5256	1109.917	2.219	772.266	7.116	2.73603	282.2424	365
83.8602	0.5130	1112.147	2.230	765.145	7.121	2.46370	310.2766	374
84.3552	0.4950	1114.399	2.250	758.026	7.119	2.22502	340.6369	383
84.8394	0.4842	1116.660	2.261	750.9	7.126	2.01857	372.3022	392

TABLE III.

AUXILIARY TABLES FOR SATURATED STEAM OF WATER (ZEUNER).

French Measures.

Pressure p in atmospheres.	Q (page 425)	τ (page 435)	r (page 393)	$\frac{r}{T}$ (page 425)	$\frac{r}{Apu}$
0.5	126.747	0.26273	549.404	1.54887	14.229
1	148.47	0.31356	536.500	1.43834	13.344
2	170.639	0.36814	521.866	1.32588	12.453
3	183.778	0.40205	512.353	1.25913	11.935
4	193.163	0.42711	505.110	1.21129	11.568
5	200.457	0.44693	499.186	1.17395	11.284
6	206.394	0.46392	494.124	1.14322	11.052
7	211.481	0.47840	489.686	1.11714	10.856
8	215.862	0.49120	485.709	1.09441	10.684
9	219.726	0.50270	482.093	1.07425	10.535
10	223.178	0.51297	478.776	1.05617	10.401
11	226.292	0.52266	475.707	1.03980	10.280
12	229.134	0.53150	472.839	1.02477	10.168
13	231.752	0.53975	470.141	1.01088	10.066
14	234.165	0.54744	467.600	0.99801	9.971

TABLE III.—*continued.*

AUXILIARY TABLES FOR SATURATED STEAM OF WATER.

English Measures.

Pressure p in atmospheres.	Q (page 425)	τ (page 435)	r (page 393)	$\frac{r}{T}$ (page 425)	$\frac{r}{Apu}$
0.5	228.144	0.33252	988.927	1.54887	14.229
1	267.246	0.38335	965.700	1.43834	13.344
2	307.150	0.43793	939.358	1.32588	12.453
3	330.800	0.47184	922.235	1.25913	11.935
4	347.693	0.49690	909.198	1.21129	11.568
5	360.822	0.51672	898.535	1.17395	11.284
6	371.509	0.53371	889.423	1.14322	11.052
7	380.606	0.54819	881.435	1.11714	10.856
8	388.553	0.56099	874.276	1.09441	10.684
9	395.507	0.57249	867.767	1.07425	10.535
10	401.720	0.58276	861.797	1.05617	10.401
11	407.326	0.59245	856.072	1.03980	10.280
12	412.441	0.60129	851.110	1.02477	10.168
13	417.154	0.60954	846.254	1.01088	10.066
14	411.497	0.61723	841.680	0.99801	9.971

TABLE IV.

TABLE OF THE PROPERTIES OF SATURATED STEAM OF WATER.*

Pressure above a vacuum in pounds per square inch.	Temperature, Fahrenheit degrees t.	QUANTITIES OF HEAT.					Weight of a cubic foot of steam in pounds.	VOLUME.		Pressure above a vacuum, in pounds per square inch.
		In British thermal units.								
		Required to raise the temperature of the water from 32° to $t°$ q.	Inner latent heat. ρ.	Outer latent heat Apu.	Total latent heat $r = \rho + Apu$.	Total heat above 32° $W = q + r$.		Of a pound of steam in cubic feet.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.	
1	102.018	70.040	981.396	61.619	1043.015	1113.055	.003027	330.4	20623	1
2	126.302	94.368	961.980	64.114	1026.094	1120.462	.005818	171.9	10730	2
3	141.654	109.764	949.725	65 655	1015.380	1125.144	.008522	117.3	7325	3
4	153.122	121.271	940.597	66.773	1007.370	1128.641	.011172	89.51	5588	4
5	162.370	130.563	933.239	67.660	1000.899	1131.462	.013781	72.56	4530	5
6	170.173	138.401	927.038	68.403	995.441	1133.842	.016357	61.14	3816	6
7	176.945	145.213	921.654	69.041	990.695	1135.908	.018908	52.89	3302	7
8	182.952	151.255	916.883	69.602	986.485	1137.740	.021436	46.65	2912	8
9	188.357	156.699	912.584	70 106	982.690	1139.389	.023944	41.77	2607	9
10	193.284	161 660	908.672	70.560	979.232	1140.892	.026437	37.83	2361	10
11	197.814	166.225	905.083	70.967	976.050	1142.275	.028911	34.59	2159	11
12	202.012	170.457	901.766	71.332	973.098	1143.555	.031376	31.87	1990	12
13	205.929	174.402	898.683	71.663	970.346	1144.748	.033828	29.56	1845	13
14	209.604	178.112	895.784	71.973	967.757	1145.869	.036265	27.58	1721	14
14.69	212.000	180.531	893.894	72.175	966.069	1146.600	.037928	26.37	1646	14.69
15	213.067	181.608	893.044	72.274	965.318	1146.926	.038688	25.85	1614	15

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* The following table is given by Mr. R. H. Buel in Weisbach's Mechanics of Engineering. Vol. II., Part II., Steam and the Steam Engine, Du Bois' Translation, pp. 107-111. It is founded upon Rankine's formulæ. To find the pressure in inches of mercury, multiply the pressure in lbs. per sq. inch by 2.036. To find the pressures in atmospheres, divide the pressure in lbs. per sq. inch by 14.7.

TABLE IV.—*continued.*

TABLE OF THE PROPERTIES OF SATURATED STEAM OF WATER.

Pressure above a vacuum in pounds per square inch.	Temperature, Fahrenheit degrees t.	QUANTITIES OF HEAT. In British thermal units. Required to raise the temperature of the water from 32° to $t°$ q.	Inner latent heat. ρ.	Outer latent heat Apu.	Total latent heat $r=\rho+Apu$.	Total heat above 32° $W=q+r$.	Weight of a cubic foot of steam in pounds.	VOLUME. Of a pound of steam in cubic feet.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.	Pressure above a vacuum, in pounds per square inch.
16	216.347	184.919	890.458	72.549	963.007	1147.926	.041109	24.33	1519	16
17	219.452	188.056	888.007	72.811	960.818	1148.874	.043519	22.98	1434	17
18	222.424	191.058	885.661	73.060	958.721	1149.779	.045920	21.78	1359	18
19	225.255	193.918	883.427	73.298	956.725	1150.643	.048312	20.70	1292	19
20	227.964	196.655	881.289	73.525	954.814	1151.469	.050696	19.73	1231	20
21	230.565	199.285	879.239	73.739	952.978	1152.263	.053074	18.84	1176	21
22	233.069	201.817	877.267	73.942	951.209	1153.026	.055446	18.04	1126	22
23	235.479	204.258	875.368	74.136	949.504	1154.762	.057812	17.30	1080	23
24	237.803	206.610	873.538	74.323	947.861	1155.471	.060171	16.62	1038	24
25	240.053	208.887	871.767	74.503	946.270	1155.157	.062524	16.00	998.4	25
26	242.225	211.089	870.052	74.678	944.730	1155.819	.064870	15.42	962.8	26
27	244.333	213.223	868.391	74.847	943.238	1156.461	.067210	14.88	928.8	27
28	246.376	215.293	866.780	75.011	941.791	1157.084	.069545	14.38	897.6	28
29	248.363	217.306	865.215	75.168	940.383	1157.691	.071875	13.91	868.5	29
30	250.293	219.261	863.700	75.319	939.019	1158.280	.074201	13.48	841.3	30
31	252.171	221.165	862.221	75.466	937.687	1158.852	.076522	13.07	815.8	31
32	254.002	223.021	860.781	75.608	936.389	1159.410	.078839	12.68	791.8	32
33	255.782	224.827	859.382	75.745	935.127	1159.954	.081152	12.32	769.2	33
34	257.523	226.594	858.018	75.878	933.891	1160.485	.083461	11.98	748.0	34
35	259.221	228.316	856.680	76.007	932.687	1161.003	.085766	11.66	727.9	35
36	260.888	230.001	855.375	76.133	931.508	1161.509	.088067	11.36	708.8	36

TABLE OF THE PROPERTIES OF SATURATED STEAM OF WATER—*continued.*

37	262.505	231.650	854.099	76.255	930.354	1162.004	.090864	11.07	690.8	37
38	264.093	233.261	852.852	76.375	929.227	1162.488	.092657	10.79	673.7	38
39	265.647	234.840	851.629	76.493	928.122	1162.962	.094946	10.53	657.5	39
40	267.168	236.386	850.432	76.608	927.040	1163.426	.097231	10.28	642.0	40
41	268.660	237.902	849.261	76.719	925.980	1163.882	.099514	10.05	627.3	41
42	270.122	239.389	848.113	76.827	924.940	1164.329	.101794	9.826	613.3	42
43	271.557	240.846	846.988	76.932	923.920	1164.766	.104071	9.609	599.9	43
44	272.965	242.275	845.884	77.035	922.919	1165.194	.106345	9.403	587.0	44
45	274.347	243.680	844.799	77.136	921.935	1165.615	.108616	9.207	574.7	45
46	275.704	245.061	843.733	77.235	920.968	1166.029	.110884	9.018	563.0	46
47	277.036	246.418	842.687	77.331	920.018	1166.436	.113149	8.838	551.7	47
48	278.348	247.752	841.659	77.425	919.084	1166.836	.115411	8.665	540.9	48
49	279.637	249.064	840.647	77.517	918.164	1167.228	.117670	8.498	530.5	49
50	280.904	250.355	839.653	77.607	917.260	1167.615	.119927	8.338	520.5	50
51	282.151	251.624	838.675	77.696	916.371	1167.995	.122181	8.185	510.9	51
52	283.381	252.875	837.710	77.784	915.494	1168.369	.124433	8.037	501.7	52
53	284.589	254.106	836.762	77.870	914.632	1168.738	.126682	7.894	492.8	53
54	285.781	255.321	835.827	77.954	913.781	1169.102	.128928	7.756	484.2	54
55	286.955	256.518	834.906	78.036	912.942	1169.460	.131172	7.624	475.9	55
56	288.111	257.695	834.001	78.117	912.118	1169.813	.133414	7.496	467.9	56
57	289.251	258.857	833.108	78.196	911.304	1170.161	.135654	7.372	460.2	57
58	290.374	260.002	832.228	78.273	910.501	1170.503	.137892	7.252	452.7	58
59	291.483	261.132	831.361	78.348	909.709	1170.841	.140128	7.136	445.5	59
60	292.575	262.248	830.507	78.421	908.928	1171.176	.142363	7.024	438.5	60
61	293.653	263.348	829.663	78.494	908.157	1171.505	.144594	6.916	431.7	61
62	294.717	264.433	828.830	78.566	907.396	1171.829	.146824	6.811	425.2	62
63	295.768	265.506	828.005	78.638	906.643	1172.149	.149052	6.709	418.8	63
64	296.805	266.566	827.191	78.709	905.900	1172.466	.151277	6.610	412.6	64
65	297.830	267.612	826.388	78.779	905.167	1172.779	.153500	6.515	406.6	65
66	298.842	268.644	825.596	78.847	904.443	1173.087	.155721	6.422	400.8	66
67	299.843	269.666	824.814	78.913	903.727	1173.393	.157940	6.332	395.2	67
68	300.831	270.674	824.042	78.978	903.020	1173.694	.160157	6.244	389.8	68
69	301.807	271.669	823.280	79.042	902.322	1173.991	.162372	6.159	384.5	69
70	302.774	272.657	822.524	79.105	901.629	1174.286	.164584	6.076	379.3	70
71	303.728	273.633	821.778	79.167	900.945	1174.578	.166794	5.995	374.8	71
72	304.669	274.597	821.041	79.228	900.269	1174.866	.169008	5.917	369.4	72
73	305.603	275.550	820.312	79.288	899.600	1175.150	.171210	5.841	364.6	73
74	306.526	276.493	819.589	79.349	898.938	1175.431	.173417	5.767	360.0	74
75	307.440	277.427	818.873	79.410	898.283	1175.710	.175623	5.694	355.5	75
76	308.344	278.350	818.166	79.469	897.635	1175.985	.177825	5.624	351.1	76
77	309.239	279.265	817.468	79.526	896.994	1176.259	.180027	5.555	346.8	77

TABLE IV.—*continued.*

TABLE OF THE PROPERTIES OF SATURATED STEAM OF WATER.

Pressure above a vacuum in pounds per square inch.	Temperature, Fahrenheit degrees t.	QUANTITIES OF HEAT. In British thermal units. Required to raise the temperature of the water from 32° to t° q.	Inner latent heat. ρ.	Outer latent heat Apu.	Total latent heat $r = \rho + Apu$.	Total heat above 32° $W = q + r$.	Weight of a cubic foot of steam in pounds.	VOLUME. Of a pound of steam in cubic feet.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.	Pressure above a vacuum, in pounds per square inch.
78	310.123	280.170	816.777	79.582	896.359	1176.529	.182229	5.488	342.6	78
79	311.000	281.066	816.090	79.639	895.729	1176.795	.184429	5.422	338.5	79
80	311.866	281.952	815.413	79.695	895.108	1177.060	.186627	5.358	334.5	80
81	312.725	282.830	814.742	79.749	894.491	1177.321	.188823	5.296	330.6	81
82	313.576	283.701	814.077	79.802	893.879	1177.580	.191017	5.235	326.8	82
83	314.417	284.562	813.419	79.856	893.275	1177.837	.193210	5.176	323.1	83
84	315.250	285.414	812.768	79.909	892.677	1178.091	.195401	5.118	319.5	84
85	316.076	286.260	812.122	79.961	892.083	1178.343	.197591	5.061	315.9	85
86	316.893	287.096	811.484	80.012	891.496	1178.592	.199781	5.006	312.5	86
87	317.705	287.927	810.850	80.063	890.913	1178.840	.201969	4.951	309.1	87
88	318.510	288.750	810.222	80.113	890.335	1179.085	.204155	4.898	305.8	88
89	319.306	289.565	809.601	80.162	889.763	1179.328	.206340	4.846	302.5	89
90	320.094	290.373	808.986	80.210	889.196	1179.569	.208525	4.796	299.4	90
91	320.877	291.176	808.375	80.258	888.633	1179.809	.210709	4.746	296.3	91
92	321.653	291.970	807.770	80.305	888.075	1180.045	.212892	4.697	293.2	92
93	322.422	292.758	807.170	80.351	887.521	1180.279	.215074	4.650	290.2	93
94	323.183	293.539	806.575	80.397	886.972	1180.511	.217253	4.603	287.3	94
95	323.939	294.314	805.985	80.442	886.427	1180.741	.219430	4.557	284.5	95
96	324.688	295.083	805.400	80.487	885.887	1180.970	.221604	4.513	281.7	96
97	325.431	295.845	804.821	80.531	885.352	1181.197	.223778	4.469	279.0	97
98	326.169	296.601	804.245	80.576	884.821	1181.422	.225950	4.426	276.3	98
99	326.900	297.350	803.675	80.620	884.295	1181.645	.228122	4.384	273.7	99
100	327.625	298.093	803.108	80.665	883.773	1181.866	.230293	4.342	271.1	100

TABLE OF THE PROPERTIES OF SATURATED STEAM OF WATER—*continued.*

101	328.345	298.832	802.544	80.709	883.253	1182.085	.232464	4.302	268.5	101
102	329.060	299.566	801.985	80.752	882.737	1182.303	.234634	4.262	266.0	102
103	329.769	300.293	801.432	80.794	882.226	1182.519	.236808	4.223	263.6	103
104	330.471	301.014	800.884	80.835	881.719	1182.733	.238972	4.185	261.2	104
105	331.169	301.731	800.339	80.875	881.214	1182.945	.241139	4.147	258.9	105
106	231.862	302.444	799.796	80.916	880.712	1183.156	.243314	4.110	256.6	106
107	332.550	303.152	799.258	80.956	880.214	1183.366	.245467	4.074	254.3	107
108	333.232	303.854	798.725	80.995	879.720	1183.574	.247629	4.038	252.1	108
109	333.911	304.551	798.196	81.034	879.230	1183.781	.249789	4.003	249.9	109
110	334.583	305.242	797.672	81.072	878.744	1183.986	.251947	3.969	247.8	110
111	335.250	305.927	797.153	81.110	878.263	1184.190	.254105	3.935	245.7	111
112	335.914	306.609	796.637	81.147	877.784	1184.393	.256283	3.902	243.6	112
113	336.573	307.285	796.125	81.184	877.309	1184.594	.258490	3.870	241.6	113
114	337.226	307.956	795.617	81.221	876.838	1184.794	.260676	3.838	239.6	114
115	337.874	308.621	795.114	81.257	876.371	1184.992	.262788	3.806	237.6	115
116	338.518	309.281	794.614	81.293	875.907	1185.188	.264887	3.775	235.7	116
117	339.159	309.939	794.114	81.330	875.444	1185.383	.267041	3.745	233.8	117
118	339.796	310.592	793.619	81.366	874.985	1185.577	.269195	3.715	231.9	118
119	340.430	311.241	793.126	81.403	874.529	1185.770	.271348	3.685	230.1	119
120	341.058	311.885	792.637	81.439	874.076	1185.961	.273500	3.656	228.3	120
121	341.681	312.524	792.152	81.474	873.626	1186.150	.275651	3.626	226.5	121
122	342.300	313.161	791.669	81.509	873.178	1186.339	.277801	3.600	224.7	122
123	342.916	313.795	791.189	81.543	872.732	1186.527	.279949	3.572	223.0	123
124	343.528	314.425	790.711	81.578	872.289	1186.714	.282097	3.545	221.3	124
125	344.136	315.051	790.236	81.612	871.848	1186.899	.284243	3.518	219.6	125
126	344.741	315.672	789.765	81.646	871.411	1187.083	.286389	3.492	218.0	126
127	345.340	316.289	789.298	81.679	870.977	1187.266	.288533	3.466	216.4	127
128	345.936	316.903	788.834	81.711	870.545	1187.448	.290677	3.440	214.8	128
129	346.530	317.518	788.374	81.742	870.116	1187.629	.292820	3.415	213.2	129
130	347.121	318.121	787.914	81.774	869.688	1187.809	.294961	3.390	211.6	130
131	347.706	318.725	787.458	81.805	869.263	1187.988	.297102	3.366	210.1	131
132	348.287	319.325	787.004	81.837	868.841	1188.166	.299242	3.342	208.6	132
133	348.867	319.922	786.554	81.868	868.422	1188.344	.301382	3.318	207.1	133
134	349.443	320.515	786.105	81.900	868.005	1188.520	.303521	3.295	205.7	134
135	350.015	321.105	785.659	81.931	867.590	1188.695	.305659	3.272	204.3	135
136	350.584	321.692	785.215	81.962	867.177	1188.869	.307797	3.249	202.8	136
137	351.149	322.274	784.775	81.992	866.767	1189.041	.309934	3.227	201.4	137
138	351.711	322.853	784.339	82.021	866.360	1189.213	.312070	3.204	200.0	138
139	352.271	323.429	783.905	82.050	865.955	1189.384	.314205	3.182	198.7	139
140	352.827	324.003	783.472	82.080	865.552	1189.555	.316338	3.161	197.3	140

TABLE IV.—*continued.*

TABLE OF THE PROPERTIES OF SATURATED STEAM OF WATER.

Pressure above a vacuum in pounds per square inch.	Temperature, Fahrenheit degrees t.	Quantities of heat. In British thermal units. Required to raise the temperature of the water from 32° to t° q.	Inner latent heat. ρ.	Outer latent heat Apu.	Total latent heat $r=\rho+Apu$.	Total heat above 32° $W=q+r$.	Weigh of a cubic foot of steam in pounds.	Volume. Of a pound of steam in cubic feet.	Ratio of volume of steam to volume of equal weight of distilled water at temperature of maximum density.	Pressure above a vacuum, in pounds per square inch.
141	353.380	324.573	783.042	82.109	865.151	1189.724	.318471	3.140	196.0	141
142	353.981	325.141	782.618	82.133	864.751	1189.892	.320603	3.119	194.7	142
143	354.478	325.705	782.188	82.166	864.354	1190.059	.322735	3.099	193.4	143
144	355.022	326.265	781.766	82.194	863.960	1190.225	.324867	3.078	192.2	144
145	355.562	326.823	781.346	82.221	863.567	1190.390	.326998	3.058	190.9	145
146	356.100	327.378	780.927	82.249	863.176	1190.554	.329128	3.038	189.7	146
147	356.636	327.930	780.510	82.277	862.787	1190.717	.331257	3.019	188.5	147
148	357.169	328.479	780.096	82.304	862.400	1190.879	.333386	3.000	187.3	148
149	357.697	329.024	779.684	82.332	862.016	1191.040	.335515	2.981	186.1	149
150	358.223	329.566	779.275	82.359	861.634	1191.200	.337643	2.962	184.9	150
160	363.346	334.850	775.296	82.616	857.912	1192.762	.358886	2.786	173.9	160
170	368.296	339.892	771.505	82.854	854.359	1194.251	.380071	2.631	164.3	170
180	372.886	344.708	767.891	83.072	850.963	1195.671	.401201	2.493	155.6	180
190	377.352	349.329	764.430	83.278	847.708	1197.032	.422280	2.368	147.8	190
200	381.636	353.766	761.111	83.463	844.578	1198.339	.443310	2.256	140.8	200
210	385.759	358.041	757.916	83.640	841.556	1199.597	.464295	2.154	134.5	210
220	389.736	362.168	754.834	83.808	838.642	1200.810	.485237	2.061	128.7	220
230	393.575	366.152	751.862	83.966	835.828	1201.980	.506189	1.976	123.3	230
240	397.285	370.008	748.988	84.115	833.103	1203.111	.527008	1.898	118.5	240
250	400.883	373.750	746.203	84.256	830.459	1204.209	.547831	1.825	114.0	250
260	404.370	377.377	743.508	84.388	827.896	1205.273	.568696	1.759	109.8	260

TABLE OF THE PROPERTIES OF SATURATED STEAM OF WATER—*continued.*

270	407.755	380.905	740.891	84.510	825.401	1206.306	.589890	1.697	105.9	270
280	411.048	384.337	738.350	84.623	822.973	1207.310	.610124	1.639	102.3	280
290	414.250	387.677	735.878	84.731	820.609	1208.286	.630629	1.585	99.0	290
300	417.371	390.963	733.470	84.835	818.305	1209.238	.651506	1.535	95.8	300
350	431.96	406.26	722.20	85.28	807.48	1213.74	.754534	1.325	82.7	350
400	444.92	419.76	712.34	85.60	797.94	1217.70	.857185	1.167	72.8	400
450	456.62	432.18	703.28	85.84	789.12	1221 30	.959536	1.042	65.1	450
500	467.42	443.58	695.01	86.01	781.02	1224.54	1.061700	.942	58.8	500
550	477.50	454.14	687.34	86.12	773.46	1227.60	1.16380	.859	53.6	550
600	486.86	464.22	680.08	86.18	766.26	1230.48	1.26586	.790	49.3	600
650	495.68	473.58	673.40	86.20	759.60	1233.18	1.36791	.731	45.6	650
700	504.14	482.40	667.11	86.19	753.30	1235.70	1.46995	.680	42.4	700
750	512.06	490.86	661.04	86.14	747.18	1238 04	1.57198	.636	39.6	750
800	519.62	498.88	655.34	86.08	741.42	1240.30	1.67401	.597	37.1	800
850	526.82	506.66	649.84	86.00	735.84	1242.50	1.77603	.563	34.9	850
900	533.66	514.08	644.71	85 91	730.62	1244.65	1 87804	.532	33.0	900
950	540.32	521.30	639.60	85.80	725.40	1246.70	1.98004	.505	31.4	950
1000	546.80	528.30	634.68	85.68	720.36	1248.66	2.08203	.480	30.0	1000

TABLE V.

$\frac{p_1}{p}$	$m = 1.185$ $x = 1$		$m = 1.125$ $x = 0.9$	
	$\left(\frac{p_1}{p}\right)^{\frac{1}{m}}$	$1-\left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}$	$\left(\frac{p_1}{p}\right)^{\frac{1}{m}}$	$1-\left(\frac{p_1}{p}\right)^{\frac{m-1}{m}}$
0.9	0.9114	0.0125	0.9106	0.0116
0.8	0.8215	0.0262	0.8201	0.0245
0.7	0.7303	0.0416	0.7283	0.0388
0.6	0.6376	0.0590	0.6350	0.0552
0 5	0.5430	0.0792	0.5400	0.0741
0.45	0.4949	0.0907	0.4917	0.0849
0.4	0.4461	0.1033	0.4429	0.0968
0.35	0.3966	0.1175	0.3933	0.1101
0.3	0.3462	0.1335	0.3430	0.1252
0.25	0.2948	0.1521	0.2916	0.1428
0.2	0.2422	0.1743	0.2393	0.1637
0.15	0.1880	0.2021	0.1852	0.1901
0.12	0.1544	0.2280	0.1519	0.2099
0.1	0.1315	0.2397	0.1292	0.2257
0.08	0.1080	0.2596	0.1059	0.2447
0.06	0.0839	0.2845	0.0820	0.2685

TABLE VI.

SATURATED STEAM OF ETHER ($C_4H_{10}O$)

French Units.

1 Temperature Centigrade t	2 Absolute temperature T	3 Pressure in millimeters p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
0	273	184.39	94.00		0.00		94.00	
10	283	286.83	98.44	4.44	5.32	5.32	93 12	0.88
20	293	432.78	102.78	4.34	10.70	5.38	92.08	1.04
30	303	634.80	107.00	4.22	16.14	5.44	90.86	1.22
40	313	907.04	111.11	4.11	21.63	5.49	89.48	1.38
50	323	1264.83	115.11	4.00	27.19	5.56	87.92	1.56
60	333	1725.01	119.00	3.89	32.80	5.61	86.20	1.72
70	343	2304.90	122.78	3.78	38.48	5.68	84.30	1.90
80	353	3022.79	126.44	3.66	44.21	5.73	82.23	2.07
90	363	3898.26	130.00	3.56	50.00	5 79	80.00	2.23
100	373	4953.30	133.44	3.44	55.86	5.86	77.58	2.42
110	383	6214.63	136.78	3.34	61.77	5.91	75.01	2.57
120	393	7719.20	140.00	3.22	67.74	5.97	72.26	2.75

English Units.

1 Temperature Fahrenheit t	2 Absolute temperature T	3 Pressure in inches p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
32	491.4	7.259	169.2		0.00		169.2	
50	509.4	11.292	177.192	7.992	9.576	9.576	167.616	1.584
68	527.4	17.039	185.104	7.912	19.26	9.684	165.744	1.872
86	545.4	24.992	192.6	7.496	29.052	9.792	163.548	2.196
104	563.4	35.710	200	7.400	38.934	9.882	161.064	2.484
122	581.4	49.796	207.198	7.198	48.942	10.008	158.256	2.808
140	599.4	67.914	214.2	7.002	59.04	10.098	155.16	3.096
158	617.4	90.744	221.004	6.804	69.264	10.224	151.74	3.42
176	635.4	119.008	227.592	6.588	79.578	10.314	148.014	3.726
194	653.4	153.476	234	6.408	90	10.422	144	4.014
212	671.4	195.018	240.192	6.192	100.548	10.548	139.644	4.356
230	689.4	244.672	246.204	6.012	111.186	10.638	135.018	4.626
248	707.4	303.908	252	5.796	121.932	10.746	130.068	4.950

TABLE VI.

SATURATED STEAM OF ETHER ($C_4H_{10}O$).

French Units

7 Outer latent heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner lat. heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Centigrade t
7.521		86.48		86.48		1.2720	67.99	0
7.721	0.200	90.72	4.24	85.40	1.08	0.8893	101.74	10
7.919	0.198	94.86	4.14	84.16	1.24	0.5706	147.50	20
8.113	0.194	98.89	4.01	82.75	1.41	0.3985	207.64	30
8.300	0.187	102.81	3.92	81.18	1.57	0.2853	284.51	40
8.475	0.175	106.64	3.83	79.45	1.73	0.2089	380.22	50
8.635	0.160	110.37	3.73	77.57	1.88	0.1561	496.88	60
8.772	0.137	114.01	3.64	75.53	2.04	0.1186	636.40	70
8.876	0.104	117.56	3.55	73.35	2 18	0.0916	801.08	80
8.937	0.061	121.06	3.50	71.06	2.29	0 0715	994.00	90
8.934	−0.003	124.51	3.45	68.65	2.41	0.0563	1220.49	100
8.850	−0.084	127.93	3.42	66.16	2.49	0.0444	1489 88	110
8.655	−0.195	131.35	3.42	63.61	2.55	0.0350	1819.11	120

English Units.

7 Outer latent heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner lat. heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Fahrenheit t
13.5378		155.664		155.664		20.6644	7.63984	32
13.8978	0.36	163 296	7.632	153.72	8.056	13.4447	11.4322	50
14.2542	0.3564	170.748	7.452	151.488	7.768	9.14039	16.5741	68
14.6034	0.3492	178.002	7.254	148.95	7.462	6.38353	23.3819	86
14.94	0.3366	185.058	7.056	146.124	7.174	4.57019	31.9693	104
15.255	0.315	191.952	6.894	143.01	6.887	3.34635	42.7242	122
15.543	0.288	198.666	6.714	139.626	6.616	2.50055	55.8329	140
15.7896	0.2466	205.218	6.552	135.954	6.328	1.89984	71.5104	158
15.9764	0.1868	211.608	6.390	132.03	6.076	1.46733	90.015	176
16.0866	0.1102	217.908	6.300	127.908	5.878	1.14534	111.6929	194
16.0812	−0.0054	224.118	6.210	123.57	5.662	0.900261	137.143	212
15.93	−0.1512	230.274	6.156	119.088	5.518	0.711288	167.4079	230
15.579	−0.351	236.43	6.156	114.498	5.410	0.560661	204.4082	248

TABLE VII.

SATURATED STEAM OF ALCOHOL (C_2H_6O).—*French Units.*

1 Temperature Centigrade t	2 Absolute temperature T	3 Pressure in millimeters p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
0	273	12.70	236.50		0.00		236.50	
10	283	24.23	244.40	7.90	5.59	5.59	238.81	−2.31
20	293	44.46	252.00	7.60	11.42	5.83	240.58	−1.77
30	303	78.52	258.00	6.00	17.49	6.07	240.51	+0.07
40	313	133.69	262.00	4.00	23.71	6.22	238.29	2.22
50	323	219.90	264	2.00	30.21	6.50	233.79	4.50
60	233	350.21	265	1.00	37.37	7.16	227.63	6.16
70	343	541.15	265.20	0.20	44.58	7.21	220.61	7.01
80	353	812.91	265.20	0.00	52.11	7.53	213.09	7.53
90	363	1189.30	266.00	0.80	59.97	7.86	206 03	7.06
100	373	1697.55	267.30	1.30	68.18	8.21	199.12	6.91
110	383	2367.64	269.60	2.30	76.74	8.56	192.86	6.26
120	393	3231.73	272.50	2.90	85.67	8.93	186.83	6.03
130	403	4323.00	276.00	3.50	94.98	9.31	181.02	5.81
140	413	5674.59	280.50	4.50	104.70	9.72	175.80	5 22
150	423	7318.40	285.30	4.80	114.82	10.12	170.48	5.32

English Units.

1 Temperature Fahrenheit t	2 Absolute temperature T	3 Pressure in inches p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
32	491.4	0.5	425.7		0.000		424.7	
50	509.4	0.9539	439.92	14.22	10.062	10.062	429.858	−5.158
68	527.4	1.7502	453.6	13.68	20.556	10.494	433.034	−3.176
86	545.4	3.0984	464.4	10.8	31.482	10.926	432.918	+0.116
104	563.4	5.2634	471.6	7.2	42.678	11.196	428.922	3.996
122	581.4	8.6575	475.2	3.6	54.378	11.700	420.822	8.100
140	599.4	13.7879	477	1.8	67.266	12.888	409.734	11.088
158	617.4	21.3052	477.36	0.36	80.244	12.978	397.098	12.636
176	635.4	32.0046	477.36	0.00	93.798	13.554	383.562	13.536
194	653.4	46.8232	478.8	1.44	107.946	14.148	370.854	12.708
212	671.4	66.8332	481.14	2.34	122.724	14.778	358.416	12.438
230	689 4	93.2147	485.28	4.14	138.132	15.408	347.148	10.268
248	707.4	127.2345	490.5	5.22	154.206	16.074	336.294	10.854
266	725.4	170.198	496.8	6.3	170.964	16.758	325.836	10.458
284	743.4	223.410	504.9	8.1	188.46	17.496	316.44	9.896
302	761.4	288.128	513.54	8 64	206.676	18.216	306.864	9.576

TABLE VII.

SATURATED STEAM OF ALCOHOL (C_2H_6O).—*French Units.*

7 Outer latent heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner latent heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Centigrade t
13.067		223.43		223.43		32.0844	6.96	0
13.465	0.398	230.94	7.51	225.35	−1.95	17.3281	13.00	10
13.969	0.504	238.03	7.09	226.61	−1.26	9.7976	23.13	20
14.429	0.460	243.57	5.54	226.08	+0.53	5.7303	39.35	30
14.797	0.368	247.20	3.63	223.49	2.59	3.4513	64.75	40
15.044	0.247	248.96	1.76	218.75	4.74	2.1832	102.54	50
15.193	0.149	249.81	0.85	212.44	6.31	1.3528	157.03	60
15.287	0.094	249.91	+0.10	205.32	7.11	0.8809	233.08	70
15.344	0.057	249.86	−0.05	197.75	7.58	0.5886	335.97	80
15.428	0.084	250.57	+0.71	190.60	7.15	0.4045	471.18	90
15.518	0.090	251.78	1.21	183.60	7.00	0.2850	644.09	100
15.654	0.136	253.95	2.17	177.21	6.39	0.2062	859.53	110
15.803	0.149	256.70	2.75	171.03	6.18	0.1525	1121.53	120
15.969	0.166	260.03	3.33	165.05	5.98	0.1152	1482.56	130
16.184	0.215	264.32	4.29	159.62	5.43	0.0889	1794.84	140
16.386	0.202	268.91	4.59	154 09	5.53	0.0698	2207.02	150

English Units.

7 Outer latent heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner latent heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Fahrenheit t
23.520		402.174		402.174		513.957	0.78207	32
24.237	0.717	415.692	13.518	405.63	−3.456	277.577	1.4607	50
25.144	0.907	428.454	12.762	407.898	−2.268	156.947	2.5989	68
25.972	0.828	438.426	9.972	406.944	+0.954	91.793	4.4216	86
26.634	0.662	444.96	6.534	402.282	4.662	55.286	7.2757	104
27.079	0.445	448.128	3.168	393.75	8.532	34.171	11.522	122
27.347	0.268	449.658	1.530	382.392	11.358	21.670	17.645	140
27.516	0.169	449.838	+0.180	369.576	12.816	14.111	26.1904	158
27.619	0.103	449.748	−0.090	355.95	13.626	9.429	37.7518	176
27.770	0.151	451.026	+1.278	343.08	12.87	6.479	52.9451	194
27.932	0.162	453.204	2.178	330.48	12.60	4.564	72.3744	212
28.177	0.245	457.11	3.906	318.978	11.502	3.305	96.5827	230
28.445	0 268	462.06	4.95	307.854	11.124	2.443	126.0228	248
28.744	0.299	468.054	5.994	297.09	10 764	1.845	160.9724	266
29.131	0.387	475.776	7.722	287.316	9.774	1.424	201.681	284
29.495	0.364	484.038	8.262	277.362	9.944	1.118	248.016	302

TABLE VIII.

SATURATED STEAM OF ACETONE (C_3H_6O).

French Units.

1 Temperature Centigrade t	2 Absolute temperature T	3 Pressure in millimeters p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
0	273	63.33	140.5		0.00		140.5	
10	283	110.32	144.11	3.61	5.10	5.10	139.01	1.49
20	293	180.08	147.62	3.51	10.29	5.19	137.33	1.68
30	303	280.05	151.03	3.41	15.55	5.26	135.48	1.85
40	313	419.35	154.33	3.30	20.89	5.34	133.34	2.04
50	323	608.81	157.53	3.20	26.31	5 42	131.22	2.22
60	333	860.96	160.63	3.10	31.81	5.50	128.82	2.40
70	343	1189.90	163.62	2.99	37.39	5.58	126.23	2.59
80	353	1611.05	166.51	2.89	43.05	5.66	123.46	2.77
90	363	2140.82	169.30	2.79	48.79	5.74	120.51	2.95
100	373	2796.20	171.98	2.68	54.61	5.82	117.37	3.14
110	383	3594.33	174.56	2.58	60.50	5.89	114.06	3.31
120	393	4551.95	177.04	2.48	66.48	5.98	110.56	3.50
130	403	5684.90	179.42	2.38	72.54	6.06	106.88	3.68
140	413	7007.64	181.69	2.27	78.67	6.13	103.02	3.86

English Units.

1 Temperature Fahrenheit t	2 Absolute temperature T	3 Pressure in inches p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
32	491.4	2.4933	252.9		0.00		252.9	
50	509.4	4.3432	259.398	6.498	9.18	9.18	250.218	2.682
68	527.4	7.0897	265.716	6.318	18.522	9.342	247.194	3.024
86	545.4	11.0256	271.854	6.138	27.99	9.468	243.864	3.330
104	563.4	16.5098	277.794	5.940	37.602	9.612	240.192	3.672
122	581.4	23.9688	283.554	5.760	47.358	9.756	236.196	3.996
140	599.4	33.8962	289.134	5.580	57.258	9.900	231.876	4.320
158	617.4	46.8467	294.516	5.382	67.302	10.044	227.214	4.662
176	635.4	63.4275	299.718	5.202	77.49	10.188	222.228	4.986
194	653.4	84.2848	304.74	5.022	87.822	10.332	216.918	5.310
212	671.4	110.0874	309.564	4.824	98.298	10.476	211.266	5.652
230	689.4	141.5101	314.208	4.644	108.9	19.692	205.308	5.958
248	707.4	179.212	318.672	4.464	119.664	10.764	199.008	6.300
266	725.4	223.8167	322.956	4.284	130.572	10.908	192.384	6.624
284	743.4	275.8936	327.042	4.086	141.606	11.034	185.436	6.948

TABLE VIII.

SATURATED STEAM OF ACETONE (C_3H_6O).

French Units.

7 Outer latent heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner lat. heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Centigrade t
8.64		131.85		131		4.2576	30.97	0
9.462	0.815	134.65	2.80	129.55	2.30	2.6748	48.43	10
10.132	0.670	137.49	2.84	127.20	2.35	1.7505	72.68	20
10.613	0.481	140.42	2.93	124.87	2.33	1.1818	105.65	30
11.006	0.393	143.32	2.90	122.43	2.44	0.8184	149.59	40
11.315	0.309	146.22	2.90	119.91	2.52	0.5796	206.89	50
11.561	0.246	149.07	2.85	117.26	2.65	0.4187	280.03	60
11.759	0.198	151.86	2.79	114.47	2.79	0.3082	371.44	70
11.921	0.162	154.59	2.73	111.54	2.93	0.2307	483.40	80
12.052	0.131	157.25	2.66	108.46	3.08	0.1755	617.84	90
12.154	0.102	159.83	2.58	105.22	3.24	0.1355	776.25	100
12.231	0.077	162.33	2.50	101.83	3.39	0.1061	959.64	110
12.281	0.050	164.76	2.43	98.28	3.55	0.0841	1168.18	120
12.302	0.021	167.12	2.36	94.58	3.70	0.0675	1401.58	130
12.302	0.000	169.39	2.27	90.62	3.96	0.0547	1658.64	140

English Units.

7 Outer latent heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner lat. heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Fahrenheit t
155.646		237.33		237.33		68.2021	3.48	32
170.316	14.670	242.37	5.04	233.19	4.14	42.8483	5.4419	50
182.376	12.060	247.482	5.112	228.96	4.23	28.0411	8.1668	68
191.034	8.658	252.756	5.274	224.766	4.194	18.9311	11.8715	86
198.108	7.074	257.976	5.220	220.374	4.392	13.1098	16.8089	104
203.67	5.562	263.196	5.220	215.838	4.536	9.2845	23.2475	122
208.098	4.428	268.326	5.130	211.068	4.770	6.7071	21.466	140
211.662	3.564	273.348	5.022	206.046	5.022	4.9370	41.7376	158
214.978	3.316	278.262	4.914	200.772	5.274	3.6955	54.3181	176
216.936	1.958	283.05	4.788	195.228	5.544	2.8118	69.4249	194
218.772	1.836	287.694	4.644	189.396	5.832	2.1705	87.2249	212
220.158	1.386	292.194	4.500	183.294	6.102	1.6996	108.4791	230
221.058	0.900	296.568	4.374	176.904	6.390	1.3472	131.2648	248
221.436	0.378	300.816	4.248	170.244	6.660	1.0812	157.4912	266
221.436	0.000	304.902	4.086	163.116	7.128	0.87662	186.3764	284

TABLE IX.

SATURATED STEAM OF CHLOROFORM ($CHCl_3$).—*French Units.*

1 Temperature Centigrade t	2 Absolute temperature T	3 Pressure in millimeters p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
0	273	59.72	67.000	1.375	0.00		67.00	
10	283	100.47	68.375	"	2.33	2.33	66.04	0.96
20	293	160.47	69.75	"	4.67	2.34	65.08	0.96
30	303	247.51	71.12	"	7.02	2.35	64.10	0.98
40	313	369.26	72.50	"	9.37	2.35	63.13	0.97
50	323	535.05	73.87	"	11.74	2.37	62.13	1.00
60	333	755.44	75.25	"	14.12	2.38	61.13	1.00
70	343	1042.11	76.62	"	16.51	2.39	60.11	1.02
80	353	1407.64	78.00	"	18.91	2.40	59.09	1.02
90	363	1865.22	79.37	"	21.32	2.41	58.05	1.04
100	373	2428.54	80.75	"	23.74	2.42	57.01	1.04
110	383	3110.99	82.12	"	26.17	2.43	55.95	1.06
120	393	3925.74	83.50	"	28.61	2.44	54.89	1.06
130	403	4885.10	84.87	"	31.06	2.45	53.81	1.08
140	413	6000.16	86.25	"	33.52	2.46	52.73	1.08
150	423	7280.62	87.62	"	35.99	2.47	51.63	1.10
160	433	8734.20	89.00	"	38.47	2.48	50.53	1.10

English Units.

1 Temperature Fahrenheit t	2 Absolute temperature T	3 Pressure in inches p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
32	491.4	2.3512	120.6		0.00		120.6	
50	509.4	3.9555	123.075	2.475	4.194	4.194	118.872	1.728
68	527.4	6.3178	125.55	"	8.406	4.212	117.144	1.728
86	545.4	9.7446	128.016	"	12.636	4.230	115.38	1.764
104	563.4	14.538	130.5	"	16.866	4.230	113.634	1.746
122	581.4	21.065	132.966	"	21.132	4.266	111.834	1.800
140	599.4	29.742	135.45	"	25.416	4.384	110.034	1.800
158	617.4	41.028	137.916	"	29.718	4.302	108.198	1.836
176	635.4	55.416	140.4	"	24.038	4.320	106.362	1.836
194	653.4	73.485	142.866	"	38 376	4.338	104.49	1.872
212	671.4	95.612	145.35	"	42.732	4.356	102.618	1.872
230	689.4	122.48	147.816	"	47.106	4.374	100.71	1.908
248	707.4	154.56	150.3	"	51.498	4.392	98.802	1.908
266	725.4	192.33	152.766	"	55.908	4.410	96.858	1.944
284	743.4	236.23	155.25	"	60.336	4.428	94.914	1.944
302	761.4	286.64	157.716	"	64.782	4.446	92.934	1.980
320	779.4	343.87	160.2	"	69.246	4.464	90.954	1.980

TABLE IX.

SATURATED STEAM OF CHLOROFORM ($CHCl_3$).—*French Units.*

7 Outer latent heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner latent heat ρ	Differences.	10 μ	11 $\frac{p}{u}$	1 Temperature Centigrade t
4.530		62.47		62.47		2.3658	26.41	0
4.734	0.204	63.64	1.17	61.31	1.16	1.4692	41.73	10
4.919	0.185	64.83	1.19	60.16	1.15	0.9559	62.93	20
5.086	0.167	66.03	1.20	59.01	1.15	0.6407	92.10	30
5.241	0.155	67.26	1.23	57.89	1.12	0.4426	130.80	40
5.382	0.141	68.49	1.23	56.75	1.12	0.3137	180.92	50
5.514	0.132	69.74	1.25	55.62	1.13	0.2276	244.85	60
5.638	0.124	70.98	1.24	54.47	1.15	0.1687	322.89	70
5.756	0.118	72.24	1.26	53.33	1.14	0.1275	418.24	80
5.870	0.114	73.50	1.26	52.18	1.15	0.0981	531.73	90
5.981	0.111	74.77	1.27	51.03	1.15	0.0768	664.48	100
6.088	0.107	76.03	1.26	49.86	1.17	0.0610	817.06	110
6.195	0.107	77.31	1.28	48.70	1.16	0.0492	989.62	120
6.299	0.104	78.57	1.26	47.51	1.19	0.0403	1181.60	130
6.404	0.105	79.85	1.28	46.33	1.18	0.0333	1391.96	140
6.506	0.102	81.11	1.26	45.12	1.21	0.0279	1619.32	150
6.609	0.103	82.39	1.28	43.92	1.20	0.0236	1861.51	160

English Units.

7 Outer latent heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner latent heat ρ	Differences.	10 μ	11 $\frac{p}{u}$	1 Temperature Fahrenheit t
8.154		112.446		112.446		37.8975	2.9676	32
8.521	0.367	114.552	2.106	110.358	2.088	23.535	4.6892	50
8.854	0.333	116.694	2.142	108.288	2.070	15.3124	7.0712	68
9.154	0.300	118.854	2.160	106.218	2.070	10.2633	10.349	86
9.433	0.279	121.068	2.214	104.202	2.016	7.0899	14.698	104
9.687	0.254	123.282	2.214	102.15	2.052	5.0251	20.330	122
9.925	0.238	125.532	2.250	100.116	2.034	3.6459	27.394	140
10.148	0.223	127.764	2.232	98.046	2.070	2.7024	36.282	158
10.361	0.213	130.032	2.268	95.994	2.052	2.0424	46.888	176
10.566	0.205	132.3	2.268	93.924	2.070	1.5714	59.749	194
10.766	0.200	134.586	2.286	91.854	2.070	1.2302	74.666	212
10.958	0.192	136.854	2.268	89.748	2.106	0.97715	91.811	230
11.151	0.193	139.158	2.304	87.66	2.088	0.78818	111.20	248
11.338	0.187	141.426	2.268	85.518	2.142	0.64396	132.77	266
11.521	0.183	143.73	2.304	83.394	2.124	0.53348	156.41	284
11.711	0.190	145.98	2.25	81.216	2.178	0.44693	181.96	302
11.896	0.185	148.302	2.322	79.056	2.160	0.37805	209.17	320

TABLE X.

SATURATED STEAM OF CHLORIDE OF CARBON (CCl_4).—*French Units.*

1 Temperature Centigrade t	2 Absolute temperature T	3 Pressure in millimeters p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
0	273	32.95	52.00		0.00		52.00	
10	283	55.97	53.44	1.44	1.99	1.99	51.45	0.55
20	293	90.99	54.86	1.42	3.99	2.00	50.87	0.58
30	303	142.27	56.23	1.37	6.02	2.03	50.21	0.66
40	313	214.81	57.58	1.35	8.06	2.04	49.52	0.69
50	323	314.38	58.88	1.30	10.12	2.06	48.76	0.76
60	333	447.43	60.16	1.28	12.20	2.08	47.96	0.80
70	343	621.15	61.40	1.24	14.30	2.10	47.10	0.85
80	353	843.29	62.60	1.20	16.42	2.12	46.18	0.92
90	363	1122.26	63.77	1.17	18.55	2.13	45.22	0.96
100	373	1467.09	64.90	1.13	20.70	2.15	44.20	1.02
110	383	1887.44	66.01	1.11	22.87	2.17	43.14	1.06
120	393	2393.67	67.07	1.06	25.06	2.19	42.01	1.13
130	403	2996.88	68.10	1.03	17.27	2.21	40.83	1.18
140	413	3709.04	69.10	1.00	29.49	2.22	39.61	1.22
150	423	4543.13	70.07	0.97	31.73	2.24	38.34	1.27
160	433	5513.14	71.00	0.93	34.00	2.27	37.00	1.34

English Units.

1 Temperature Fahrenheit t	2 Absolute temperature T	3 Pressure in inches p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
32	491.4	1.2974	93.6		0.00		93.6	
50	509.4	2.2035	96.192	2.592	3.582	3.582	92.61	0.99
68	527.4	3.5823	98.748	2.556	7.182	3.600	91.566	1.044
86	545.4	5.6012	101.214	2.466	10.836	3.654	90.378	1.188
104	563.4	8.4572	103.644	2.430	14.508	3.672	89.136	1.242
122	581.4	12.377.	105.984	2.340	18.216	3.708	87.768	1.368
140	599.4	17.615	108.288	2.304	21.96	3.744	86.328	1.440
158	617.4	24.455	110.52	2.232	25.74	3.78	84.78	1.548
176	635.4	33.200	112.68	2.16	29.556	3.816	83.124	1.656
194	653.4	44.184	114.786	2.106	33.39	3.834	81.396	1.728
212	671.4	57.760	116.82	2.034	37.26	3.87	79.56	1.836
230	689.4	74.310	118.818	1.998	41.166	3.906	77.652	1.904
248	707.4	94.24	120.726	1.908	45.108	3.942	75.618	2.034
266	725.4	117.98	122.58	1.854	49.086	3.978	73.494	2.124
284	743.4	146.03	124.38	1.800	53.082	3.996	71.298	2.196
302	761.4	178.86	126.126	1.746	57.114	4.032	69.012	2.286
320	779.4	217.05	127.8	1.674	61.2	4.086	66.6	2.412

TABLE X.

SATURATED STEAM OF CHLORIDE OF CARBON (CCl_4).—*French Units.*

7 Outer lat. heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner latent heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Centigrade t
3.442		48.56		48.56		3.2581	14.90	0
3.585	0.143	49.86	1.30	47.87	0.69	1.9969	23.97	10
3.728	0.143	51.13	1.27	47.14	0.73	1.2775	36.90	20
3.864	0.136	52.37	1.24	46.35	0.79	0.8470	54.72	30
3.997	0.133	53.58	1.21	45.52	0.83	0.5802	78.46	40
4.120	0.123	54.76	1.18	44.64	0.88	0.4087	109.22	50
4.236	0.116	55.92	1.16	43.72	0.92	0.2952	148.11	60
4.339	0.103	57.06	1.14	42.76	0.96	0.2178	196.31	70
4.429	0 090	58.17	1.11	41.75	1.01	0.1638	254.95	80
4.505	0.076	59.27	1.10	40.72	1.03	0.1252	325.28	90
4.564	0.059	60.34	1.07	39.64	1.08	0.0970	408.59	100
4.606	0.042	61.40	1.06	38.53	1.11	0.0761	506.30	110
4.629	0.023	62.44	1.04	37.38	1.15	0.0603	619.94	120
4.631	0.002	63.47	1.03	36.20	1.18	0.0482	751.17	130
4.614	−0.017	64.49	1.02	35.00	1.20	0.0388	902.06	140
4.577	−0.037	65.49	1.00	33.76	1.24	0.0314	1074.28	150
4.517	−0.060	66.48	0.99	32.48	1.28	0.0255	1271.39	160

English Units.

7 Outer lat. heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner latent heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Fahrenheit t
6.1956		8.7408		87.408		52.1912	1.6748	32
6.453	0.2574	8.9748	0.2340	86.166	1.242	31.9881	2.6935	50
6.7104	0.2574	9.2034	0.2286	84.852	1.314	20.4641	4.1463	68
6.9552	0.2448	9.4266	0.2232	83.43	1.422	13.568	6.1487	86
7.1946	0.2394	9.6444	0.2178	81.936	1.494	9.2942	8.8163	104
7.416	0.2214	9.8568	0.2124	80.352	1.584	6.5469	12.273	122
7.6248	0.2088	10.0656	0.2088	78.696	1.656	4.7287	16.643	140
7.8102	0.1854	10.2708	0.2052	76 968	1.728	3.489	22.059	158
7.9722	0.1620	10.4706	0.1998	75.15	1.818	2.6239	33.097	176
8.104	0.1318	10.6686	0.1980	73.296	1 854	2.0056	36.551	194
8.2152	0.1112	10.8612	0 1926	71.352	1.944	1.5538	45.912	212
8.2908	0.0756	11.052	0.1908	69.354	1.998	1.2190	56.891	230
8.3322	0.0414	11.2392	0.1872	67.284	2.070	0.96594	69.661	248
8.3358	0.0036	11.4246	0.1854	65.16	2.124	0.77212	84.407	266
8.3052	−0 0306	11.6082	0.1836	63.00	2.16	0.62153	102.47	284
8.2386	−0.0666	11.7882	0.1800	60.768	2.232	0.5030	120.71	302
8.1306	−0 1080	11.9664	0.1782	58.464	2.304	0.40848	142.86	320

TABLE XI.

SATURATED STEAM OF BISULPHIDE OF CARBON (CS_2).—*French Units.*

1 Temperature Centigrade t	2 Absolute temperature T	3 Pressure in millimeters p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
0	273	127.91	90.00		0.00		90.00	
10	283	198.46	91.42	1.42	2.36	2.36	89.06	0.94
20	293	298.03	92.76	1.34	4.74	2.38	88.02	1.04
30	303	434.62	94.01	1.25	7.13	2.39	86.88	1.14
40	313	617.53	95.18	1.17	9.54	2.41	85.64	1.24
50	323	857.07	96.27	1.09	11.96	2.42	84.31	1.33
60	333	1164.51	97.28	1.01	14.41	2.45	82.87	1.44
70	343	1552.09	98.20	0.92	16.86	2.45	81.34	1.53
80	353	2032.53	99.04	0.84	19.34	2.48	79.70	1.64
90	363	2619.08	99.80	0.76	21.83	2.49	77.97	1.73
100	373	3325.15	100.48	0.68	24.34	2.51	76.14	1.83
110	383	4164.06	101.07	0.59	26.86	2.52	74.21	1.93
120	393	5148.79	101.58	0.51	29.40	2.54	72.18	2.03
130	403	6291.60	102.01	0.43	31.96	2.56	70.05	2.13
140	413	7603.96	102.36	0.35	34.53	2.57	67.83	2.22
150	423	9095.94	102.62	0.26	37.12	2.59	65.50	2.33

English Units.

1 Temperature Fahrenheit t	2 Absolute temperature T	3 Pressure in inches p	4 Total heat W	Differences.	5 Heat of liquid q	Differences.	6 Total latent heat r	Differences.
32	491.4	5.0359	162		0.00		162	
50	509.4	7.8134	164.556	2.556	4.248	4.248	160.308	1.692
68	527.4	11.733	166.968	2.412	8.532	4.284	158.436	1.872
86	545.4	17.111	169.218	2.250	12.834	4.302	156.384	2.052
104	563.4	24.312	171.324	2.106	17.172	4.338	154.152	2.232
122	581.4	33.743	173.286	1.962	21.528	4.356	151.758	2.394
140	599.4	45.742	175.104	1.918	25.938	4.410	149.166	2.592
158	617.4	61.106	176.76	1.656	30.348	4.410	146.412	2.754
176	635.4	80.021	178.272	1.512	34.812	4.464	143.46	2.952
194	653.4	103.11	179.64	1.368	39.294	4.482	140.346	3.114
212	671.4	130.91	180.864	1.224	43.812	4.518	137.052	3.294
230	689.4	163.94	181.926	1.062	48.348	4.536	133.578	3.474
248	707.4	202.71	182.844	0.918	52.92	4.572	129.924	3.654
266	725.4	247.7	183.618	0.774	57.528	4.608	126.090	3.834
284	743.4	299.87	184.248	0.630	62.154	4.626	122.094	3.996
302	761.4	358.11	184.816	0.568	66.816	4.662	117.9	4.194

TABLE XI.

SATURATED STEAM OF BISULPHIDE OF CARBON (CS_2).—*French Units.*

7 Outer latent heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner latent heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Centigrade t
7.213		82.79		82.79		1.7585	47.08	0
7.452	0.239	83.97	1.18	81.61	1.18	1.1711	69.69	10
7.676	0.224	85.08	1.11	80.34	1.27	0.8031	100.04	20
7.881	0.205	86.13	1.05	79.00	1.34	0.5654	139.71	30
8.068	0.187	87.11	0.98	77.57	1.43	0.4074	190.40	40
8.239	0.171	88.03	0.92	76.07	1.50	0.2997	253.82	50
8.390	0.151	88.89	0.86	74.48	1.59	0.2247	331.51	60
8.523	0.133	89.68	0.79	72.82	1.66	0.1712	425.21	70
8.638	0.115	90.40	0.72	71.06	1.76	0.1325	536.20	80
8.734	0.096	91.07	0.67	69.24	1.82	0.1040	665.77	90
8.812	0.078	91.67	0.60	67.33	1.91	0.0826	814.79	100
8.869	0.057	92.20	0.53	65.34	1.99	0.0644	993.82	110
8.906	0.037	92.67	0.47	63.27	2.07	0.0539	1173.00	120
8.922	0.016	93.09	0.42	61.13	2.14	0.0442	1382.35	130
8.917	−0.005	93.44	0.35	58.91	2.22	0.0366	1611.15	140
8.886	−0.031	93.73	0.29	56.61	2.30	0.0305	1858.36	150

English Units.

7 Outer lat. heat Apu	Differences.	8 Steam heat J	Differences.	9 Inner latent heat ρ	Differences.	10 μ	11 $\frac{\rho}{u}$	1 Temperature Fahrenheit t
12.9834		149.022		149.022		28.1692	5.2902	32
13.4136	0 4302	151.146	2.124	146.898	2.124	18.7597	7.8309	50
13.8168	0.4032	153.144	1.998	144.612	2.286	12.865	11.241	68
14.1858	0.3690	155.034	1.890	142.2	2.412	9.0571	15.699	86
14.5224	0.3366	156.798	1.764	139.626	2.574	6.5261	21.395	104
14.8302	0.3078	158.454	1.656	136.926	2.700	4.8008	28.521	122
15.102	0.2718	160.002	1.548	134.064	2.862	3.5994	37.251	140
15.3414	0.2394	161.424	1.422	131.076	2.988	2.7424	47.780	158
15.5484	0.2070	162.72	1.296	127.908	3.168	2.1225	60.251	176
15 7212	0.1728	163.926	1.206	124.632	3.276	1.666	74.811	194
15.8616	0.1404	165.006	1.080	121.194	3.438	1.3231	91.556	212
15.9642	0.1026	165.96	0.954	117.612	3.582	1.0637	110.55	230
16.0308	0.0666	166.806	0.846	113.886	3.726	0.86342	131.81	248
16.0596	0.0288	167.562	0.756	110.034	3.852	0.70808	155.33	266
16.0506	−0.0090	168.192	0.630	106.038	3.996	0.5863	181.04	284
15.9948	−0.0558	168.714	0.522	101.898	4.140	0.48857	208.82	302

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